On the Optimal Majority Rule

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Abstract

We develop a simple model that accounts for the widely spread intuition that as committees get large, (well chosen) majority rules are preferable to unanimity. The model is one of collective search in which members do not control the proposal put to a vote. The main drawback of unanimity is that it makes it too difficult to find a proposal acceptable by all, which in turn induces extra costly delays in comparison with majority rules. The best majority rule is the one that solves best the trade-off between speeding up the decision process and avoiding the risk of adopting too inefficient proposals.

1 Introduction

It is widely accepted that when a committee is too large and must adopt decisions by unanimity, it does not function well.[1] This is, in essence, what

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[1] This view is expressed in various ways in a number of classic writings. For example, Black (1958, page 99) writes:

> "The larger the size of majority needed to arrive at a new decision on a topic, the smaller will be the likelihood of the committee reaching a decision that alters the existing state of affairs."

has led the EU to adopt the Lisbon treaty in recent years. As the EU has grown larger, it has become clear that maintaining the requirement that decisions should be approved by unanimity would lead to much inaction, and the Lisbon treaty was precisely proposed to correct this deficiency (by lowering the majority requirement for a number of decisions).

Despite the wide acceptance of such a claim, the bargaining literature has difficulties providing a rationale for it. For example, following the legislative bargaining literature (pioneered by Baron and Ferejohn (1989)) and viewing committees as bargaining over the division of a pie of fixed size, unanimity is found to be no worse than any other majority rule (in fact all majority rules are welfare equivalent).[2] Even more striking: if one extends the basic Baron-Ferejohn's setup to allow for the size of the pie to evolve according to a stochastic process, then the unanimity rule provides a welfare efficient outcome (see Merlo and Wilson (1998)) and other majority rules typically lead to welfare inferior outcomes (see Eraslan and Merlo (2002)).

We revisit this essential question by applying a collective search framework first developed in Compte and Jehiel (2004-2009) (see also Albrecht et al. (2009) – which is discussed further below – for other applications of this setup). In the collective search model, the members of the committee do not control the proposals put to a vote. Their strategic decision consists in voting on whether they are in favor of the proposal or whether they prefer waiting for a better alternative. If the proposal put to a vote receives the support of the required majority, it is implemented. Otherwise, the search process continues.

Such a collective search framework is we believe well suited to model a number of collective decision processes such as those taking place in the EU or in other organizations representing diverse interests, public or private.

And Buchanan and Tullock (1962) express related concerns about the cost of unanimity rules.

[2]This is because an agreement is reached immediately in all cases.

More precisely, whenever financial capacities are scarce, as is the case in virtually all organizations, implementing a tentative proposal or financing a tentative project would come at the cost of making future proposals harder (or even impossible) to implement. Moreover, a tentative proposal generally comes together with how the rents it generates are distributed among the various concerned parties. To the extent that proposals are not under the (perfect) control of parties, the collective decision making in such situations is well described as a collective search process.[3]

Observe that the collective search model is flexible enough to accommodate the idea that the various possible proposals may correspond to different aggregate payoff or welfare (the sum of utilities of the various committee members or the size of the pie in Eraslan and Merlo's framework) so that it can accommodate the idea that the size of the pie may be stochastic (as in Merlo and Wilson's work). A key difference between collective bargaining and collective search though is that under the collective bargaining approach all possible partitions of the pie are simultaneously available, while under the collective search approach, a proposal at a given date determines both the size of the pie and how the pie would be divided among the committee members: some other proposals for partitioning the pie will eventually arise, but only through later draws. With patient players however, this difference between the two approaches would not seem to matter a great deal, as in principle each member could at little cost wait for a division he would like to see proposed. What this paper shows is that the two approaches actually lead to predictions that differ substantially.[4]

[3] In the collective search model to be analyzed later, there is room for only one project. We conjecture that similar insights would carry over in setups allowing for flows of projects and bounded capacity.

