

On the Value of Competition in Procurement Auctions*

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Abstract

This short paper shows that in an affiliated value setting more bidders at the auction stage need not induce a higher expected welfare in either ascending price or second price auctions. We highlight the roles of asymmetries between bidders and of the multidimensional character of the private information in deriving this result.

Key words: auctions, affiliated value, asymmetries, competition, efficiency.

1 Introduction

This short paper is concerned with the welfare effect¹ of competition in one-object auction contexts in which bidders' valuations are interdependent and bidders' private information may be multi-dimensional.

Specifically, we focus on two auction formats: the sealed-bid second price auction and the open ascending price auction. We consider contexts in which bidders' valuations have a *private* and a *common* element, and in which bidders know their

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¹Bulow-Klemperer (1996) study the revenue effect of competition in symmetric private value settings.

own private element and are *differentially informed* about the common element.² For each auction format, we are interested in the welfare effect of adding one more bidder at the auction stage when the identity of participants, bidders' valuation and information structures are all assumed to be common knowledge among bidders.

Our main observation is that in an affiliated value setting more bidders at the auction stage need not induce a higher expected welfare in either ascending price or second price auctions. We highlight the roles of asymmetries between bidders and of the multidimensional character of the private information in deriving this result.

Our first result concerns the *symmetric* case. For a reasonably wide class of situations in which all bidders are ex ante symmetric, we show that in either second price or ascending price auctions, the participation of an extra bidder is always good for welfare.³

We next explore the welfare effect of having one more bidder in *asymmetric* cases.⁴ Our main insight is that both in second price and in ascending price auctions there are situations in which expected welfare is lower when one more bidder participates in the auction.⁵ In such contexts, more competition at the auction stage deteriorates welfare.⁶

The situations with this property analyzed in this paper all share the feature that

²Our interest in multi-dimensional settings is of practical importance, since there is no reason to believe that the private information held on the private element and on the common element are related in a deterministic fashion (see Maskin 1992, and also Pesendorfer-Swinkels 1998, Jackson 1999 and Jehiel-Moldovanu 1999).

³This holds true despite the fact that in a multidimensional setup the good need not be allocated to the ex post efficient bidder (see Maskin 1992 for an example and Jehiel and Moldovanu 1999 for a general treatment).

⁴Our interest in asymmetric setups is motivated by applications. In procurement contracts, incumbent firms are presumably better informed about the common value element. Other asymmetries may concern technological aspects: Bidding firms may vary with respect to their choice of technology; For those firms using the same technology (a subset of all bidders), the cost structures are likely to share some common value element.

⁵This result would also hold true in first price auctions. However, it is less surprising, since even in the private value paradigm with one-dimensional private signals, one could generate such examples (this is a standard argument against the use of first price auctions). What our paper shows is that even second price or ascending price auctions may have this feature in a broader setup.

⁶The role of asymmetries is also highlighted in a revenue comparison context by Bulow-Klemperer (1998).

the additional bidder has some extra information that is relevant to other bidders and that the other bidders do not have. When the additional bidder has no such information (that is relevant to other bidders), it can be shown (for a wide class of situations) that both in ascending price and in second price auctions, expected welfare is higher when this extra bidder participates in the auction.

The situations we identify illustrate *two different sources of welfare loss* according to whether second price or ascending price auctions are considered.

In the second price auction situations we analyze, the participation of the extra bidder deteriorates welfare because the extra bidder gets the object too often.

The intuition is as follows. The additional bidder (an incumbent) is informed about a common element that the other bidders (who are least two) do not know (they are entrants, say). When the incumbent bidder is not present, the entrant bidders all take the common value element to be equal to its expected value, and the efficient entrant bidder gets the object. When the incumbent bidder participates, he gets the object - even if the entrant bidders have a higher valuation - when the realization of the common element is sufficiently high:⁷ In order to avoid the incumbent bidder getting the object for high realizations of the common element, the entrant bidders would have to bid very high so that (because they are least two) they would end up paying a high price even for low realizations of the common value element, thus resulting in expected losses for entrant bidders. This cannot hold in equilibrium. Thus, when the value of the additional informed bidder is below that of the other uninformed bidders with a sufficiently large probability, we may conclude that the participation of the additional bidder deteriorates expected welfare.

In ascending price auctions, we identify another source of welfare loss. Here the mere presence of the additional bidder modifies the course of competition between the remaining bidders even though (in the basic example) this additional bidder never acquires the object. The point is that when the remaining bidders are differentially informed about the common element, the price at which the extra bidder drops out does not convey the same information to the remaining bidders, and the induced competition between the remaining bidders is then biased in a way that can be detrimental to welfare, as we show.

