On the Role of Arbitration in Negotiations*

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March 1995

Abstract

Two parties who discount the future negotiate on the partition of a pie of size one. Each party may in turn either make a concession to the other on what has not been conceded yet or call the arbitrator. In case of arbitration, each party endures a fixed cost c, and what has not been conceded yet is shared equally between the two parties. The negotiation stops when either there is nothing left to be conceded or there is arbitration. The game is dominance solvable, and its solution has the following properties: 1) The equilibrium concessions are gradual and cannot exceed $4c$, which results in delays; 2) The strategic behavior of the parties may involve "wars of attrition" because at some point each party is willing not to be the first to concede.

Key words: Arbitration, Negotiation, Delay.

Journal of Economic Literature Classification Numbers: C72, C78, D60.

*We thank David Frankel, Steve Matthews, Ariel Rubinstein, Jean Tirole, Yoram Weiss and seminar participants at Tel Aviv University and Séminaire Fourgeaud (Paris) for helpful comments.

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1 Introduction

We are interested in the following problem: *Two parties negotiate on the partition of a pie in presence of a third party, the arbitrator. What is the outcome of the negotiation? What is the effect of the arbitrator?*

This paper is an attempt to understand this problem where in the tradition of Rubinstein (1982) the bargaining procedure is viewed as a sequential process with perfect information and without imposed deadline.

Bargaining situations where arbitration is used are legion. For example, arbitration is used in such diverse contexts as landlord-tenant disputes, divorce proceedings (see Mnookin and Kornhauser 1979), the dissolution of partnerships, or in the business community where disputes between equal-ranking employees may be arbitrated by the superior (see Bonn 1977). Also in the context of international trade, GATT may arbitrate tariff wars between two or more countries when those do not manage to find a negotiated agreement. Some of the above examples may require that arbitration be formal. However, sometimes it may be less formal and rely on mediators and referees. This is the case, for example, for territorial negotiations between two conflicting countries when if an agreement is not reached through regular negotiations, an international organization like UNO may play the role of the mediator.

The negotiation process with arbitration that we will consider has the following features: 1) At any point of the process, a party can call the arbitrator. 2) Calling the arbitrator is costly to the parties. 3) The arbitrator observes the sequence of actions made by the parties during the process of bargaining and use them in her choice of the arbitrated outcome. 4) If the negotiation is to be arbitrated, the arbitrated outcome is accepted by both parties.

Our objective is to analyze the dynamics of bargaining when both parties are committed to such a negotiation process with arbitration.

Following Stevens (1966), the industrial relations literature acknowledges the role of arbitration in negotiations[1] and comes in support of the features of the negotiation process described above. In particular, Stevens considers crucial that each party can on its own call the arbitrator and thereby "impose a cost of disagreement on the other".[2] Besides, the industrial relations literature identifies various potential behaviors of arbitrators: in order to derive the arbitrated outcome, the arbitrator may or may not take into account the final offers made by the parties.[3] It also identifies different costs of arbitration: those include direct fees, the cost that risk averse parties face when they are uncertain about the arbitrated outcome (see Farber and Katz 1979, and Mnookin and Kornhauser 1979), the cost of implementation since an arbitrated agreement may be more difficult to implement than a negotiated one.

We adopt the following modelling strategy. Two parties $i = 1, 2$ who discount the future negotiate on the partition of a pie of size one. Each party has to make in turn a concession to the other on what has not been conceded yet or may call the arbitrator. The parties enjoy the concessions they receive only when the negotiation stops. The current total concession to a party is the sum of all past (partial) concessions to that party. In case of arbitration, each party endures a fixed cost c, and what has not been conceded yet is shared equally between the two parties.[4] The negotiation stops

[1] Also in the context of pretrial negotiations, Mnookin and Kornhauser (1979) explicitly argue that the legal framework may have an impact on negotiations, even though the parties do not go to the court.

[2] This assumption should be compared with the alternative one that both players should agree on the call of the arbitrator. Our approach is to assume that the parties have agreed on a negotiation process with arbitration, and that this agreement allows any party to trigger arbitration. This agreement may be implicit (disputes between equal-ranking employees), explicit (landlord tenant disputes, dissolution clause in a partnership contract), or compulsory (wage negotiations when strikes are prohibited).

[3] Farber and Bazerman (1986) show evidence that arbitrators are very heterogeneous and partly take into account the final offers made by the parties and partly rely on their own understanding of the case to be arbitrated.

[4] As reported by the industrial relations literature, this corresponds to a particular form of Conventional Arbitration. Conventional Arbitration is to be opposed to Final Offer Arbitration where the final offer that is closest to the arbitrator's view is eventually imposed. Still, what matters

when either there is nothing left to be conceded or there is arbitration. Moreover, the arbitrator has an *impasse solving function*: when the total amount of concessions made by both parties during T^* consecutive periods is less than ε, the arbitrator identifies an *impasse*, and if after another time T the parties have not got out of the impasse, the arbitrator selects at random one of the parties with probability $1/2$, and forces that party to choose between getting out of the impasse (i.e., conceding more than ε) or using arbitration.

This game is dominance solvable, and its solution has the following properties:

1. The equilibrium concessions made by the parties are gradual and cannot exceed $4c$.

2. Either there is immediate arbitration (for c small) or the negotiation lasts for at least $1/4c$ periods (for c large).

3. When there is no arbitration, each party makes in turn a partial concession up to a point where it is credible that he will not concede further unless the other does.

4. The strategic behavior of the parties may involve "wars of attrition" because at some point no party is willing to be the first to concede.[5]

To assess the role played by arbitration in our results, assume first that arbitration is so costly (c very large) that no party is ever willing to use it. Then an agreement is reached in only two periods, and the standard partition of Rubinstein (1982) is achieved. In particular each player is willing to be the first to concede.

is the parties' perception of the arbitrator's behavior. Hence even in the Final Offer Arbitration framework, if the parties are aware of the heterogeneity of arbitrators (see Farber and Bazerman), they will perceive that calling the arbitrator will lead to either one or the other final offer, and in expectation (under symmetry assumptions) this may well correspond to an equal sharing of what has not been conceded. It follows that our assumption is compatible with both forms of arbitration.

[5] Wars of attrition are solved thanks to the arbitrator's action when identifying an impasse.

In contrast, when arbitration costs are less prohibitive, the arbitration procedure induces two types of inefficiencies: it may induce delays, and arbitration is sometimes used in equilibrium. Besides, we observe that the efficiency of the bargaining outcome is not a monotonic function of the arbitration cost c: when c is large, the agreement is reached without the arbitrator, and the more efficient the arbitration procedure (the smaller c), the more the agreement is delayed (which results in a less efficient outcome); when c is small, arbitration is used and a more efficient arbitration results in a more efficient equilibrium outcome.

At first glance one might be surprised that a party may be willing to make a concession to the other party, since this reduces the share he can hope to get. However, since the parties can take advantage of the concessions they receive only after the negotiation stops, a party who concedes makes the other party feel more impatient. Therefore, even though conceding apparently weakens one's bargaining position, it may induce further concessions by the other party and overall be beneficial. That argument underlies the logics of equilibrium concessions when arbitration costs are prohibitive: party 1 concedes up to a point where party 2 is sufficiently impatient to concede the rest of the pie; when the parties use the same discount factor close to one, this results in two consecutive concessions of approximately half the pie. When arbitration costs get lower though, conceding half the pie to party 2 turns out to be a bad idea for party 1 because party 2 may now prefer to call the arbitrator instead of conceding the rest of the pie. Thus the threat of the use of the arbitrator forces equilibrium concessions to be gradual, which in turn results in delays.

We shall see that - in equilibrium - at a point where a party stops conceding, it is credible for that party to concede nothing until the other party concedes (see feature 3 above). A consequence of that observation is that the other party subsequently makes a concession because she has no hope that the original party will make a further concession. That reasoning of the parties is to be related to Schelling (1960)'s view of bargaining (pp 21-22): "*Why does he* [a party] *concede? Because he thinks the other will not.*" [He thinks so because she has made her commitment or *threat* not to make further concessions credible.][6] Entering into the dynamics of the bargaining[7] has allowed us to endogenize the players' ability to make such *threats* not to concede further *credible*, and identify the possibility of delays. Another observation that we shall make is that when arbitration costs are not prohibitive, the set of positions at which it is credible for a party to concede nothing is enlarged. The other party may then have in some cases to concede a large share in order to avoid such positions and induce him to continue the process of alternate concessions. It may therefore deter her from making a concession in the first place. In contrast with the case where arbitration costs are prohibitive, such a situation may arise simultaneously for both players. No player is then willing to make the first step and a *war of attrition* results.[8]

We wish to make a final comment about the commitment idea present in our model. We have already argued that, prior to the negotiation, the parties are committed to the bargaining process with arbitration. This mutual commitment implies that once a party makes a concession, he cannot claim back that share of the pie later on. In other words, the parties are *implicitly* committed to their earlier concessions.[9] Although this paper focuses on some negative (in terms of efficiency) effects of the arbitrator, a positive role of arbitration is to ensure that the concessions made by

[6]As in Ordover and Rubinstein (1984) though, Schelling does not in general view concessions as being partial, and once a party makes a concession, the negotiation terminates. Schelling is well aware that his view fits better the case of indivisible objects with no possibility of (monetary) compensation. Our view is that while perhaps only total concessions are available in contexts such as nuclear wars very much studied by Schelling, in many other contexts such as those described above, partial concessions (with the idea of compromises) are available as well, and our modelling of concessions seems reasonable (see also Fershtman 1989 who considers partial concessions in a continuous-time differential game).