[4] Our finding shares some similarities with Diamond's observation that small search costs on consumers' side would lead competing oligopolists to charge the monopoly price as if there were no competition, in sharp contrast with the analysis of the frictionless case (Bertrand competition). Here, we show that even as members are very patient (so

Our collective search setup will be used to characterize simply the welfare associated with the various majority rules in the limit of large committees and patient members. We shall characterize the optimal majority rule, and in particular, we shall establish (as a corollary) that the unanimity rule is not the optimal rule. Inefficiencies in our setup can take two forms. Either a proposal that is not welfare optimal is approved or there is excessive delay. The unanimity rule always induces inefficiencies in the form of delay (in particular, inefficiencies continue to arise in the unanimity case even when all proposals are welfare-equivalent), thereby confirming the intuition that unanimity would lead to much inaction in large committees. As one decreases the majority requirement starting from unanimity, one eventually reaches a point at which mostly those proposals with maximal welfare are accepted and almost all of them are accepted. This is the optimal majority rule. Under further symmetry assumptions on how the welfare is distributed among committee members, this optimal majority rule corresponds to the simple majority rule.[5] As one lowers further the majority requirement beyond the optimal majority rule, then inefficiencies arise again and they typically take the form that welfare inefficient proposals may be adopted too early.

While the latter form of inefficiency obtained for majority requirements less stringent than the optimal majority rule is similar to the inefficiency derived in the collective bargaining model of Eraslan and Merlo (2002) (for rules other than unanimity), the inefficiency of the unanimity rule (as well as the inefficiency obtained under majority requirements more stringent than the optimal one) is specific to the collective search approach of our model, and does not arise in the collective bargaining approach.[6]

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that search frictions may be considered to be small), the collective search model and the collective bargaining model have very different predictions.

[5]More precisely, a majority rule is characterized by a scalar α, i.e. the fraction of players required for the proposal to pass. The optimal majority rule corresponds to setting α equal to the probability that the idiosyncratic part of the rent is positive.

[6]Aghion and Bolton (2003) consider a two-period collective bargaining model that

2 The Model

We consider a committee consisting of n members, labeled $i = 1, ..., n$. At any date $t = 1, ...,$ if a decision has not been made yet, a new proposal is drawn and examined. A proposal is denoted u. The set of proposals is denoted U. If the proposal u is implemented, it gives member i utility u_i. The utilities $(u_i)_{i=1}^n$ of the proposal u may vary from one proposal to the next. Each u_i belongs to $[\underline{u}, \overline{u}]$, and we assume that proposals at the various dates $t = 1, ..$ are drawn *independently* from the same distribution with continuous density $f(\cdot) \in \Delta([\underline{u}, \overline{u}]^n)$.

Upon arrival of a new proposal u, each member decides whether to accept that proposal. We consider various majority rules. Under the k-majority rule, the game stops whenever at least k out of the n members vote in favor of the proposal.

We normalize to 0 the payoff that parties obtain under perpetual disagreement, and we let δ denote the common discount factor of the committee members. That is, if the proposal u is accepted at date t, the date 0 payoff of member i is $\delta^t u_i$. Observe that we allow that $\underline{u}$ be negative, that is, we do not impose that proposals deliver payoffs above the status quo payoffs to all members.

Strategies and equilibrium. In principle, a strategy specifies an acceptance rule that may at each date be any function of the history of the game. We will however restrict our attention to *stationary* equilibria of this game,

shares some similarities with our basic insight. More precisely, when transfers are sufficiently costly, Aghion and Bolton find in their setup that the unanimity rule is not optimal as less stringent majority rules may allow smaller groups to adopt welfare improving projects without the consent of the hurt agents who would resquest (costly) compensations (thereby making the project non-profitable when the transfer costs are big enough).

Compared to their model, our collective search approach allows us to obtain similar insights about the inefficiency of unanimity even in the limit as search frictions get small, thereby dispensing with the assumption of exogenous large transfer costs.

where each member adopts the same acceptance rule at all dates.[7]

Given any stationary acceptance rule σ_{-i} followed by members j, $j \neq i$, we may define the expected payoff $\bar{v}_i(\sigma_{-i})$ that member i derives given σ_{-i} from following his (best) strategy. An optimal acceptance rule for member i is thus to accept the proposal u if and only if

$$u_i \geq \delta \bar{v}_i(\sigma_{-i}),$$

which is stationary as well (this defines the best-response of member i to σ_{-i}).