⁷This holds true despite the fact that entrant bidders adjust their bidding strategy to the presence of the incumbent bidder.

In Section 2 we describe the model. In Section 3 we analyze the value of competition in second price and ascending price auctions. We first derive a positive result for the symmetric case. We next explore the asymmetric case. Some discussion appears in Section 4.

2 The model

Payoff structure: There is one object for sale. We consider n potential bidders $i \in N = \{1, \dots, n\}$. When a bidder does not get the object, he gets a payoff normalized to zero.

The value of the object to bidder i is assumed to depend on a private element θ_i and on a vector of K characteristics $w = (w^1, \dots, w^K)$ of the object for sale.⁸ This value is denoted by $v_i(\theta_i, w)$.

Information structure: Each bidder i knows his private element θ_i , and has some private (partial) information on w . The set of variables $\theta_i, w^k, i \in N, k \in K$ are distributed according to a joint density denoted by $f(\cdot)$. This density is assumed to be common knowledge among all bidders.

We describe bidder i 's information about the common characteristics by defining for each bidder i the set $H_i \subset K$ of characteristics of which bidder i knows the realization. In case $H_i = \emptyset$, bidder will be said to be *uninformed*. In case $H_i = K$, bidder i will be said to be *fully informed*. In all other cases, bidder i will be said to be *partially informed*.

This informational differentiation between bidders seems particularly relevant for the distinction between incumbents and potential entrants in a procurement auction: Incumbent firms are likely to know more of the characteristics of the object for sale than potential entrants do.

Auction formats: The good is to be sold through an auction procedure. We will consider two auction formats: the second price sealed-bid auction and the ascending price auction, and we will mostly focus on equilibria that do not use dominated

⁸With some abuse of notation, K will sometimes also denote the set of all characteristics k .

strategies.⁹

The second price auction is defined as follows. Each bidder i simultaneously sends a bid b_i to the seller. The bidder with maximal bid, i.e. $i_0 = \arg \max_i b_i$ gets the good and pays the second highest bid, i.e. $\max_{i \neq i_0} b_i$ to the seller.¹⁰

The ascending price auction is defined as follows.¹¹ The price starts at a low level, say 0, at which each bidder is present. The price gradually increases. Each bidder may decide to quit at every moment. When a bidder quits, this is commonly observed by every bidder. The auction stops when there is only one bidder left. The object is allocated to that bidder at the current price. A strategy for each bidder specifies a price at which it quits as a function of current public information and private information.¹²

Policy issues: We are interested in whether or not promoting the maximum participation at the auction stage is good for expected welfare.

If the object is allocated to bidder i , (ex post) the social value is given by $v_i(\theta_i, w)$, which thus measures ex post welfare.¹³ For each auction format, and for any given strategy profile σ of the bidders, ex ante welfare will thus be measured by (remember that i_0 denotes the winner of the auction):

$$E[v_{i_0}(\theta_{i_0}, w) \mid \sigma] = \sum_{i \in N} \Pr\{i_0 = i \mid \sigma\} E[v_i(\theta_i, w) \mid \sigma, i_0 = i].$$

⁹Equilibria in dominated strategies always exist in this type of auctions (even in the simple private value paradigm). They are in general considered as implausible because they are poorly robust to mistakes in the bidding behavior of other bidders (see also comment 4 page 15).

¹⁰If there are several bidders with maximal bids, one of them is selected at random with equal probability to get the good, and pays that bid to the seller.

¹¹We present here the continuous time/price version of the ascending price auction. This raises some technical difficulties regarding the definition of equilibria in undominated strategies. The equilibria we will refer to are the limits as $\varepsilon > 0$ tends to 0 of the equilibria in undominated strategies of the corresponding game in which time is discrete and after each round the price increases by the increment ε .

¹²In case all the remaining bidders quit at the same date, one of them is selected at random with equal probability to get the object. He then pays the current price.

¹³Efficiency refers here to productive efficiency (since we abstract from market structure considerations).

3 The Symmetric Case

In this Section we assume that all bidders share the same valuation function v_i , which we will denote by v . When all bidders are informed about the same characteristics ($H_i = H_j \forall i, j$), both the second price auction and the ascending price auction clearly select the ex post efficient bidder (the bidder with largest θ_i). Thus, the participation of an additional bidder may only increase welfare.

We will now analyze the more interesting case in which bidders are not informed about the same characteristics.¹⁴ We first define a relatively broad class of symmetric settings of this sort.