[7]Our dynamic approach of concessions should be contrasted with the static one of Crawford (1982).

[8]The impasse solving function of the arbitrator serves to select the (endogenously defined) less patient party to concede first. When in addition to not willing to make the first step both parties prefer the arbitrated outcome to the one they obtain by conceding, the arbitrator is called, and an inefficiency results.

[9]The idea of commitment also appears in Fershtman and Seidman (1993), see below.

the parties are effectively fulfilled.

The remainder of the paper is organized as follows. In Section 2 we describe the model. In Section 3 the main results are presented. The construction of the solution is presented in section 4. It is followed by a discussion in Section 5. Concluding remarks are gathered in Section 6.

2 The Model

There are three agents: two parties $i = 1, 2$ and an arbitrator A. The parties are bargaining on the partition of *a pie of size one* which will be partitioned after the negotiation process stops. Each party moves in turn every other period. When it is her turn to move, party i can either make a *concession* to party j, where j stands for the party other than i, or she may call the arbitrator. In that case, the arbitrator chooses a partition (to be described below) that depends on past concessions. The negotiation stops when either there is nothing left to be conceded or the arbitrator is called by one of the parties. The parties are assumed to discount the future. Except otherwise mentioned, we will consider the same discount factor δ for both parties.[10] Note that our framework differs from Rubinstein' s (1982) bargaining model in two respects: 1) The parties do not make partition offers, but concessions; 2) The parties have the option to call the arbitrator.

Formally, we denote by $C_i^k \geq 0$, $k \geq 0$, the concession made by party i in period k. We assume that player 1 (resp. 2) can only make concessions in even (resp odd)-numbered periods, so that: $C_1^{2k+1} = C_2^{2k} = 0$ for all $k \geq 0$. At the beginning of period t, the *total concession* to party j is the sum of all the concessions made by party i to party j in earlier periods:

$$X_j^t = \sum_{k<t} C_i^k \tag{1}$$

[10] As is now familiar, $1 - \delta$ can be interpreted as the exogenous probability that the negotiation process breaks down during one period.

and what has not been conceded yet is:

$$X^t = 1 - X_1^t - X_2^t \tag{2}$$

Equation (1) says that concessions are cumulative; therefore a party may only concede a share of what has not been conceded yet. Also, since concessions are positive, the total concession made to a party may only increase over time: concessions cannot be claimed back. In particular, if the share X_j^t has been conceded by party i, then party j will get at least the share X_j^t if an agreement is to be reached.

At period t, if it is party i's turn,

1. Party i may either concede the rest of the pie: $C_i^t = X^t$. In that case, the negotiation stops, party i receives the share X_i^t of the pie, and party j receives the share $1 - X_i^t$.

2. Or she may make a partial concession $C_i^t \in [0, X^t)$. In that case, the negotiation proceeds to the next period $t+1$. The period $t+1$ total concession levels are $X_i^{t+1} = X_i^t$ and $X_j^{t+1} = X_j^t + C_i^t$.

3. Or she may call the arbitrator A. In that case, what has not been conceded yet is shared equally between the parties, and each party endures a fixed cost c. The final partition is then $(X_i^t + X^t/2, X_j^t + X^t/2)$.

When the negotiation process stops at t, the final shares X_1 and X_2 add up to 1 and the associated (period 0) payoffs are given by: 1) $(\delta^t X_i^t, \delta^t X_j^t)$ if the arbitrator is not called – we say then that the agreement is *negotiated*– and 2) $(\delta^t[X_i^t - c], \delta^t[X_j^t - c])$ if the agreement is *arbitrated*. Note that in the case of arbitration, the parties' period t payoffs sum up to $1 - 2c$, which is less than one; $2c$ measures the efficiency loss induced by arbitration.

In addition to the above described action of the arbitrator A, we consider an extra function of the arbitrator, the *impasse solving function*. The aim of this function is to facilitate the process of concessions in situations where the process is too slow.

Roughly, it uses the threat of arbitration to force further concessions (remember that the parties are *a priori* willing to avoid arbitration because it is costly). We will see that in some instances though, this function is not sufficient to avoid potentially great inefficiencies. We will also mention how our results should be modified when the function does not prevail. Specifically the impasse solving function is defined as follows: When the total amount of concessions made by both parties during $T^*(> 1)$ consecutive periods is less than ε, the arbitrator identifies an *impasse*. To get out of the impasse, a concession of at least ε (ε sufficiently small) has to be made by one of the parties. In an impasse phase, time proceeds continuously, and each party may intervene (to get out of the impasse) at any time.[11] If after another time T (T sufficiently large) the parties have not got out of the impasse, the arbitrator selects at random one of the parties with probability 1/2, and forces that party to choose between getting out of the impasse (i.e., conceding more than ε) or using the arbitrator's sharing device with the effect on payoffs as described above. The game then proceeds as before the impasse.

Comments

1. The fact that parties can call the arbitrator makes arbitration an outside option. However, in contrast with the literature on bargaining with outside options (see Shaked and Sutton 1984), the option value depends on the actions previously chosen by the parties during the process. That dependence is a key element in our framework.

2. Our modelling of concessions is to be related to commitment ideas. Fershtman and Seidman (1993) also consider commitment ideas; they assume a party cannot accept a partition offer that is less favorable to some partition she has

[11] When both players decide to concede at the same time, we assume that only one randomly chosen player is required to announce his concession.

previously rejected. An analogy between the two forms of commitment can be developed. However, Fershtman and Seidman insist on the role of imposed deadlines, and are not concerned with the possibility of arbitration.

3. Since the pie is partitioned only after the negotiation stops, we implicitly assume that the parties do not immediately enjoy the concessions they receive during the process: the concessions become effective only when a full agreement is reached. We will discuss in Section 5 the alternative framework where the parties can immediately enjoy the concessions they receive.

4. Making concessions is not the only way by which players could try to achieve an agreement. A more standard way could be to have them make partition offers, or - more generally - proposals for mutual partial concessions. Whatever be the formulation adopted, what matters is the way offers or proposals affect the arbitrator's choice. For example, the arbitrator may consider that a party who offers an half and half partition is ready to concede, say, half the pie to the other party (where concessions have the meaning for the arbitrator defined above). So although in practice concessions might be quite sophisticated, our modelling permits us to address how the presence of an arbitrator affects the dynamics of concessions.

5. Contrary to partition offers, concessions need not be agreed upon by both parties: they can be viewed as unilateral decisions (of the currently moving party). That makes concession processes perhaps easier to implement than their partition-offer counterparts.

6. Some form of impasse solving function seems natural and common practice in arbitration, even though it is also of interest to analyze what happens when the arbitrator has no such function (see below). We have found it more realistic to assume that during an impasse, time proceeds continuously and the parties may intervene at every moment. However, this is inessential, and results similar to

ours would be obtained if the alternating-move framework continued to prevail during impasses. Technically, the impasse solving function of arbitration helps select an outcome in the wars of attrition that may arise for some ranges of concession levels.

7. In general, arbitration is costly either because the parties are risk-averse and uncertain about the arbitrated outcome, or because the arbitrator needs time to implement the arbitrated outcome, or because he gets fees when asked to decide on a partition (see introduction). Our assumption that each party endures a fixed cost c (i.e., that arbitration costs are additive) fits better the last interpretation, whereas the first (risk-aversion) and second (delay) interpretations would result in multiplicative specifications. We have considered the additive specification because it simplifies the exposition without altering the main qualitative features of the model. Other specifications are discussed in Section 5.

In view of the analysis, it is convenient to introduce the following notation:

1. P designates the current *position* in the negotiation: $P = (X, \rho)$, where X is the current amount that has not been conceded yet, and $\rho = X_2 - X_1$ is the difference between the total concession to party 2, X_2, and the total concession to party 1, X_1. The set of bargaining positions P is denoted F.