Stationary equilibrium acceptance rules are thus characterized by a vector $v = (v_1, .., v_n)$ such that member i votes in favor of u if $u_i \geq \delta v_i$ and votes against it otherwise. For any k-majority rule and value vector v, it will be convenient to refer to $A_{v,k}$ as the corresponding *acceptance set*, that is, the set of proposals that get support from at least k members when failing to agree today yields member i a continuation payoff of v_i (from the viewpoint of next period):[8]

$$A_{v,k} = \{u \in U, \exists K \subset \{1, ..., n\}, \ \mid K \mid = k, \ u_i \geq \delta v_i \text{ for all } i \in K\}. \tag{1}$$

Equilibrium consistency then requires that

$$v_i = \Pr(u \in A_{v,k}) E[u_i \mid u \in A_{v,k}] + [1 - \Pr(u \in A_{v,k})] \, \delta v_i \tag{2}$$

or equivalently

$$v_i = \frac{\Pr(u \in A_{v,k})}{1 - \delta + \delta \Pr(u \in A_{v,k})} E[u_i \mid u \in A_{v,k}]. \tag{3}$$

[7]To avoid coordination problems that are common in voting (for example, all players always voting "no"), we will also restrict attention to equilibria that employ no weakly dominated strategies (in the stage game). These coordination problems could alternatively be avoided by assuming that votes are sequential.

[8]For any finite set B, $\mid B \mid$ denotes the cardinality of B.

A stationary equilibrium is characterized by a vector v and an acceptance set $A_{v,k}$ that satisfy (1)-(2). It always exists, as shown in Compte and Jehiel (2004-2010).

3 On the cost of too stringent majority rules

The objective of this paper is to understand the disadvantage of having too stringent majority requirements as the number n of committee members get large. Before we address this by specifying further the distribution of proposals, we provide a simple example illustrating that unanimity may be undesirable in some cases.

The simplest illustration of this is obtained in the following symmetric setup for which symmetric (stationary) equilibria are considered.

> **Claim:** *Assume proposals u are all such that the welfare $W(u) = \sum_i u_i$ is constant, say with value $w > 0$, and proposals are drawn according to a uniform distribution on $U = \left\{u \mid \sum_i u_i = w\right\}$. Then expected welfare increases when the majority requirement is decreased.*

When proposals are welfare equivalent, the majority rule affects the expected welfare only to the extent that it speeds up the agreement. Intuitively, the claim holds because under less stringent majority rule, the acceptance set gets bigger.

To show this formally, observe that by symmetry, the acceptance threshold δv_i is the same for all members and depends only on the majority requirement k. Denote by v_k^*, the per-member welfare obtained in equilibrium under the $k-$majority rule, and by π_k^* the equilibrium probability of agreement. Assume by contradiction that one can have $k_1 > k_2$ and

$v^*_{k_1} \geq v^*_{k_2}$. Given that $\Pr(u \in A_{v,k})$ is decreasing with k and v, we would have $\pi^*_{k_1} = \Pr(u \in A_{v^*_{k_1},k_1}) < \pi^*_{k_2} = \Pr(u \in A_{v^*_{k_2},k_2})$. Since the expected welfare v^*_k is an increasing function of the probability of agreement π^*_k,[9] we must have $v^*_{k_2} > v^*_{k_1}$, contradicting the premise that $v^*_{k_1} \geq v^*_{k_2}$.

In more general settings, when the welfare associated with the various proposals may vary (and members may possibly be ex ante asymmetric), a different conclusion may arise. To get further insight about the pros and cons of tightening the majority rule, we decompose the various effects of the majority requirement on the ex ante welfare as measured by the expected sum of utilities obtained by all committee members in equilibrium.

Specifically, define $W(u) \equiv \sum_i u_i$ as the welfare associated with proposal u, by v^k the equilibrium value profile associated with the k-majority rule, and by $W_k \equiv \sum_i v^k_i$ the associated ex ante welfare. Summing expression (3) over all members i yields:

$$W_k \equiv \frac{\Pr(u \in A_{v^k,k})}{1 - \delta + \delta \Pr(u \in A_{v^k,k})} E[W(u) \mid u \in A_{v^k,k}] \tag{4}$$

Expression (4) shows that there may be two factors reducing welfare as compared with the maximum possible welfare level $\bar{W} = \max_{u \in U} W(u)$:

- There may be delays, because it may take some time before a proposal gets accepted: the smaller the term $\frac{\Pr(A_{v^k,k})}{1-\delta+\delta \Pr(A_{v^k,k})}$, the more severe the decrease in welfare due to delays.