Definition 1 *Assume $K \geq N$. A setting is said to be symmetric if: 1) All bidders have the same valuation function v ; 2) Each bidder i knows θ_i and w^i , that is, $H_i = \{i\}$ for all i ; 3) The variables (θ_i, w^i) are i.i.d. among bidders and independent from w^k , $k > N$: they are distributed according to $g(\cdot)$ on $[\underline{\theta}, \bar{\theta}] \times [\underline{w}, \bar{w}]$; 3) The valuation function v is separable in each bidder i 's information, and symmetric with respect to the other common value characteristics $k \neq i$. That is, there are functions $u(\theta_i, w^i)$ and $\phi(w^k)$ such that:*

$$v(\theta_i, w) = u(\theta_i, w^i) + \sum_{k \in N - \{i\}} \phi(w^k).$$

Note that in a symmetric setting as described above, we may define

$$h(\theta_i, w^i) = u(\theta_i, w^i) - \phi(w^i),$$

and bidder i is the ex post efficient bidder whenever $h(\theta_i, w^i)$ is largest among bidders. The following Proposition establishes that both in the second price and the ascending price auctions (and by restricting attention to symmetric equilibria), expected welfare increases with the number of bidders.

Proposition 1 *Consider the symmetric setting. Suppose that 1) $\gamma_N(z) = z + (N - 1)E[\phi(w^k) \mid u(\theta_k, w^k) \leq z] + E[\phi(w^k) \mid u(\theta_k, w^k) = z]$ is (strictly) increasing in z , and 2) $\eta(z) = E[h(\theta_i, w^i) \mid u(\theta_i, w^i) = z]$ is a (strictly) increasing function of z .*

¹⁴Then it can be shown that all mechanisms induce ex post inefficiencies (see Maskin 1992 for an example and Jehiel and Moldovanu 1999 for a general investigation of this issue).

Then for any $m \leq N$, the sealed bid second price auction with m bidders and the ascending price auction with m bidders each have a unique symmetric equilibrium. Furthermore, the aggregate expected welfare in this equilibrium increases with the number m of bidders.

The intuition for Proposition 1 (which is proven in Appendix) is as follows. Consider the second price sealed bid auction. The equilibrium bid of bidder i should aggregate the multidimensional private information (θ_i, w^i) held by bidder i . The separability of $v(\cdot, \cdot)$ ensures that each bidder i 's equilibrium behavior should be a function of $u(\theta_i, w^i)$. Condition 1 of Proposition 1 then ensures that a symmetric equilibrium allocates the good to $\arg \max_i u(\theta_i, w^i)$. Whenever condition 2 holds, the aggregate value $u(\theta_i, w^i)$ is affiliated with the welfare criterion as measured by $h(\theta_i, w^i)$. More competition at the auction stage is then good for welfare in expectation.

4 The Asymmetric Case

Symmetry plays an important role in the argument given above. We now investigate asymmetric settings, and we analyze whether the conclusion that more bidders at the auction stage is welfare-enhancing holds true.

Analyzing asymmetric settings in auctions is in general very hard because in equilibrium bidding strategies are the result of a sophisticated inference process. Besides, the addition of one more bidder may completely change this inference process making the comparison very difficult. Our relatively simple information structure will nevertheless allow us to carry out these comparisons for two kinds of informational asymmetries.

Asymmetric setting 1 : $K \geq 1; n = 3, H_1 = H_2 = \emptyset; H_3 = K$.

Asymmetric setting 2 : $K \geq 2; n = 3, H_1 = \{1\}, H_2 = \emptyset, H_3 = K$.

In both settings, we will be interested in the effect of allowing bidder 3 to participate. For simplicity, we will assume throughout this section that all the variable θ_i, w^k are independent from one another.

The two settings have in common that the extra bidder is fully informed of the common value element; this extra bidder may thus be thought of as an incumbent in both settings. However, the private information held by bidders other than the extra bidder is not the same in settings 1 and 2. In setting 1, bidder 1 and 2 are totally uninformed of the common element (they may thus be thought of as entrants); in setting 2, they are differentially informed of the common element (bidder 2 is uninformed, while bidder 1 is partially informed).

Our results are as follows. In asymmetric setting 1, we will show through an example that the participation of the extra bidder 3 may deteriorate (ex ante) welfare if the object is allocated with a *sealed bid second price auction*. In contrast, if the object is allocated with an ascending price auction, the participation of the informed bidder may only improve welfare.

In asymmetric setting 2, we will show that the comparison between the ascending price auction and the sealed bid second price auction may be reversed: we exhibit an example such that when the informed bidder participates, expected welfare deteriorates when the auction format is the *ascending price auction*, and such that expected welfare does *not* deteriorate when the auction format is the *sealed bid second price auction*.