2. $A_i(P)$ is the set of positions *accessible* by party i (through a concession) from position P. Formally,

$$A_1(P = (X, \rho)) = \{P' = (X', \rho') \text{ } s.t. \text{ } X' + \rho' = X + \rho \text{ } and \text{ } 0 \leq X' \leq X\} \quad (3)$$

$$A_2(P = (X, \rho)) = \{P'' = (X'', \rho'') \text{ } s.t. \text{ } X'' - \rho'' = X - \rho \text{ } and \text{ } 0 \leq X'' \leq X\} \quad (4)$$

3. A *path* is a sequence of distinct bargaining positions $Q = (P_n, \ldots, P_1, P_0)$ that players reach when they alternate in making concessions, until the final position

$P_0 = (0, \rho_0)$ is reached. An $i-$path is a path such that player i makes the last concession; we have:

$$P_0 \in A_i(P_1); \ldots; P_{2k-1} \in A_j(P_{2k}); \ldots; P_{2k} \in A_i(P_{2k+1})$$

At the time the final position is reached, players get the payoffs:

$$(v_1(P_0), v_2(P_0)) = (\frac{1-\rho_0}{2}, \frac{1+\rho_0}{2}) \tag{5}$$

Note that when the path Q is to be followed and Q is composed of $n+1$ distinct positions $-$we say that the *length* of Q is $n-$, the agreement is reached in $n-1$ periods, and player i's payoff is equal to $\delta^{n-1}\ v_i(P_0)$, where P_0 is the final position in Q.

4. Player i can reach a path Q from a position P when:

$$\exists k, s.t., A_i(P) \cap [P_k P_{k-1}] \neq \emptyset \tag{6}$$

where $[P_k P_{k-1}]$ denotes the segment joining P_k to P_{k-1}. Assume that once Q is reached, later equilibrium concessions are described by Q.[12] Then by reaching Q -we say then that player i concedes to Q- player i can secure $\delta^m v_i(P_0)$ where m is the smallest integer satisfying (6).

3 Results

3.1 Gradual Concessions and Delay

We start our analysis by showing that because of the presence of the arbitrator, making large concessions is a dominated strategy. A player who calls the arbitrator gets a share of the remaining pie. When the value of that share exceeds the cost of calling the arbitrator, that is, when $X/2 > c$, the player would rather call the

[12]For example, if the position reached in Q is in the interior of $[P_k P_{k-1}]$ the next concession is from that position to P_{k-1} (by the appropriate player).

arbitrator than concede the rest of the pie to the other. It follows that the largest *final* concession is $2c$. More generally, a player who chooses to make a partial concession has to be aware of two facts: 1) he could have called the arbitrator instead of conceding; 2) his opponent may choose to call the arbitrator afterwards. This observation gives the following result:

Proposition 1 *It is a strictly dominated strategy to concede more than* $4c$.

Proof. We assume players' current concession levels are X_1 and X_2. Without loss of generality, assume that it is player 1's turn to move. Given his belief about future play, he may compute the expected payoffs u_1, u_2 to players 1 and 2, respectively. Assume player 1 concedes the share $C_1 < X$. Whatever be his belief, u_1 and u_2 should add up to δ at most since an agreement is not reached before the next period. Besides, player 1 should expect to get at least what he can get by calling the arbitrator:

$$u_1 \geq \frac{X}{2} + X_1 - c > \delta(\frac{X}{2} + X_1 - c) \tag{7}$$

Similarly, in the next round, player 2 (as expected by 1) should get at least what he can get by calling the arbitrator:

$$u_2 \geq \delta[\frac{X - C_1}{2} + X_2 + C_1 - c] \tag{8}$$

Observing that $X + X_1 + X_2 = 1$ (see 2), $u_1 + u_2 \leq \delta$, and adding (7) and (8) gives $C_1 \leq 4c$. ∎

The bound on the size of any (rational) concession found in Proposition 1 results in a delayed agreement if arbitration is not used. More precisely, it gives a lower bound on the number of rounds necessary to achieve a negotiated agreement (as opposed to an arbitrated one). When this lower bound is large, the inefficiency induced by the delay can be larger than the inefficiency associated with the arbitrated solution. We should then expect that both players prefer to call the arbitrator rather than attempt to achieve a negotiated agreement. The following result, the proof of which is in the Appendix, makes precise that intuition. Let n_0 denote the smallest integer for which the inefficiency induced by a delay of n_0 periods exceeds the inefficiency associated with arbitration: $n_0 \equiv \min\{n, 1-\delta^n > 2c\}$. We have:

Proposition 2 *If $1 \geq X > (n_0+1)4c$, then at position $P = (X, \rho)$ both players prefer to call the arbitrator.*

An immediate corollary of this Proposition is that for a given discount factor δ, there exists a lower bound on c below which at the start of the game (where $X = 1$) both players choose to call the arbitrator rather than negotiate an agreement. In other words, we have shown that 1) If arbitration is not too inefficient, the parties immediately call the arbitrator, and 2) Otherwise, if the negotiation takes place, the parties enter a process of gradual concessions. (We may then conjecture from Proposition 1 that the more efficient arbitration is, i.e., the lower c, the more gradual concessions are, which results in longer delays.)

It should be noted that the results of Propositions 1 and 2 hold whether or not the impasse solving function prevails: it is the concession-dependent sharing device of the arbitrator that is responsible for the gradual dynamics of equilibrium concessions.

3.2 The solution

We will show in Section 4 that the game described in Section 2 is dominance solvable and we now state the main properties of the solution. We have seen above that the presence of the arbitrator imposes that concessions cannot be too large. A key feature of the solution is that except when the cost of arbitration is prohibitive relative to the size of the pie, arbitration constraints are always binding on the equilibrium path. That is, in case a negotiated agreement is reached, parties concede up to a position where the other party is indifferent between conceding further and calling the arbitrator.[13]

In order to state this result formally, we say that a path $Q = (P_n, \ldots, P_1, P_0)$ is *extremal* when the following properties hold:

$$\text{For } k \neq n, \text{ if for some } j,\ P_k \in A_j(P_{k+1}), \text{ then } v_i^a(P_k) = \delta^{k-1} v_i(P_0) \tag{9}$$

$$\text{If for some } j,\ P_{n-1} \in A_j(P_n), \text{ then } v_j^a(P_n) \leq \delta^{n-1} v_j(P_0) \tag{10}$$

Condition (9) ensures that a player always concedes to a position where the other player is indifferent between conceding further and calling the arbitrator. Condition (10) ensures that the player who has to move at the initial position P_n prefers the negotiated agreement to the arbitrated one. Given a final position $P_0 = (0, \rho_0)$, the extremal $i-$paths Q leading to P_0 are constructed backwards using condition (9). Note that the length of (the number of positions in) Q is bounded, since at some point either the boundary of the feasible set is reached, or calling the arbitrator becomes preferable. Hence for any final position P_0 there exists a unique extremal $i-$path with maximal length leading to P_0; it is denoted by $Q_i^e(P_0)$.

The solution is depicted in Figure 1, and its main properties are gathered in the following Proposition:

(Insert Figure 1.)

Proposition 3 *Consider an initial position $P = (X, \rho)$. The solution has the following properties, where the last two properties allow us to characterize the solution for $X > 6c$:*[14]

[13] Throughout the paper, we assume that when a player is indifferent between conceding and calling the arbitrator, he chooses to concede. More generally, we shall assume that when a player is indifferent between several actions, he chooses the most efficient one. These assumptions are standard in models with a continuum of actions.

[14] Compare first what each player i can achieve (at best) by conceding to a path $Q \in \mathcal{Q}$ (see below) with what he can achieve by calling the arbitrator. Eliminate accordingly his dominated strategy. When at P such eliminations result in both players prefering arbitration, the arbitrator is called at P whoever turn it is. When at least one player does not prefer arbitration, that player

1. *Either the arbitrator is called immediately, or the agreement is reached through an alternating sequence of strictly positive concessions.*

2. *If he concedes, a player always reaches a position at which he would find it optimal to concede nothing, that is, wait for the other player to concede.*

3. *If player i concedes, he concedes to an extremal $i-$path $Q_i^e(P_0)$ for some final position $P_0 = (0, \rho_0)$, and players remain on that path thereafter.*

4. *In addition, if $X > 6c$, the set of equilibrium paths that a player may consider reaching is:*

$$\mathcal{Q} = \{Q_i^e(P_0), \mid \rho_0 \mid \leq \nu\} \cup \{Q_2^e(P_0), \rho_0 > 4c - 3\nu\} \cup \{Q_1^e(P_0), \rho_0 \leq -4c + 3\nu\}$$

where $\nu = \frac{1-\delta}{1+\delta}$.