- The acceptance set may contain inefficient proposals in the sense that some proposals in $A_{v^k,k}$ may not belong to $\arg\max_{u \in U} W(u)$. The reduction in welfare is all the more severe that $A_{v^k,k}$ is far away from $\arg\max_{u \in U} W(u)$.

Why majority rules may dominate?

Intuitively, for a given profile of acceptance thresholds, as one reduces the majority requirement k, the acceptance set increases, hence delays are

[9] This is because $E[u_i \mid u \in A_{v,k}] = w/n$ so (3) implies $v^*_k = \frac{\pi^*_k}{1-\delta+\delta\pi^*_k} \frac{w}{n}$.

reduced. However, the acceptance set may now allow for proposals that are further away from the Pareto frontier, thereby inducing a welfare loss.

In case proposals are mostly efficient as in the simple example provided above, the term $E[W(u) \mid u \in A_{v^k,k}]$ in Expression (4) remains close to $\bar{W}$ for all k, hence the only source of decrease in welfare is delay. Majority rules then dominate the unanimity rule because they reduce delay. Of course the comparison may be reversed when proposals may be significantly inefficient and members are patient enough (so that delay costs become negligible).

Comparison with first-best.

To conclude this Section, we compare the equilibrium outcomes with the first best. Note that given the search friction (i.e. the randomness of proposals), waiting may be socially desirable,[10] and the maximum welfare that members can *jointly* obtain may be strictly smaller than $\bar{W}$. Let $\bar{w}$ be that welfare value.[11] The socially efficient acceptance set $\bar{A}$ would thus consist of the proposals u for which $W(u) \geq \delta\bar{w}$:

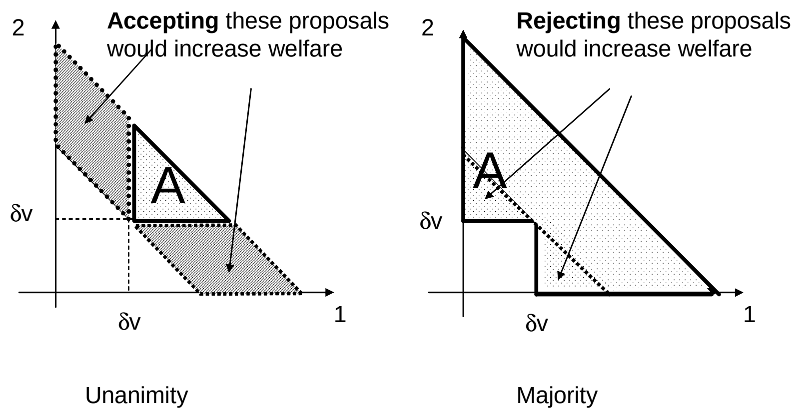
$$\bar{A} = \{u \mid W(u) \geq \delta\bar{w}\}.$$

All k-majority rules whatever k are socially inefficient because they induce acceptance sets that cannot take the form $\bar{A}$. For any tentative v, k-majority rules either exclude proposals that are welfare superior to $\delta W(v)$ (this is for example the case under the unanimity rule), or they include proposals that are welfare inferior to $\delta W(v)$. When the majority requirement is increased, inefficiencies of the second type are reduced, but inefficiencies of the first

[10] Because the current proposal may be too inefficient.

[11] It is the value obtained in a standard one-agent search model in which this agent's utility is $W(x)$, x arrives according to $f(\cdot)$ and the discount factor is δ.

type are generated.



The Figure above provides an illustration in a simple three member setting in which member 3 would always accept (because, say, all proposals are equivalent for him). The figure draws the acceptance set in the space of members 1 and 2's preferences under the unanimity rule (Left) and under the majority rule (Right).

4 Optimal majority rule in large committees

We now turn to our main question of interest, that is, how the majority rule should be set in large committees so as to maximize welfare. We will consider symmetric settings and symmetric (stationary) equilibria so that the ex ante payoff is the same for all members, and welfare can be identified with any member's payoff.

Specifically, member i assesses proposal u according to:

$$u_i = x + \theta_i$$

The common part x is drawn in each period according to a density $g(\cdot)$ on $[\underline{x}, \overline{x}]$. We assume that the conditional expectation $z \to E(x \mid x > z)$ is a smoothly differentiable function of z with slope no greater than 1. This holds true for the uniform distribution and for many more densities $g(\cdot)$ with bounded variations.