4.1 Negative value of competition in second price auctions

Basic Example 1

1. We consider asymmetric setting 1 with three bidders $i = 1, 2, 3$. Bidder i 's valuation is given by :

$$v_i(\theta_i, w) = \theta_i + \sum_{k \in K} w^k.$$

2. Bidders 1 and 2 are uninformed of w ($H_1 = H_2 = \emptyset$) whereas bidder $n = 3$ is fully informed of w ($H_3 = K$).
3. The variables θ_i , $i = 1, 2, 3$ and w are assumed to be drawn from independent distributions denoted by $f_i(\cdot)$, $i = 1, 2, 3$ and $g(\cdot)$, with supports $[\underline{\theta}_i, \bar{\theta}_i]$, $i = 1, 2$ and $[\underline{w}, \bar{w}]$, respectively. We assume that the informed bidder 3 is always less efficient than the two uninformed bidders $i = 1, 2$. That is,

$$\Pr\{\max_{i < 3} \theta_i > \theta_3\} = 1. \tag{1}$$

Result 1: *In basic example 1, and if the auction format is the second price auction, the participation of bidder 3 deteriorates expected welfare.*

When bidder 3 is absent:

Suppose first that bidder 3 is absent. The equilibrium bid of the uninformed bidder $i = 1, 2$ with private element θ_i is

$$\theta_i + E(w). \quad (2)$$

Thus, the second price auction allocates the good efficiently, to the bidder with highest θ_i .

When all three bidders are present:

Characterizing the equilibrium of the second price auction when all three bidders are present is not straightforward (see Compte-Jehiel 1999 for an example). But, we now show by contradiction that in equilibrium (whatever its exact form) the participation of bidder 3 must deteriorate welfare.

Suppose (by contradiction) that the participation of bidder 3 does not deteriorate welfare. Then bidder 3 must get the object with probability 0 (because bidder 3 is never the efficient bidder). So assume that (in equilibrium) bidder 3 never gets the object. Since bidders 1 and 2 choose their bids independently, one of the two uninformed bidders, say bidder 1, must choose to bid $b_1 \geq \bar{\theta}_3 + \bar{w}$ with probability 1 (otherwise, $\max\{b_1, b_2\}$ would be smaller than or equal to some $b < \bar{\theta}_3 + \bar{w}$ with positive probability, and bidder 3 would be able to secure positive expected profits, contradicting the premise that he does not get the object in equilibrium).

Now observe that whenever bidder 2 wins, he must pay a price at least equal to b_1 , hence at least equal to $\bar{\theta}_3 + \bar{w}$. However, bidder 2's expected value from winning the object is $\theta_2 + Ew$ (because bidder 3 is supposed not to get the object and because bidder 1's bid does not convey any information on w). When

$$\theta_2 + Ew - (\bar{\theta}_3 + \bar{w}) < 0, \quad (3)$$

bidder 2 with private element θ_2 will not acquire the object, since otherwise he would make losses. Thus bidder 1 should acquire the object. However, bidder 2 (rather than bidder 1) may be the efficient bidder, since condition (3) does not imply that $\theta_2 < \theta_1$. To summarize, in any event where

$$\theta_1 < \theta_2 \text{ and } \theta_2 + Ew < \bar{\theta}_3 + \bar{w},$$

the object would be allocated to bidder 1 even though he is not the ex post efficient bidder. Clearly, since $Ew < \bar{w}$, this event may have positive probability even when condition (1) holds. Expected welfare is then negatively affected. ■

Comments:

1) In the above basic example, bidder 3 was assumed to never be the ex post efficient bidder. This simplified the exposition of the argument because then the outcome when bidder 3 is absent is the ex post efficient outcome. However, it is easy to check (by a continuity argument) that the participation of bidder 3 may still deteriorate welfare even if sometimes bidder 3 is the efficient bidder.¹⁵

2) The participation of *two* (or more) uninformed bidders who compete for the object is key to our result. In Compte-Jehiel (1999), we show that if there is only *one* uninformed bidder, the addition of a bidder who is perfectly informed of the common element (like bidder 3 in the basic example) always improves welfare. This is easily seen in the case where the informed bidder has a valuation lower than that of the uninformed bidder with probability 1 (as in the basic example), since it is then an equilibrium for the uninformed bidder to bid very high and for the informed to bid his own value.¹⁶ In the general case where the informed bidder 2 may be more efficient than the uninformed bidder 1, the intuition is as follows. The welfare change associated with bidder 1 (with a given realization θ_1) varying his bid from b_1 to b'_1 coincides with the change in bidder 1's expected payoff associated with that bid variation.¹⁷ Since a very high bid of bidder 1 induces the same allocation as when bidder 2 does not participate, the welfare change associated with participation of bidder 2 coincides with the change in bidder 1's expected payoff associated with bidder 1 varying his bid from a very high bid to his equilibrium bid. This latter