The first property should not be surprising: if player i concedes to a position where player j calls the arbitrator, player i would have benefited from calling the arbitrator instead of conceding.

The second property can easily be understood: player i should not be willing to concede to a position P' from which he would strictly prefer to concede further to P'', since player i could have conceded to P'' in the first place. This property illustrates Schelling's (1960) view according to which a player concedes because it is credible that the other player will not concede *further unless he does.* Two issues remain unanswered at this stage though: 1) What makes a player's threat to concede nothing credible? and 2) How much does a player need to concede in order to make it credible that he will not concede further unless the other does? The third feature of the solution shows that the players' threat to call the arbitrator is what drives the dynamics of concessions: when the agreement is negotiated, a player concedes (in equilibrium) to a position on an extremal path, which implies that except may

compares conceding to $\mathcal{Q}$ with waiting for his opponent to concede to $\mathcal{Q}$, unless the opponent prefers arbitration (in which case he concedes); if he does not prefer waiting, he concedes to $\mathcal{Q}$.

be for the first concession, each player concedes to a position where his opponent is indifferent between continuing the process of alternate concessions and calling the arbitrator. Roughly, that player does not concede less because he cannot expect his opponent to compensate for the smaller concession: if she did, she would end up on a less favorable extremal path,[15] and she would then prefer to call the arbitrator. Besides, he does not concede more because the only effect would be to reach an extremal path that is less favorable to him.

The fourth property implies that when the agreement is negotiated and arbitration costs are not prohibitive, the final shares are either close to 1/2 or the difference between the final shares is bounded away from 0 (larger than $4c - 3\nu$). An implication (see Figure 1) is that there exists a domain such that both players choose (in equilibrium) to call the arbitrator even though each of them could have conceded to an extremal path leading to a Pareto superior agreement (potentially much superior to calling the arbitrator). However, such an extremal path would lead to a final position $P_0 = (0, \rho_0)$ s.t. $\nu < |\rho_0| < 4c - 3\nu$ and the fourth feature says that it is not an equilibrium path. That is because at some point following such a path would turnout to be not credible. Thus neither player would accept to concede in the first place (the underlying strategic arguments will be analyzed in the next Section).

4 The construction of the solution

The construction of the solution works backwards, through iteration of dominance relations. When $X = 0$, the bargaining process ends and players' payoffs are given by their final bargaining positions (see (5)). Given any other initial bargaining

[15] This is a corollary of the following monotony property: the value to party i when it is his turn to move at P' is no less that that at P whenever P' results from a concession of party j from P, e.g., $P' \in A_j(P)$. Observe that the monotony property can be proven independently of the construction (see also the proof of Lemma 1). Roughly, this can be established by observing that from P' whenever party i projects onto the (equilibrium) path followed from P, he can secure at least what he gets from P.

position, we distinguish three options among which a player has to choose: *calling the arbitrator* (**CA**), *conceding* up to a bargaining position where the solution has already been computed (**C**), or *remaining* in a domain where the solution has not been computed yet (**R**). We are able to extend the set of bargaining positions for which we know the solution when we can establish that, at least for one of the two players, say player i, either calling the arbitrator (**CA**) or conceding (**C**) dominates the last option (**R**). The reason is that: 1) when it is player i's turn to move, he either concedes (**C**) or calls the arbitrator (**CA**) according to what is best for him; 2) when it is player j's turn to move, her payoffs are well defined even when she concedes nothing (**R**), because in the next period player i either concedes (**C**) or calls the arbitrator (**CA**).

In order to give some intuition for this technique, we present how the argument works in the end game, when the size of the pie remaining to be shared is small.

4.1 The end game

We know players' payoffs when a negotiated agreement has been reached, that is, on the domain $D = \{P = (X, \rho) \in F,\ X = 0\}$. When the bargaining position is $P = (X, \rho)$ with $X > 0$, player i gets:

1. $X_i + \frac{X}{2} - c$ if he calls the arbitrator (**CA**),
2. X_i if he concedes the rest of the pie (**C**).
3. At best $\delta(X_i + X)$ if he does not concede everything, since the best he can expect is that player j concedes everything in the next period (**R**).

Consequently, the second option (**C**) dominates the first one (**CA**) when the cost of arbitration exceeds the size of the pie remaining to be shared, i.e. $X \leq 2c$. The second option (**C**) also dominates the third one (**R**) when $X_i \geq \delta(X_i + X)$, or equivalently, $(1-\delta)X_i \geq \delta X$. That is, player i prefers to stop the bargaining process immediately by conceding the rest of the pie when the cost of waiting $(1-\delta)X_i$ exceeds the maximum gain from waiting δX.

Those inequalities define a domain where conceding everything is a dominant strategy for player i. We denote by D_i that domain. A simple computation shows that, for player 1:

$$D_1 = \{(X, \rho) \in F,\ X \leq \frac{1-\delta}{1+\delta}(1-\rho) \text{ and } X \leq 2c\}$$

Similarly, for player 2, we have:

$$D_2 = \{(X, \rho) \in F,\ X \leq \frac{1-\delta}{1+\delta}(1+\rho) \text{ and } X \leq 2c\}$$

Observe that when ρ is positive, player 2 is willing to concede a larger share of the pie than player 1 is. The reason is that when player 1 has conceded more to player 2 than player 2 has conceded to player 1, player 2's cost of waiting is larger than player 1's.

The next crucial step is to show that once a player's behavior has been derived on a domain, we can infer the other player's behavior on that same domain. We have shown that in the domain $D_2/(D_2 \cap D_1)$, it is a dominant strategy for player 2 to concede the rest of the pie. Now consider a bargaining position $P = (X, \rho) \in D_2/(D_2 \cap D_1)$ and assume it is player 1's turn to move. Since $P \notin D_1$, player 1 strictly prefers to concede nothing rather than concede everything. Besides, making any partial concession $C_1 < X$ would result in player 2 conceding the rest in the following period - since $(X - C_1, \rho + C_1) \in D_2$. Therefore a positive concession C_1 would only decrease player 1's final share, and it is a strictly dominant strategy for player 1 to concede nothing.

The results of this Subsection are summarized in the following Figure:

(Insert Figure 2)

4.2 The general argument

It remains to show how the solution found in the end game can be extended to a larger domain. An important argument for that extension relies on the iterate application of strict dominance relations to nearby domains. The main outcome of this Subsection is that once it is credible for a player to concede nothing, then the domain on which that property holds can be extended. This extension applies until the other player prefers arbitration to conceding, that is, up to the point where that player can credibly threaten not to concede unless the other does.

To get some intuition for the argument, imagine that on the domain where the share of the pie not conceded yet X is smaller than $\overline{X}$, player 1 concedes it.[16] Then, when X is larger than $\overline{X}$ but smaller than $\overline{X}+\eta$ (η small), player 2 has the option to concede η and force player 1 to concede the rest afterwards. Hence player 1 has little to gain from waiting for player 2's concession, and he should concede right away so as to avoid wasting one period (η should be small relative to $1-\delta$). As a result the domain on which player 1 concedes everything can be extended locally and so on iteratively. Only his threat to call the arbitrator may put an end to this iteration. The rest of this Subsection makes the argument more general with respect to the domain where the solution is already known.

We consider a set D and we assume that for any bargaining position $P \in D$ the payoffs $(v_i(P), \overline{v}_j(P))$ that players i and j obtain when it is player i's turn to move are uniquely defined. Our aim is to extend the solution to the set $\overline{A}(D)$ of positions such that any party exiting from $\overline{A}(D)$ necessarily concedes at least to D:

$$\overline{A}(D) \equiv \{P \in F \backslash D, \forall i, A_i(P) \cap D \neq \emptyset\}$$

First, we consider a bargaining position in $\overline{A}(D)$ and compare the payoff a player may obtain by conceding (at least) to D (i.e., by not remaining in $\overline{A}(D)$) with the largest payoff he may obtain by remaining in $\overline{A}(D)$. To do that, we define an extension of

[16] Of course, such an assumption is not satisfied in our framework (see Figure 2), but it helps present the intuition for our argument.

the functions $(v_i, \overline{v}_j)$ to $\overline{A}(D)$ and denote by $(v_i^D, \overline{v}_j^D)$ the extension: for $P \in \overline{A}(D)$, $v_i^D(P)$ is the maximum payoff player i may obtain by conceding (at least) to D (when it is his turn to move), and $\overline{v}_j^D(P)$ is the largest payoff player j obtains in that case.[17] More precisely, consider the smallest concession that allows player i to reach D and denote by $\pi_i^D(P) \in A_i(P) \cap D$ the resulting bargaining position. Since the sets of bargaining positions not in $\overline{A}(D)$ accessible by player i from P and $\pi_i^D(P)$ respectively are identical, we have:[18]

$$\forall P \in \overline{A}(D),\ v_i^D(P) \equiv v_i(\pi_i^D(P)) \text{ and } \overline{v}_j^D(P) \equiv \overline{v}_j(\pi_i^D(P))$$

A dominance relation that is key to the construction of the solution is the following:

$$v_i^D(P) > \delta \overline{v}_i^D(P) \tag{11}$$

It says that player i prefers to exit from $\overline{A}(D)$ rather than concede nothing and have player j exit from $\overline{A}(D)$ in the next period. Even when $v_i^D(P) > \delta \overline{v}_i^D(P)$ though, we cannot a priori exclude that player i would want to remain in $\overline{A}(D)$ so that eventually, a position more favorable to him is reached. The following Lemma derives conditions under which it is sufficient that (11) holds to infer that remaining in $\overline{A}(D)$ is strictly dominated by conceding to D. The proof of the Lemma is in the spirit of Proposition 1, and is relegated to the Appendix.