The idiosyncratic parts θ_i are drawn independently across periods and across members according to a smooth density $h(\cdot)$ on $[-1, 1]$. We assume that $E(\theta_i) = 0$,[12] and we define α_0 as the probability that the idiosyncratic part is positive:

$$\alpha_0 \equiv \Pr(\theta_i > 0).$$

When θ_i is symmetric around 0, we have $\alpha_0 = \frac{1}{2}$.

4.1 Equilibrium analysis

We wish to analyze how the various majority rules compare in terms of expected welfare as the number of members grows large. Specifically, we will compare the ex ante payoff obtained by every member in equilibrium under the various majority rules. In making this comparison, we shall set the discount factor close to 1 (patient members) and make the number of members grows arbitrarily large.

When the number of members grows large, whether a proposal is accepted or not depends almost exclusively on the realization of x (this is due to the law of large numbers, as we shall see). In subsequent results we refer to $\alpha = \frac{k}{n}$ as the majority rule where k is the majority requirement defined in Section 2. For every α and δ, there will be a threshold x^* such that, as n grows large, only proposals such that $x > x^*$ are accepted. Our objective below is to characterize x^*.

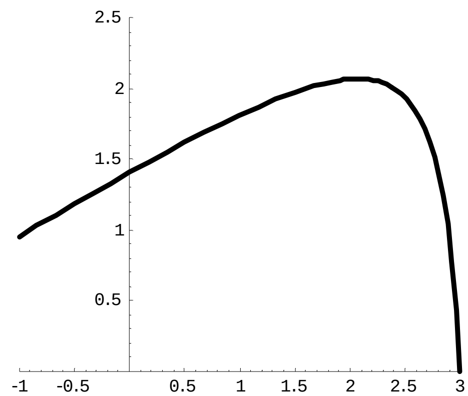
Let us first define $v(x^*, \delta)$ as the expected payoff that any member receives if all proposals such that $x > x^*$ are accepted and only such proposals are accepted. We have:

$$v(x^*, \delta) \equiv \frac{\Pr(x > x^*)}{1 - \delta + \delta \Pr(x > x^*)} E(x \mid x > x^*) \tag{5}$$

[12] This is just a normalization, since if $E(\theta_i) \neq 0$ we can add $E(\theta_i)$ to x.

Note that our assumption that $z \to E(x \mid x > z)$ has slope less than 1 implies that $z \to v(z, \delta)$ has slope less than 1. Also note that $x^* = \underline{x}$ corresponds to the case where all proposals are accepted (so $v(\underline{x}, \delta) = E(x)$), while $x^* = \overline{x}$ corresponds to the case where no proposals are accepted (so $v(\overline{x}, \delta) = 0$).

The following figure draws $v(., \delta)$ for a discount factor $\delta = 0.95$, assuming that x is uniformly distributed on $[-1, 3]$, and θ_i is uniformly distributed on $[-1, 1]$.



The function $v(., \delta)$

The figure illustrates that starting from $\underline{x}$, a more stringent acceptance threshold x^* increases welfare, up to the point where this would reduce so much the probability of acceptance that welfare starts decreasing.

The complete characterization of the equilibrium as n grows large goes as follows. For every α, define $\theta^*(\alpha)$ as the threshold that solves:

$$\Pr(\theta_i > \theta^*(\alpha)) = \alpha$$

or equivalently:

$$1 - H(\theta^*(\alpha)) = \alpha$$

where $H(\cdot)$ denotes the cumulative of $h(\cdot)$. The threshold $\theta^*(\alpha)$ is thus set so that each member i has a probability α to have his idiosyncratic part θ_i exceed $\theta^*(\alpha)$. As n grows large, by the law of large number, α will approximately correspond to the fraction of members for which θ_i exceeds $\theta^*(\alpha)$. Observe that when α gets close to 1, $\theta^*(\alpha)$ gets close to -1, while when α gets close to 0, $\theta^*(\alpha)$ gets close to 1. For the uniform distribution for example, $H(\theta) = (1+\theta)/2$ so $\theta^*(\alpha) = 1 - 2\alpha$.