¹⁵Indeed, one may check that our argument may be adapted to the case where bidder 3 is the efficient bidder with a very small probability, say ε . When bidder 3 is absent, expected welfare is $O(\varepsilon)$ away from maximum welfare. When bidder 3 participates, consider $b^\nu = \bar{\theta}_3 + \bar{w} - \nu$ and choose $\nu \gg \varepsilon^{1/5}$. One of the uninformed bidders, say bidder 1, must bid above b^ν with probability $1 - \nu$ at least [otherwise bidder 3 could secure expected profits comparable to $\nu^5 \gg \varepsilon$, hence expected welfare would be more than $O(\varepsilon)$ away from maximum, thus lower than when bidder 3 does not participate]. So when bidder 2 wins, he must pay at least b^ν with probability $1 - \nu$. However his expected gain from winning cannot be lower than $\theta_2 + Ew$ by more than $O(\nu)$.

¹⁶The uninformed bidder is not hurt any longer when he bids very high (he will end up paying the bid of the informed bidder, which is below the value of the good to him).

¹⁷This is because the informed bidder 2 bids his own valuation.

change must be non negative, as the equilibrium bid of bidder 1 is optimal for bidder 1, by definition. Thus participation of the informed bidder may only improve expected welfare.

3) Result 1 does not carry over to the case of ascending price auctions. In Compte-Jehiel (1999), we actually prove that in the context of setting 1, and to the extent that the valuation of the uninformed bidders is affiliated with that of the informed bidder, the participation of the informed bidder always (weakly) improves expected welfare. This is easily seen in the basic example. In an ascending price auction, bidders 1 and 2 can wait for bidder 3 to drop out. The price at which bidder 3 drops out gives a finer assessment about the value of the common element to bidders 1 and 2, and the ensuing subgame yields the efficient outcome as in the case where bidder 3 is absent.

4.2 Negative value of competition in ascending price auctions

Basic Example 2

1. We consider asymmetric setting 2 with three bidders $i = 1, 2, 3$ and two characteristics $K = \{1, 2\}$. For each bidder i ,

$$v_i(\theta_i, w) = \begin{cases} \theta_i + w^1 + w^2 & \text{if } i \in \{1, 3\} \\ \theta_i + w^2 & \text{if } i = 2 \end{cases}$$

1. Bidder 3 is fully informed of $w = \{w^1, w^2\}$; Bidder 2 is totally uninformed. Bidder 1 is partially informed of w ; he only knows w^1 .
2. All variables θ_i , $i = 1, 2$, and w^1 , w^2 are assumed to be drawn from independent distributions denoted by $f_i(\cdot)$, $i = 1, 2$ and $g_k(\cdot)$, $k = 1, 2$, with supports $[\underline{\theta}_i, \bar{\theta}_i]$, $i = 1, 2$ and $[\underline{w}^k, \bar{w}^k]$, $k = 1, 2$, respectively. We assume that $\underline{\theta}_3 = \bar{\theta}_3 = 0$, $\underline{\theta}_1 > 0$, and $\bar{w}^1 + \bar{w}^2 < \underline{\theta}_2$.

A simple interpretation of this setup is as follows: w^2 represents a purely common value characteristic that applies to all bidders while w^1 represents a common characteristic that applies to bidders 1 and 3 only, for example because bidder 2 is known to use a technology different from that of bidders 1 and 3.

Concerning the assumptions on the distributions of the parameters note that they imply that the informed bidder 3 is never the ex post efficient bidder. We will analyze the equilibria in undominated strategies of the ascending price auction and obtain the following result:

Result 2: *In basic example 2, and if the auction format is the ascending price auction, the participation of bidder 3 deteriorates expected welfare.*

When bidder 3 is absent:

Note that the private information held by $i = 1, 2$ is irrelevant for the determination of the valuation of bidder $j \neq i$, $j \in \{1, 2\}$. The auction can thus be analyzed as a private value ascending price auction: the efficient bidder among $i = 1, 2$ gets the object.¹⁸ Since $\underline{\theta}_1 > 0$ and $\bar{\theta}_3 = 0$, the informed bidder 3 is always welfare inferior to bidder 1, and therefore the ascending price auction without the informed bidder 3 allocates the good to the ex post efficient bidder (among all three bidders).

When all three bidders are present:

We first observe that the good will not be allocated more efficiently than when the informed bidder 3 is absent (since then it is allocated to the ex post efficient bidder). We will prove that it does strictly worse, thus showing that the addition of the informed bidder 3 deteriorates expected welfare.