Lemma 4 *Assume that, when player i (resp. j) concedes to D from a position in $\overline{A}(D)$, a negotiated agreement is reached in k_i (resp. k_j) periods, and that $k_i + 2 \geq k_j + 1$. Then, if $v_i^D(P) > \delta \overline{v}_i^D(P)$, remaining in $\overline{A}(D)$ is a strictly dominated strategy for player i.*

[17] If D is not compact, $v_i^D(P) = \sup\{\ v_i(P'), P' \in D \cap A_i(P)\}$ and $\overline{v}_j^D(P) = \limsup\{\overline{v}_j(P'), v_i(P') \to v_i^D(P)\}$.

[18] Notice that when a player exits from $\overline{A}(D)$ he does not necessarily concede to a position in D. Yet if from $P \in \overline{A}(D)$, it is optimal for a player to concede to $P' \notin D$, it is also optimal for that player to concede to P' from $\pi_i^D(P)$. As a result, we need only know the payoffs on the frontier of D to compute the extensions on $\overline{A}(D)$.

In general however, for a position P far away from D, there is little hope that $v_i^D(P) > \delta\overline{v}_i^D(P)$ hold since a large concession would be required to reach D. Presumably, each player would then prefer that the other player makes such a concession. The following Lemma derives conditions under which (11) holds locally, near the frontier of D. Roughly, we assume that a small concession by the other player may only translate into a small change in the value of conceding to D - this is a continuity assumption. Hence locally, when the best a player can expect is that the other player concedes to the frontier of D, it is not worth waiting. The Lemma subsequently establishes that by iterating the same local argument, we can (uniquely) extend the solution to $\overline{A}(D)$. For expositional reasons, we assume first that arbitration constraints are binding for neither player on $\overline{A}(D)$, that is, $\forall i, v_i^D(P') \geq v_i^a(P'), \forall P' \in \overline{A}(D)$; and we let $\overline{A}_j^\eta(D) = \{P \in \overline{A}(D),\, d_j(P, D) \leq \eta\}$, where $d_j(P, D) = d(P, \pi_j^D(P))$ and $d(P, P')$ denotes the distance between P and P' defined by: $d(P, P') = \max\{\mid X_1 - X_1' \mid, \mid X_2 - X_2' \mid\}$.

Lemma 5 *In addition to the assumptions of Lemma 1, assume that: 1) On the frontier of D, player i chooses to concede, and conceding nothing for player j strictly dominates making a positive concession;2) v_i^D satisfies a Lipschitz condition:*

$$\exists h, \forall P, P' \in A_j(P), \mid v_i^D(P) - v_i^D(P') \mid \leq\ h\, d(P, P')\ ;$$

3)When player i is to concede (at least) to D from any position $P \in \overline{A}(D)$, then player j's unique optimal choice is to concede nothing from $P \in \overline{A}(D)$, that is,

$$\forall P' \in A_j(P) \cap \overline{A}(D), P' \neq P, \overline{v}_j^D(P') < \overline{v}_j^D(P).$$

Then, from any $P \in \overline{A}(D)$ player i does strictly prefer to concede (at least) to D rather than wait in $\overline{A}(D)$, and consequently player j concedes nothing. Therefore the solution can be extended to $\overline{A}(D)$, and for all positions P in $\overline{A}(D)$:[19]

$$v_i(P) \equiv v_i^D(P);\ \overline{v}_j(P) \equiv \overline{v}_j^D(P)\ \text{and}\ v_j(P) \equiv \delta\overline{v}_j^D(P);\ \overline{v}_i(P) \equiv \delta v_i^D(P).$$

[19]To infer player i's behavior on $\overline{A}(D)$, it is enough that *his* arbitration constraint is not binding.

Proof. Consider a position $P \in \overline{A}_j^{\eta}(D)$. If player j were to concede at least to D, then he would choose the position $P' = \pi_j^D(P) \in A_j(P)$ on the frontier of D: if player j conceded more (i.e. to some position $P'' \in D$, $P'' \neq P'$) then it would also be optimal for player j to concede to P'' from P', contradicting the assumption that conceding nothing at P' is strictly dominant. Hence $\overline{v}_i^D(P) = \delta v_i(P') \equiv \delta v_i^D(P')$. For η sufficiently small, the Lipschitz condition implies that $v_i^D(P')$ is close to $v_i^D(P)$. Thus $\forall P \in \overline{A}_j^{\eta}(D)$:

$$\delta \overline{v}_i^D(P) = \delta^2 v_i^D(P') < v_i^D(P)$$

and Lemma 1 now implies that player i chooses to concede (at least) to D. Given that player i concedes to D, player j's unique optimal choice is to concede nothing.

The above argument can next be applied to $D \cup \overline{A}_j^{\eta}(D)$, and so on iteratively. (The Lipschitz condition guarantees that a finite number of iterations is sufficient to cover $\overline{A}(D)$.) ■

The above Lemmas do not allow us to deal with domains where the arbitration constraint is binding for one player. Such domains will be dealt with thanks to the following Lemma which is proven in the Appendix:

Lemma 6 *In addition to the assumptions of Lemma* 1*, assume that 1)* $v_i^D(\cdot)$ *satisfies the Lipschitz condition of Lemma* 2*; 2)* $\forall P \in \overline{A}(D)$, $v_j^a(P) > v_j^D(P)$ *and* $v_i^a(P) < v_i^D(P)$. *Then the solution can be extended to the domain of positions* P *such that* $v_j^a(P) > \delta \overline{v}_j^D(P)$*; on that domain, player* i *concedes (at least) to* D*, and player* j *calls the arbitrator:*

$$v_i(P) \equiv v_i^D(P), \overline{v}_j(P) \equiv \overline{v}_j^D(P) \text{ and } v_j(P) \equiv v_j^a(P), \overline{v}_i(P) \equiv v_i^a(P).$$

4.3 When Arbitration Costs are Prohibitive

To illustrate the technique of construction, we briefly consider the case where c is larger than $1/2$ so that no party ever considers calling the arbitrator, and therefore arbitration constraints are binding for neither player. From the analysis of the end game, we know the solution on the domain $D_1 \cup D_2$ described in Figure 2. Consider the domain D_2^* described Figure 3 below. When player 2 concedes to D_2^*, he actually prefers to concede the rest of the pie. When player 1 concedes to D_2^*, he prefers to concede to the frontier of D_2^* and player 2 concedes the rest afterwards. Thus a negotiated agreement is reached in 1 or 2 periods depending on whether player 2 or player 1 exits from $\overline{A}(D_2^*)$: the assumption of Lemma 1 holds. Besides, in $\overline{A}(D_2^*)$, if player 2 is to concede the rest of the pie, player 1 strictly prefers to concede nothing. So Lemma 2 applies. As a result, from any bargaining position $P \in \overline{A}(D_2^*)$, player 2 concedes the rest of the pie, and on that domain, player 1 strictly prefers to wait.

A similar argument applies to the domain symmetric to D_2^*. The domain D on which the solution is now defined is drawn in Figure 3.

(Insert Figure 3)

To complete the construction, observe that on the remaining domain players obtain $(v_i^D(P), \overline{v}_j^D(P)) = (v^*, \overline{v}^*)$ when player i concedes to D, where $v^* = \frac{\delta}{1+\delta}$ and $\overline{v}^* = \frac{\delta^2}{1+\delta}$. Since $v^* > \delta\overline{v}^*$, Lemma 1 implies that both players decide to concede at least to D from any bargaining position in $\overline{A}(D)$.

To summarize each player concedes up to the bargaining position where the other player concedes the rest of the pie. The reasons why the latter concedes the rest of the pie are: 1) Having received a large concession, he becomes more impatient than the other. 2) Because the concession was large enough, he does not have anymore the option to put the other in a position where she would become more impatient.