Now assume that v^* is the equilibrium expected payoff received by each member. A member votes in favor of a proposal x whenever

$$x + \theta_i > \delta v^*. \tag{6}$$

For a given x, the number of members in favor of the proposal is thus a random variable: it corresponds to the number of realized value of θ_i for which (6) holds. As n grows large however, the proposal receives the support of a share of members approximately equal to $\Pr(\theta_i > \delta v^* - x)$ with probability close to 1. Thus, given the majority rule α, proposal x goes through whenever $\Pr(\theta_i > \delta v^* - x) > \alpha$, or equivalently whenever $\theta^*(\alpha) > \delta v^* - x$. That is, whenever:

$$x > \delta v^* - \theta^*(\alpha).$$

So as mentionned earlier, in equilibrium, as n grows arbitrarily large, a proposal x is accepted if and only if it exceeds some threshold x^*.[13] Thus by definition of $v(., \delta)$ (see (5)) we must have:

$$v^* = v(x^*, \delta)$$

Three cases may thus be distinguished, depending on whether the threshold x^* is interior, or $x^* = \bar{x}$ (no proposal accepted), or $x^* = \underline{x}$ (all proposals accepted).

In the first case (interior solution), the threshold x^* must solve:

$$\delta v(x^*, \delta) = x^* + \theta^*(\alpha) \tag{7}$$

[13]The statement only holds at the limit where n grows very large. Proposition 1 will make a precise statement for the case where n is large but fixed.

If $\underline{x} - \delta Ex + \theta^*(\alpha) < 0 < \bar{x} + \theta^*(\alpha)$, equation (7) has a solution x^*, and since $z \to v(z, \delta) - z$ is decreasing, that solution is unique.

In the second case (no proposals accepted), the candidate equilibrium is $x^* = \bar{x}$. Perpetual disagreement is indeed an equilibrium when

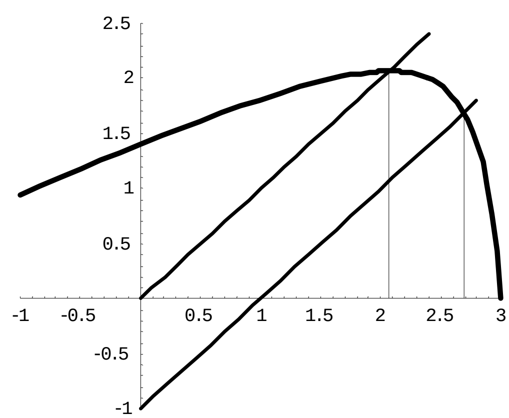
$$\bar{x} + \theta^*(\alpha) \leq 0$$

Finally, in the third case (all proposals are accepted), the candidate equilibrium is $x^* = \underline{x}$, and immediate agreement on whatever proposal is indeed an equilibrium when

$$\underline{x} - \delta E(x) + \theta^*(\alpha) \geq 0.$$

It follows that for each value of α, one and only one of the three cases above applies, thus the threshold x^* is uniquely defined.

Going back to the above case of uniform distribution, the following figure explains graphically how the threshold x^* is obtained for the unanimity rule $\alpha = 1$ (in which case $\theta^*(\alpha) = -1$), and for the majority rule $\alpha = 1/2$ (in which case $\theta^*(\alpha) = 0$).



Deriving x^*

As one decreases the majority requirement α, $\theta^*(\alpha)$ increases, hence so does the line $x + \theta^*(\alpha)$. As the figure above illustrates, the threhold x^*_α shifts to the left. Under the assumptions of the figure ($\delta = 0.95$) and starting from the unanimity rule, this shift increases welfare.

As mentioned earlier, the analysis above has been made for the case where n is arbitrarily large. The next proposition makes a formal statement for cases where n is large but fixed. It is proven in Appendix.

Proposition 1. *Fix α and for every n consider the majority requirement $k(n) = Int(\alpha n)$ where $Int(\alpha n)$ denotes the integer that is closest to αn. For every $\varepsilon > 0$, there exists $\overline{n}$ such that for every $n > \overline{n}$, the expected equilibrium payoff obtained by every member in the $k(n)$ majority requirement $w(\delta, n)$ satisfies $|w(\delta, n) - v(x^*, \delta)| < \varepsilon$ where x^* is the threshold just defined.*

4.2 Comparative statics with patient members

As the above Figure illustrates, Proposition 1 can be used to compare the welfare obtained under different majority scenarios. We now do these comparisons assuming that members are patient.