We first note that it is a (weakly) dominant strategy for bidder 3 with private information (w^1, w^2) to drop out (since $\theta_3 = 0$) at:

$$b_3(w^1, w^2) = w^1 + w^2.$$

Let b_3 denote the price at which the informed bidder 3 drops out. Given this strategy, and since $\underline{\theta}_1 > 0$ and $\bar{w}^1 + \bar{w}^2 < \underline{\theta}_2$, it is a weakly dominant strategy for bidder 1 and 2 (with private information θ_1, w^1 and θ_2 , respectively) to wait for bidder 3 to drop out, and for bidder 1 to remain in the auction until the price reaches the level:

$$b_1(\theta_1, w^1, b_3) = \theta_1 + b_3.$$

Note that bidder 1 perfectly infers the value of w_2 from b_3 , so for the allocation to be efficient bidder 2 would have to perfectly infer the value of the object in

¹⁸The strategy for bidder 1 (with private information θ_1, w^1) is to drop out at price $\theta_1 + w^1 + E(w^2)$ (if bidder 2 is still present). The strategy for bidder 2 (with private information θ_2) is to drop out at price $\theta_2 + E(w^2)$.

equilibrium. We now check however that bidder 2 (with private information θ_2) can only imperfectly infer the value of the object in equilibrium. In equilibrium, she remains in the auction until the price reaches the level:

$$b_2 = \theta_2 + E[w^2 \mid w^1 + w^2 = b_3 \text{ and } b_1(\theta_1, w^1, b_3) = b_2].$$

Since bidder 1's drop out price does not depend on w^1 , and since the random variable θ_1, w^1, w^2 are independent, bidder 2 drops out at price:

$$b_2(\theta_2, b_3) = \theta_2 + E[w^2 \mid w^1 + w^2 = b_3],$$

which confirms that bidder 2 in equilibrium only imperfectly infers the value of w^2 . Bidder 1 (resp. bidder 2) obtains the good whenever

$$b_1(\theta_1, w^1, b_3) \underset{\text{(resp. } < \text{)}}{>} b_2(\theta_2, b_3),$$

and the allocation is ex post inefficient for example when:

$$\theta_2 + E[w^2 \mid w^1 + w^2 = b_3] < \theta_1 + w^1 + w^2 < \theta_2 + w^2.$$

■

Comments:

1) The induced allocation when all three bidders participate need not be ex post efficient because in equilibrium, the inferences made by bidder 1 and 2 about the value of the purely common value w^2 differ. As a result, both types of mistakes may occur in equilibrium: the object may be allocated to bidder 1 although bidder 2 is the efficient bidder (this occurs when bidder 2 underevaluates w^2); and the object may be allocated to bidder 2 although bidder 1 is the efficient bidder (this occurs when bidder 2 overevaluates w^2).

To illustrate this point, consider the case in which w^1 and w^2 are drawn independently from the same distribution (not necessarily from the uniform distribution). Then $E[w^2 \mid w^1 + w^2 = b_3] = b_3/2$, and thus

$$b_2(\theta_2, b_3) = \theta_2 + b_3/2.$$

Bidder 1 (resp. bidder 2) gets the object whenever

$$\theta_1 - \theta_2 \underset{\text{(resp. } < \text{)}}{>} -\frac{w^1 + w^2}{2}.$$

On the other hand, bidder 1 is more (resp. less) efficient than bidder 2 whenever

$$\theta_1 - \theta_2 \begin{matrix} > \\ (resp. <) \end{matrix} -w^1.$$

Thus, whenever

$$\theta_1 - \theta_2 > -\frac{w^1 + w^2}{2} \text{ and } \theta_1 - \theta_2 < -w^1$$

or

$$\theta_1 - \theta_2 < -\frac{w^1 + w^2}{2} \text{ and } \theta_1 - \theta_2 > -w^1$$

the good is allocated to the welfare inferior bidder among $\{1, 2\}$ resulting in a welfare loss of $|\theta_1 + w^1 - \theta_2|$ as compared with the situation in which bidder 3 does not participate in the auction.

2) The reason why the participation of the informed bidder deteriorates expected welfare in basic example 2 is somewhat different from that in the second price auction basic example 1. In contrast with example 1, expected welfare does not deteriorate here because the suboptimal informed bidder acquires the object. It deteriorates because his mere presence modifies the competition between bidders 1 and 2. And it does so because bidders 1 and 2 are differentially informed, and therefore the information conveyed by the strategy of the extra informed bidder is not the same¹⁹ for the two bidders $i = 1, 2$ in equilibrium. It should be noted that if bidder 1 had been uninformed of w^1 as well as of w^2 , then as explained in comment 3 page 11, the addition of bidder 3 would have improved expected welfare.