4.4 When arbitration costs are smaller

4.4.1 The effect of arbitration constraints

We now turn to the case where arbitration costs are smaller, i.e., $c < 1/2$. When we apply Lemma 5, we can only infer players' behavior up to the point where they prefer to call the arbitrator rather than concede. Hence, in $\overline{A}(D_2^*)$, player 2 chooses to concede the rest of the pie unless $X > 2c$. Figure 4 describes the domain D on which the solution is now defined.

(Insert Figure 4)

In order to illustrate how to use Lemmas 2 and 3, consider the subset $E \subset D$ defined by $E \equiv \{P = (X, \rho), X \leq 2c, \rho \geq \rho^*\}$, where ρ^* is implicitly defined by $v_2((2c, \rho^*)) = v_2^a((2c, \rho^*)) = v^* = \frac{\delta}{1+\delta}$. (At $(2c, \rho^*)$ player 2 is indifferent between calling the arbitrator and conceding the rest of the pie, which gives her v^*.) On $\overline{A}(E)$, player 2 strictly prefers to call the arbitrator rather than concede (at least) to E, since she would then find it optimal to concede the rest of the pie. (It cannot be optimal for player 2 to concede to a position where player 1 waits for player 2 to concede the rest of the pie.) Lemma 3 applies and it follows that on $E^a = \overline{A}(E) \cap \left\{P, v_2^a(P) > \delta \overline{v}_2^E(P)\right\}$, player 1 concedes (at least) to E while player 2 calls the arbitrator. On the frontier of E^a player 2 is indifferent between calling the arbitrator and conceding nothing, but conceding a positive amount is strictly dominated. Therefore on the domain $\overline{A}(E) \setminus E^a$ Lemma 2 applies, and as long as the arbitration constraint is not binding for player 1 the latter concedes to E and player 2 waits for player 1 to concede to E.

4.4.2 Why we are led to a war of attrition

In contrast with the prohibitive arbitration cost case, when c is not too large ($c < 1/6$) there arise positions such that each player prefers the other to make the first concession and the construction technique cannot be applied. The following Figure shows a position, denoted $P^* = (X^*, \rho^*)$, where such a situation occurs.

(Insert Figure 5)

On the domain where the construction technique can be applied, two forces are driving the construction:

1. Below ρ^*, for positions in K, player 2 prefers to concede (at least) to D rather than wait for player 1 to concede (at least) to D or call the arbitrator.

2. Above ρ^*, for positions in G (or H), not remaining in G amounts to conceding

to D, and player 2 prefers arbitration to conceding to D. Therefore Lemma 6 applies, and in G player 1 concedes to D while player 2 calls the arbitrator. At the frontier of G player 2 is indifferent between conceding nothing and calling the arbitrator (that is the definition of G). Like in the previous Subsection, Lemma 2 can next be applied to domain H, where player 1 concedes to D and player 2 concedes nothing. (The frontier of H is such that player 1 is indifferent between conceding to D and calling the arbitrator.)

In other words, for positions in K, G or H, at least one player finds it optimal to concede (at least) to D, either because that strategy dominates the others (in K), or because, thanks to the arbitrator, the other player can credibly threaten not to concede to D (in G) or to wait for his opponent's concession (in H).

The position $P^* = (X^*, \rho^*)$ is precisely the position such that neither argument applies, that is, the position for which player 2 is indifferent between conceding to D, having the other concede at least to D and calling the arbitrator:[20]

$$v_2^D(P^*) = \delta \overline{v}_2^D(P^*) = v_2^a(P^*) = v^* = \frac{\delta}{1+\delta} \tag{12}$$

In addition, at P^*, the arbitration constraint is not binding for player 1, and it is readily verified that player 1 strictly prefers that player 2 concedes to D rather than concede at least to D himself.[21]

For positions P in B or C, the situation is even more severe, and we now show that the dominance relation $v_i^D(P) > \delta \overline{v}_i^D(P)$ holds for neither player $i = 1, 2$. When a player decides not to remain in $B \cup C$, his optimal concession is to concede at least to D.[22] However, the concession necessary to reach D is so large that each player

[20] ρ^* has been derived in the previous Subsection. X^* is next derived from $v_2^D(P^*) = \delta \overline{v}_2^D(P^*)$, say.

[21] This follows from (12) and the fact that once D is reached, only one concession remains to be made before the agreement is reached, which implies that $v_1^D(P^*) + \overline{v}_2^D(P^*) = \delta = v^* + \overline{v}^*$, which in turn implies $v_1^D(P^*) = \delta - v^*/\delta < \delta \overline{v}^* = \delta \overline{v}_1^D(P^*)$ (because $\delta - \frac{1}{1+\delta} < \delta \frac{\delta^2}{1+\delta} \Leftrightarrow 0 < (1-\delta)(1-\delta^2)$).

[22] For example, if player 1 conceded to H, then player 2 would not concede but wait for player 1 to concede to D. Hence player 1 should concede immediately to D rather than waste two periods.

would prefer to wait and see the other player make such a concession: When player 2 concedes to D, players get:

$$(v_2^D(P), \overline{v}_1^D(P)) = (v^*, \overline{v}^*) = (\frac{\delta}{1+\delta}, \frac{\delta^2}{1+\delta})$$

When player 1 concedes to D, players get $(v_1^D(P), \overline{v}_2^D(P))$, where $v_1^D(P)$ and $\overline{v}_2^D(P)$ satisfy:

$$v_1^D(P) < v_1^D(P^*) < \delta\overline{v}_1^D(P^*) = \delta\overline{v}^* = \delta\overline{v}_1^D(P)$$
$$\text{and } \delta\overline{v}_2^D(P) > \delta\overline{v}_2^D(P^*) = v^* = v_2^D(P)$$

Of course, a player could threaten the other to call the arbitrator. However when $\rho \leq \rho^*$, that is, when P belongs to B, neither player prefers arbitration to conceding, and the threat of arbitration is not credible. Therefore neither player is willing to concede nor has the ability to force the other to concede, and a war of attrition results.

4.4.3 The resolution of the war of attrition

The preceding Subsection has identified the domain $B \cup C$ from which neither player is willing to make a concession: each player would rather see the other make that concession. When there is no impasse solving function for the arbitrator the war of attrition results in a multiplicity issue depending on whether 1 or 2 makes the concession to D. This Subsection shows that the impasse solving function permits us to solve that war of attrition. The intuition is that players know what happens when no concession has been made at the end of the impasse phase. Hence they may compute their expected payoff when such an event occurs. Comparing that value with the value of conceding to D measures how costly waiting is to each player. When those costs of waiting differ, the solution should favor the more patient player, that is, the player whose cost of waiting is lowest.[23] The next result confirms that intuition.

[23] The idea that asymmetries may help select an equilibrium in war of attrition games is also present in Ghemawat and Nalebuff (1985), Fudenberg and Tirole (1986), Whinston (1986), and Jehiel and Moldovanu (1994).

Consider the following war of attrition game (WA): time is continuous and players get $e^{-rt}(v_1, \overline{v}_2)$ when player 1 concedes at time t, and $e^{-rt}(\overline{v}_1, v_2)$ when player 2 does, where $v_i < \overline{v}_i$.[24] That is, each player prefers that the other makes the concession. At time T, if no player has conceded yet, one of the two players is selected at random with probability half and is required to concede. We have the following result:

Lemma 7 *For T sufficiently large, if*

$$\frac{v_i}{v_i + \overline{v}_i} > \frac{v_j}{v_j + \overline{v}_j} \tag{13}$$

or equivalently if $v_i \overline{v}_j > v_j \overline{v}_i$, the game (WA) is dominance solvable and player i chooses to concede at $t = 0$.

Proof. At time T (immediately prior to the player' s selection) player i's (expected) payoff is $\frac{v_i + \overline{v}_i}{2}$. Consider a time $t < T$ such that no concession has occurred yet. Let t_i denote the time satisfying:

$$e^{-r(T-t_i)} \frac{v_i + \overline{v}_i}{2} = v_i \tag{14}$$

When $t > t_i$, and whatever be player j strategy, player i strictly prefers to wait rather than concede. From inequality (13), $t_i > t_j$. Now consider a time $t \in (t_j, t_i)$. For all $t' > t$, it is a dominant strategy for player j to wait until T. Hence, at t, player i strictly prefers to concede. Knowing that, player j does not concede at any time $t_0 < t$ such that $e^{-r(t-t_0)} \overline{v}_j > v_j$. And so on. Hence player i chooses to concede at $t = 0$. ■

This Lemma applies directly to the set of positions $B^{\varepsilon} \equiv \{P \in B, d(P, P^*) \leq \varepsilon\}$. For such positions, arbitration constraints are not binding. Thus when a player is given the choice between conceding more than ε - which amounts to conceding to D- and calling for arbitration, he prefers the former option. Lemma 7 implies that once

[24] When both players concede at the same time, we assume that one randomly chosen player is required to announce his concession. Observe that because time proceeds continuously v_i is compared to $\overline{v}_i$ not $\delta \overline{v}_i$.

players are in the terminal phase, player 2 concedes if $v_1^D(P)\overline{v}_2^D(P) < v_2^D(P)\overline{v}_1^D(P) = v^*\overline{v}^*$, which holds because 1) $v_1^D(P) < \overline{v}^* < \frac{\delta}{2}$ and 2) $v_1^D(P) + \overline{v}_2^D(P) = \overline{v}^* + v^* = \delta$, since the agreement is reached after two concessions when either party 1 or 2 concedes.