We start by considering majority rules α that are more stringent than α_0 (recall that $\alpha_0 \equiv \Pr(\theta_i > 0)$). For any such majority rule α, we have $\theta^*(\alpha) \leq 0$, so Proposition 1 tells us that the threshold x^* should satisfy

$$\delta v(x^*, \delta) = x^* + \theta^*(\alpha) \leq x^*$$

Now, for values of x^* bounded away from $\bar{x}$, $\delta v(x^*, \delta)$ is arbitrarily close to $E[x \mid x > x^*]$, hence it exceeds x^*. It follows that x^* must get close to $\bar{x}$ as the discount factor δ tends to 1.[14]

The consequence for welfare is immediate. Assuming for simplicity that $\overline{x} > 1$ (so that even under the unanimity rule, perpetual disagreement cannot be an equilibrium), we obtain that when the majority rule is $\alpha \geq \alpha_0$,

[14]Note that in this thought experiment, we have first taken n to infinity and then considered the limit as δ goes to 1, which fixes the order in which the limits should be taken for this exercise to be valid.

members (each) get an expected payoff equal to

$$\bar{x} + \theta^*(\alpha)$$

So when the majority requirement coincides with α_0, $\theta^*(\alpha) = 0$ and every member gets in expectation a payoff equal to $\overline{x}$. This is clearly the maximum welfare that one can hope to get in this problem, and it clearly dominates the welfare obtained under more stringent majority rules. In particular, under the unanimity rule, each member obtains $\bar{x} - 1$.

In all cases above, only proposals such that x is close to $\overline{x}$ are accepted, but while under the majority requirement α_0 this is achieved at almost no additional cost, under more stringent majority rules there is an extra delay cost which can increase up to 1 under unanimity. This delay cost of 1 is the one that is necessary to obtain the approval of all members as opposed to only an α_0 majority of them.

As now one decreases the majority requirement below α_0, we have $\theta^*(\alpha) > 0$. For any such majority requirement, Proposition 1 tells us that $\delta v(x^*, \delta) = x^* + \theta^*(\alpha)$, which implies, since $v(x^*, \delta) \leq \bar{x}$, that x^* must be bounded away from $\bar{x}$. This further implies that when members are patient, $\delta v(x^*, \delta)$ is close to $E(x \mid x > x^*)$. Hence, x^* solves

$$x^* + \theta^*(\alpha) = E(x \mid x > x^*). \tag{8}$$

Equation (8) confirms that the threshold x^* is strictly below $\overline{x}$. Expected welfare, which coincides with $E(x \mid x > x^*)$ is below $\bar{x}$ as well, and there are thus inefficiencies as compared with the α_0 majority rule. Note that this time the inefficiency takes the form of having members accept too many proposals as compared to the optimal case, which is very different from the inefficiency due to delay identified in the unanimity case.[15] Too many proposals are

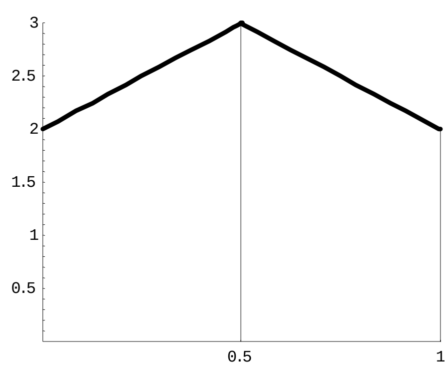
[15]This inefficiency (unlike the delay inefficiency) is similar to the inefficiency identified in Eraslan and Merlo (2002).

accepted because members are afraid that proposals even worse for them are adopted by a sufficient majority. Note that this fear of expropriation may in turn lead members to have an overall negative welfare whenever $E(x \mid x > x^*) < 0$ (remember 0 is the status quo payoff). Also note that as the majority requirement decreases below α_0, the threshold x^* decreases and the inefficiency increases.

To summarize,

Proposition 2. As the number of members gets large and δ goes to 1, expected welfare is maximized for the α_0 majority rule. Any majority rule $\alpha > \alpha_0$ induces inefficiencies in the form of delay. Any majority rule $\alpha < \alpha_0$ induces inefficiencies in the form of having too many proposals being accepted.