3) If we consider the second price auction instead of the ascending price auction, the final allocation is ex post efficient even when all three bidders are present at the auction. Thus, the second price auction performs better in this case than the ascending price auction when all three bidders are present.²⁰

¹⁹This has some (vague) connection with the failure of the linkage principle observed by Perry-Reny (1999) in a multi-object context. Perry-Reny is concerned with revenue, but in both cases some extra information (here that conveyed by the behavior of bidder 3) is detrimental to the criterion (here welfare).

²⁰To see this, observe that in a second price auction, bidders 1 and 2 would bid:

$$b_1(\theta_1, w^1) = \theta_1 + w^1 + Ew^2 \text{ and } b_2(\theta_2) = \theta_2 + Ew^2,$$

respectively (because the condition $\bar{w}^1 + \bar{w}^2 < \underline{\theta}_2$ implies that the informed bidder 3 cannot get the object in equilibrium). It follows that the final allocation is ex post efficient.

4) We confined ourselves to strategies that are not (weakly) dominated. For example, the informed bidder 3 (with private information w^1, w^2) never gets the object and therefore his incentive to drop out at price $b_3 = w^1 + w^2$ is weak. However, the setup could easily be enriched so that bidders have strict incentives to conform to the strategies defined above. For example, for each $i = 1, 2$, suppose that (with probability $1 - \varepsilon$), bidder i is as defined in basic example 2, and (with probability ε) bidder i has a valuation π_i (known to bidder i only) and distributed on $(\underline{w}_1 + \underline{w}_2, \max(\bar{\theta}_1, \bar{\theta}_2) + \bar{w}_1 + \bar{w}_2)$. Then bidder 3 has a strict incentive to drop out at price $b_3 = w^1 + w^2$ because there is a chance that the other two bidders drop out at any price. Similarly, given bidder 3's strategy and because there is a chance that bidder 2 drops out at any price, bidder 1 has strict incentives to wait for bidder 3 to drop out and to drop out himself at $\theta_1 + b_3$. Clearly, for ε small enough, the same conclusion as in Result 2 carries over in this enriched setup even without the restriction to weakly undominated strategies. Moreover, in this enriched setup the equilibrium is uniquely defined.

5 Discussion

This paper has shown that when bidders have multidimensional signals (on a private, a common and possibly a partially common element), the addition of one bidder at the auction stage may deteriorate expected welfare in asymmetric cases in either the second price or the ascending price auction. One should thus be cautious when recommending to systematically promote the maximum participation in procurement like auctions.²¹

A systematic analysis of when the addition of a bidder deteriorates welfare should be the subject of future research. It may be noted though that the basic examples we exhibited share the feature that the additional bidder has more information on the common element than the other bidders. This is no coincidence: When the extra bidder has no information affecting the valuations of others it can be shown that his

²¹Another important reason for why more competition (or more participation) at the auction stage may not enhance efficiency is that of market structure considerations (because then the valuation may include preemption motives and give rise to war of attrition phenomena, see Jehiel and Moldovanu 2000).

participation is always good for welfare both in second price and in ascending price auctions (see Compte-Jehiel 1999).

The basic examples had the additional feature that the allocation without the extra bidder is ex post efficient. Its purpose was mainly to simplify the argument (since the outcome when the extra bidder does not participate cannot be improved upon). When this feature is not met, the welfare value of the extra bidder is likely to be less negative than suggested in this paper. First his participation may allow to allocate the object to this extra bidder when he is welfare superior. Second even if the additional bidder is welfare inferior, his participation may allow the other bidders to have a sharper assessment of the common element (through the extra information conveyed by the equilibrium bidding strategy of the additional bidder), which in turn may induce a welfare superior final allocation.^{22,23}

6 Appendix

Proof of Proposition 1:

It is standard to show that under the three first conditions a) there exists a unique symmetric equilibrium and b) equilibrium bids are strictly increasing functions of $u(\theta_i, w^i)$.²⁴ Given this property, the object is allocated to the bidder with highest

²²As an illustration, consider a setup with 3 bidders $i = 1, 2, 3$. Bidder $i = 1, 2$ knows $\theta_i > 0$ and w^i . Bidder 3 knows w^1 and w^2 and $\theta_3 = 0$. Bidder i 's valuation is $\theta_i + w^1 + w^2$. The parameters $\theta_i, w^i, i = 1, 2$ are independent from each other. Consider the ascending price auction. When bidder 3 participates at the auction, bidder 3 drops out at price $b_3 = w^1 + w^2$ and the good is eventually allocated to the ex post efficient bidder. When bidder 3 does not participate in the auction, there are ex post inefficiencies, and therefore the participation of bidder 3 is good for welfare.