Since in the terminal (impasse) phase player 2 concedes to D, in the period preceding the start of the terminal phase, player 2 (if it is her turn to move) prefers to concede while player 1 (if it is his turn to move) prefers to wait. The reasoning can be extended backwards to conclude that on the domain B^ε, player 2 concedes (immediately) while player 1 waits.

Given that we know players' behavior in B^ε and H, we may derive players' behavior on the domains $\overline{C}^\varepsilon = \overline{A}(H \cup B^\varepsilon)$ and $\overline{B}^\varepsilon = \overline{A}(K \cup B^\varepsilon)$ depicted in Figure 5 from the standard construction argument. For player 2, exiting from $\overline{C}^\varepsilon$ amounts to conceding to D. But then, player 2 prefers arbitration. Hence by applications of Lemmas 2 and 3 we conclude that player 1 chooses to concede in $\overline{C}^\varepsilon$ (and player 2 waits). In $\overline{B}^\varepsilon$, Lemma 2 applies, that is, player 2 cannot gain much from waiting, and therefore concedes.

Finally, at $P^* + (\varepsilon, 0)$, we are back to a situation similar to that at P^*, where the optimal concession larger than ε is a concession to D. Again, neither player is willing to make the first step. Iterating the above arguments allows us conclude that in domain C player 1 concedes to D (and player 2 waits), while in domain B player 2 concedes (and player 1 waits).

However, that iterative construction stops at the position $P^{**} = (X^{**}, \rho^*)$ where X^{**} is the largest share such that from P^{**}, player 2 is still indifferent between conceding to D and calling the arbitrator, i.e., $v_2^D(P^{**}) = v_2^a(P^{**})$. (It is the last position for which conceding to D yields $\delta\overline{v}^*$, see Figure 5.) Define $\widehat{A}^\varepsilon(P^{**})$ as the set of positions P such that 1) there exists a (feasible) path from P to P^{**} and 2) $d(P, P^{**}) \leq \varepsilon$. Then at each $P \in \widehat{A}^\varepsilon(P^{**})$ player 2 prefers arbitration to conceding to D; even though players are still potentially in a "war of attrition like" situation, player 2 can credibly threaten to call the arbitrator unless player 1 concedes. Hence player 1 concedes unless he prefers to call the arbitrator too (by Lemma 3). The latter case arises for positions in $\widehat{A}^{\varepsilon}(\widetilde{P})$ where $\widetilde{P} = (\widetilde{X}, \widetilde{\rho})$ satisfies:

$$P^{**} \in A_1(\widetilde{P}) \text{ and } v_1^D(\widetilde{P}) = v_1^a(\widetilde{P})$$

For positions in $\widehat{A}^{\varepsilon}(\widetilde{P})$ each player is willing to call the arbitrator whoever turn it is.

4.4.4 The final steps of the construction ($X > 6c$)

For X larger than $\widetilde{X}$, the typical situation is depicted in Figure 6: for ρ in a neighborhood of $2c$, both players call the arbitrator; when ρ is larger than $2c$, a negotiated agreement gives a share at least equal to $1/2 + 2c$ to player 2; when $\mid \rho \mid$ is smaller than $2c$, the final share a player gets in a negotiated agreement belongs to $(1/2 - \nu, 1/2 + \nu)$. We now show that those properties continue to hold for larger X. Consider the domain A, A', B, B', and C depicted in Figure 6. On the domains A and A', Lemma 2 applies: for example in A, player 2 cannot gain much by waiting, hence he concedes. On the domains B, B' and C, Lemmas 2 and 3 apply:

In B, player 1 prefers to call the arbitrator rather than concede to D; since player 2 prefers to concede to D rather than call the arbitrator, player 2 concedes to D. As a result, player 1 waits.

In C, both players prefer to call the arbitrator rather than concede to D, hence they both call the arbitrator.

The presence of the arbitrator makes it credible that players will never cross the lines $\rho = \pm 2c$, which gives rise to three distinct domains on which bargaining may take place. Because on each of these domains the solution is derived by iterative application of Lemma 2 and 3 only, we get that a player's commitment to concede nothing ceases *if and only if* it becomes credible for the other player that she will call the arbitrator unless he concedes. It follows that on each of these domains, the dynamic of concessions is entirely driven by the arbitration constraints.

5 Discussion

In section 2 we have mentioned various sources of arbitration costs. We now briefly review how different specifications of arbitration costs would affect the conclusion of our model. We then discuss the implication of asymmetries in the relative patience and the relative arbitration costs of the two parties. We next discuss what happens if the parties can immediately take advantage of the concessions they receive. Finally we present an alternative interpretation of the model for which there is no need for an explicit presence of the arbitrator.

5.1 Alternative arbitration costs

Alternative arbitration costs include delays of implementation and risk aversion with respect to the uncertainty of the arbitrated outcome. More explicitly, as far as the former costs are concerned, the arbitrator may need τ periods to implement the arbitrated outcome. The payoffs to the parties are then discounted by δ^τ. As far as the latter costs are concerned, remember that an interpretation of the split-the-difference sharing device of our arbitration mechanism is that the arbitrator chooses one of the final offers, but the parties are uncertain about which one. If the parties are risk-averse, that uncertainty results in a cost. Such a (risk-aversion related) arbitration cost should obviously be higher when the difference between the two offers (here X_1 and $1 - X_2$) is larger, or equivalently when what has not been conceded yet to either party, X, is larger. A simple specification that incorporates these two kinds of costs corresponds to $v_i^a(P) = \delta^\tau(X_i + \alpha\frac{X}{2})$, where $0 \leq \alpha \leq 1$, and $1 - \alpha$ is the magnitude of the arbitration cost due to risk-aversion.

By an argument similar to the one used in Proposition 1, one can show that with the above specification of arbitration costs, at $P = (X, \rho)$ if arbitration is not used and the concession is not total, the equilibrium concession cannot be larger than $\frac{1-\alpha}{1-\alpha/2}X$. One can next show that either there is immediate arbitration or the negotiated agreement is delayed. However, a difference with Section 3 is that the equilibrium concessions decrease over time.[25]

5.2 The effect of asymmetries

In the standard bargaining model without arbitration, it is well known (see Rubinstein 1982) that the outcome is driven by the relative patience of the two parties. The same conclusion holds in our concession model when arbitration costs are prohibitive. However, it is no longer true when arbitration costs are smaller. The argument is straightforward. Assume, for example, party 1 is infinitely more patient than party 2. When arbitration costs are prohibitive, party 2 concedes the whole pie. When arbitration costs are smaller though, party 2 is still the last one to make a concession but, because of the arbitration constraint, that concession cannot exceed $2c$. Going backwards, we conclude that players alternate in making concessions of about $4c$. Hence the relative patience of players is only critical for who makes the last concession, but it is then how $4c$ compares to the size of the pie that determines the number of concessions leading to the negotiated agreement, which in turn determines who starts conceding and eventually the outcome of the negotiation.

The cost of the arbitration procedure is another possible source of asymmetry. Let γ_i denote the cost borne by party i when *he calls* the arbitrator. A careful inspection of the proof of Proposition 1 shows that what matters is the cost borne by the party who *calls* the arbitrator; So except for the last concession which cannot exceed $2\gamma_i$, the outcome of the negotiation process is similar to the one analyzed in Section 3 with arbitration cost c, where $c = \frac{1}{2}(\gamma_1 + \gamma_2)$. An interesting application is the case where only one party, say party 1, has access to the arbitration procedure

[25]In the special case where $\alpha = 1$ (no risk-aversion), a simple inspection shows that when τ is not too large, the outcome is not very far from the one with additive arbitration costs, c, where $c = \frac{1-\delta^\tau}{2}$. (The reason is that on the equilibrium path with additive costs c, $X_i + \frac{X}{2}$ remains almost constantly equal to $\frac{1}{2}$.) In the special case where $\tau = 0$ (no implementation cost), since calling the arbitrator always dominates conceding the rest of the pie, the arbitrator is always called (whatever α), and a substantial inefficiency may result. However, that conclusion is not robust, and fails as soon as $\tau \neq 0$. ($\tau = 0$ can hardly receive empirical support !)

($\gamma_2 = \infty$). Then an agreement is reached in two periods and the outcome with prohibitive arbitration costs is obtained.

Finally, risk aversion might differ across parties. In the spirit of Subsection 5.1, we could specify arbitration costs as follows: $v_i^a(P) = \delta^\tau(X_i + \alpha_i \frac{X}{2})$, where $0 \le \alpha_i \le 1$. Again, a straightforward modification of the proof of Proposition 1 suggests that equilibrium concessions are larger for the more risk averse party, so that the outcome of the negotiation is more favorable to the less risk averse party.

5.3 When concessions are immediately available

So far the parties could benefit from the concessions they receive only after the negotiation stops. We may alternatively assume that party i immediately enjoys a share s_i of the concessions he receives. It should be noted first that the results of Propositions 1 and 2 still hold whatever be s_i. Therefore the idea of gradual concessions and the emergence of delays are robust to such a specification. Second as soon as $s_i < 1$, we obtain results similar to the ones obtained above (for which $s_i = 0$) because the parties are still impatient to terminate the negotiation in order to take advantage of the share $(1 - s_i)$ of the concessions they received. However when $s_i = 1$ for $i = 1, 2$ one may suspect that the parties are no longer impatient to terminate the negotiation. As a matter of fact the structure of the end game is now different from the one analyzed in subsection 4.1: Since no party is willing to make the final concession a war of attrition results. When party 2 has a lower discount rate than party 1 has ($\delta_2 < \delta_1$), the impasse solving function of the arbitrator induces party 2 to concede the rest of the pie whenever $X < 2c$. For earlier concessions the construction technique developed in subsection 4.2 can be applied to show that the parties alternate making concessions of approximately $4c$ when δ_1, δ_2 approach 1. (That corresponds to the upper part of Figure 1.)

5.4 An alternative interpretation

We have seen above how the fear that the other party might call the arbitrator could force the parties to make small concessions rather than bigger ones. That property is responsible for the richness of the dynamics in our negotiation process. We wish to point out that a similar property may hold even for negotiations where there is no explicit presence of an arbitrator. Consider the following framework (which finds its motivation, for example, in GATT negotiations). The situation is the same as that described in section 2 except that 1) There is no arbitrator, and instead of calling the arbitrator the parties have the opportunity to leave the table of negotiations; When a party does so she benefits from the total concessions she received (so does the other party for the concessions he received); What has not been conceded to either party during the negotiation process is left for future negotiations (negotiations may take place every year, say). 2) Entering into a negotiation process costs c to each party. It is readily verified that equilibrium concessions are gradual in that framework too.

6 Conclusion

In this paper we have investigated some (indirect) effects of arbitration on negotiations. We have shown that reasonable arbitration procedures could explain in a perfect information setting 1) delays[26] in negotiated agreements, 2) that (rational) concessions are gradual, and 3) the emergence of wars of attrition. Since we have only emphasized negative effects of arbitration, it may seem that arbitration is essentially a source of inefficiency. We leave to future work the task to explore the

[26]Even if we abstract from the impasse solving function of the arbitrator, all rational outcomes display delays. That is to be contrasted with Fernandez and Glazer (1991) and Haller and Holden (1990) who sustain delayed outcomes in labor negotiations thanks to the existence of multiple equilibria (which do not display delay). See also Ma and Manove (1993) and Jehiel and Moldovanu (1992) (1993) for alternative explanations for delays in negotiations with complete information. For a survey of delays in negotiations with incomplete information, see Kennan and Wilson (1993).

positive effects arbitration.

Finally, our model of negotiation with arbitration should be related to Williamson's view of incomplete contracting (see also Gibbons 1988). Williamson (1975) argues that all contingencies cannot be foreseen at the stage of the initial contract. Therefore the initial contract can only specify for unforeseen contingencies some general principles, for example (as explicitly mentioned by Williamson) that the parties negotiate in presence of an arbitrator. This incomplete contract idea provides a justification for the parties' commitment to the negotiation process with arbitration that we have assumed throughout the paper.

Appendix

Proof of Proposition 2 We build on the proof of Proposition 1, and likewise assume that player 1 starts by conceding $C_1 \geq 0$ rather than calling the arbitrator. (We already know that player 1 does not concede the rest of the pie since $X > 2c$.) Let X_i^* denote player i's final share of the pie when an agreement is reached, let n denote the number of rounds necessary to reach it, and let p_A denote the probability that the agreement is reached through arbitration. Player i's payoff u_i as introduced in the proof of Proposition 1 is defined by:[27]

$$u_i = E[\delta^n X_i^* \mid \text{ no arbitration}](1 - p_A) + E[\delta^n(X_i^* - c) \mid \text{arbitration}]p_A \tag{15}$$

Since $X > 4(n_0 + 1)c$, Proposition 1 implies that at least $n_0 + 1$ rounds are necessary to reach a negotiated agreement. When arbitration occurs, it occurs with a one-period delay at least since player 1 starts by conceding initially. Since $X_1^* + X_2^* = 1$ and $\delta^{n_0} \leq 1 - 2c$, (15) implies:

$$u_1 + u_2 \leq \delta^{n_0+1}(1 - p_A) + \delta[1 - 2c]p_A \leq \delta[1 - 2c] \tag{16}$$

[27]The expectation is computed given player 1's belief about n and X_i^*.

Besides, adding (7) and (8) (which is *a fortiori* true when $C_1 = 0$) gives

$$u_1 + u_2 \geq [\frac{X}{2} + X_1 - c] + \delta[\frac{X}{2} + X_2 - c] > \delta[1 - 2c]$$

which contradicts (16). Consequently, player 1 calls the arbitrator. ■

Proof of Lemma 1 Without loss of generality, we assume $i = 1$. We denote by (u_1, u_2) the expected payoffs obtained by player 1 and 2, according to player 1's belief and we let π_i denote the probability that player i eventually concedes to D. Assume that player 1 does not concede immediately to D and concedes to a position $P' \in \overline{A}(D) \cap A_1(P)$ (possibly $P' = P$). Then we have:

$$u_1 + u_2 \leq \pi_1 \delta^{k_1+2} + (1 - \pi_1)\delta^{k_2+1} \tag{17}$$

Player 1 knows that when he concedes to P', player 2 can at least secure $v_2^D(P')$ by conceding to D. Hence, we have:

$$u_2 \geq \delta v_2^D(P') \tag{18}$$

Consider the path Q reached by player 2 when he concedes optimally to D from P. From any position $P' \in \overline{A}(D) \cap A_1(P)$, either player 2 can reach Q, or player j can concede everything without reaching Q. In both cases, we have:[28]

$$v_2^D(P') \geq v_2^D(P) \tag{19}$$

Besides, by definition of k_2, we have:

$$v_2^D(P) + \overline{v}_1^D(P) = \delta^{k_2} \tag{20}$$

Combining (17-20) gives:

$$u_1 \leq \pi_1[\delta^{k_1+2} - \delta^{k_2+1}] + \delta\overline{v}_1^D(P) \leq \delta\overline{v}_1^D(P)$$

Hence it is a dominant strategy for player 1 to concede at least to D immediately. ■

[28] In the latter case, player j gets a larger share than at Q in a shorter amount of time.

Proof of Lemma 3 The start of the argument is the same as in Lemma 2. Consider a position $P \in \overline{A}_j^\eta(D)$. Given that player j will not exit from $\overline{A}_j^\eta(D)$ (since by assumption 3 he prefers arbitration), the best player i can expect by not conceding to D is $\delta^2 v_i^D(P')$ for some $P' \in \overline{A}_j^\eta(D)$. The continuity assumption guarantees that this is less than $v_i^D(P)$ for η sufficiently small. Therefore at $P \in \overline{A}_j^\eta(D)$, player i concedes to D. Given that player i concedes to D, player 2 prefers to call the arbitrator rather than remain in $\overline{A}_j^\eta(D)$:

$$v_j^a(P) > \delta \overline{v}_j^D(P) \geq \delta \overline{v}_j^D(P') \quad \forall P' \in A_j(P) \cap \overline{A}_j^\eta(D)$$

where the last inequality follows from the monotony property 19 and condition 20 which results from the assumption of Lemma 1. The solution is now extended to $\overline{A}_j^\eta(D)$. The argument can next be applied to $D \cup \overline{A}_j^\eta(D)$ and so on iteratively as long as player j prefers to wait rather than call the arbitrator. ■

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