²³Another interesting extension of the paper concerns the endogenization of the information held by bidders, for example by allowing the bidders to decide prior to the auction whether or not to acquire information (about the private and/or common elements). Such an extension would more plausibly require that at the auction stage, bidders do not know exactly whether or not the other bidders have acquired information, and therefore the nature of the information structure is substantially different from that studied in this paper.

²⁴In a sealed bid second price auction, player i 's equilibrium bid satisfies:

$$b_i^*(\theta_i, w^i) = u(\theta_i, w^i) + \sum_{k \leq m} E[\phi(w_k) \mid \max_{j \neq i} u(\theta_j, w^j) = u(\theta_i, w^i)] + \sum_{k > m} E[\phi(w_k)] \quad (4)$$

which is equal to $\gamma_m(u(\theta_i, w^i)) + (N - m)E[\phi(w_k)]$ because the pairs (θ_j, w^j) are iid. Bids thus increases with $u(\theta_i, w^i)$ because γ is an increasing function.

$u(\theta_i, w^i)$. Net of the common element $\sum_{k \in N} \phi(w^k)$, the expected welfare is equal to:

$$\begin{aligned} G &= E[h(\theta_{i_0}, w^{i_0}) \mid i_0 = \arg \max_i u(\theta_i, w^i)] \\ &= \int_z E[h(\theta_{i_0}, w^{i_0}) \mid i_0 = \arg \max_i u(\theta_i, w^i), u(\theta_{i_0}, w^{i_0}) = z] h(z) dz, \end{aligned}$$

where $h(z) = -\frac{d}{dz}H(z)$, with

$$H(z) = \Pr\{\max_i u(\theta_i, w^i) \geq z\}$$

By symmetry, we have

$$E[h(\theta_{i_0}, w^{i_0}) \mid i_0 = \arg \max_i u(\theta_i, w^i), u(\theta_{i_0}, w^{i_0}) = z] = E[h(\theta_k, w^k) \mid u(\theta_k, w^k) = z \geq \max_{j \neq k} u(\theta_j, w^j)],$$

and because the random variables (θ_i, w_i) are independent from one another, we obtain

$$G = \int_z E[h(\theta_k, w^k) \mid u(\theta_k, w^k) = z] h(z) dz. \quad (5)$$

Since $E[h(\theta_k, w^k) \mid u(\theta_k, w^k)] = \eta(z)$, (5) implies:

$$G = \int_z \eta(z) h(z) dz = \eta(\underline{z}) + \int_z \eta'(z) H(z) dz$$

Since $\eta'(z) \geq 0$, and since for any z , $H(z)$ increases with the number of bidders, we conclude that welfare increases with the number of bidders. ■

References

- [1] Bulow, J. and P. Klemperer (1996) "Auctions versus Negotiations", *American Economic Review*, **86**, 180-194
- [2] Bulow, J. and P. Klemperer (1998): "Prices and the Winner's Curse," mimeo Nuffield College, Oxford.

In an ascending price auction, bids are also increasing function of $z_i = u(\theta_i, w^i)$. If n bidders have not dropped out yet, then bidder i 's bidding function is equal to (up to an additive constant), $\gamma_n(z_i)$, which is also increasing in z_i if γ_N is. (This follows the standard arguments developed in Milgrom and Weber 1982).

- [3] Compte, O. and P. Jehiel. (1999): "On the Value of Competition in Procurement Auctions", mimeo CERAS and UCL ([http:// else.econ.ucl.ac.uk/ papers/ compet14.pdf](http://else.econ.ucl.ac.uk/papers/compet14.pdf))
- [4] Dasgupta, P. and E. Maskin (1998): "Efficient Auctions," forthcoming *Quarterly Journal of Economics*.
- [5] Jackson, M. O (1999): "Efficiency and Information Aggregation in Auctions with Costly Information," mimeo.
- [6] Jehiel, P. and B. Moldovanu (1999): "Efficient Design with Interdependent Valuations," mimeo.
- [7] Jehiel, P., and B. Moldovanu (2000): "License Auctions and Market Structure," mimeo
- [8] Maskin, E. (1992): "Auctions and privatizations," in *Privatization* ed. by H. Siebert.
- [9] Milgrom, P. and R. J Weber(1982): "A Theory of Auctions and Competitive Bidding", *Econometrica*, **50**, 1089-1122
- [10] Perry, M. and P. Reny (1999): "On the Failure of the Linkage Principle," *Econometrica*.
- [11] Pesendorfer, W. and J. Swinkels (1998): "Efficiency and Information Aggregation in Auctions," forthcoming *American Economic Review*.