The following figure summarizes how welfare varies as a function of α, assuming as before uniform distributions.[16]



Expected (per member) welfare as a function of α

Comment. At first, the conclusion of Proposition 2 may seem at odds with the observation made in Albrecht et al. (2009) that as members get

[16] For uniform distributions, $\alpha_0 = 1/2$ and $\theta^*(\alpha) = 1 - 2\alpha$. When $\alpha > 1/2$, welfare is equal to $\bar{x} + \theta^*(\alpha)$. When $\alpha < 1/2$, welfare is equal to $x^* + \theta^*(\alpha)$, where $x^* + \theta^*(\alpha) = (\bar{x} + x^*)/2$, implying that $x^* + \theta^*(\alpha) = \bar{x} - \theta^*(\alpha)$.

very patient, the best rule is the unanimity rule (see also Compte and Jehiel (2004) for an early statement of the same insight in a less general class of examples). Yet, in Proposition 2, the number of members is supposed to be large, and the conclusion that the majority rule α_0 is optimal holds true only in the limit as n goes to infinity more quickly than δ goes to 1 (which is a more concrete way of interpreting the order of limits in the above analysis).

4.3 Collective search vs collective bargaining

In our collective search framework, the best majority rule is interior, as we have just shown. For comparative purpose, we now consider the corresponding collective bargaining framework, and we will show there that the more stringent the majority requirement the better (thereby confirming insights from Merlo and Wilson (1998) and Eraslan and Merlo (2002)).

The collective bargaining we consider is as follows. A pie is to be divided among the various committee members. As long as agreement on how to partition the pie has not been reached, a new pie is drawn. In every period, the size z of the pie is drawn at random from a distribution with density $g(\cdot)$ on $[\underline{z}, \bar{z}]$. The draws at the various periods are independent of each other. Members are equally patient (with discount factor δ), and they are each selected to make a proposal with the same probability. A proposal consists of a splitting of the current pie among the various members with the constraint that every member should receive a non-negative share of the pie. After the proposal is made, there is a vote. The sharing is implemented and bargaining stops if the proposal receives the support of at least $k = Int(\alpha n)$ members. Otherwise, one moves to the next period, which has the same structure.

We consider stationary equilibria of the above collective bargaining game. As in the case of collective search, we consider the case of large n. Calling $v^b(\alpha, \delta)$ the expected equilibrium welfare obtained by members under the

α-majority rule, we obtain that pies of size z get implemented whenever a fraction α of the members can each be allocated a payoff equal to $v^b(\alpha,\delta)/n$, that is, taking the limit as n grows large, whenever:

$$z > \delta\alpha v^b(\alpha,\delta).$$

Thus, $v^b(\alpha,\delta)$ is the solution to

$$v^b(\alpha,\delta) = v(\alpha v^b(\alpha,\delta),\delta)$$

where the function $v(\cdot,\cdot)$ is the one defined in (5).

Taking the limit as δ goes to 1, the welfare gets close to $E(z \mid z > z^b(\alpha))$ where $z^b(\alpha)$ is the unique solution to:[17]

$$\alpha E(z \mid z > z^b(\alpha)) = z^b(\alpha).$$

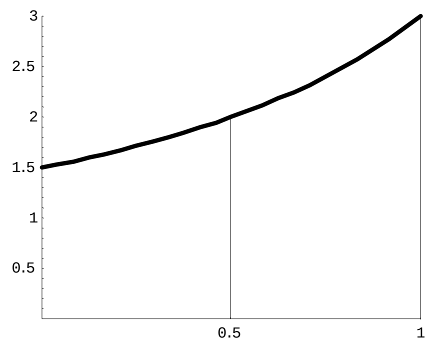
It is readily verified that $\alpha \to z^b(\alpha)$ is increasing in α and converges to $\bar{z}$ as α converges to 1. The corresponding welfare (which is equal to $E(z \mid z > z^b(\alpha))$) is also decreasing in α. To summarize,

Proposition 3. As the number of members gets large and δ goes to 1, in the collective bargaining model, expected welfare is maximized for the unanimity rule. Expected welfare is an increasing function of the majority rule α.

To illustrate Proposition 3, assume that z is uniformly distributed on $[\underline{z},\bar{z}]$. We have that $z^b(\alpha) = \max(\frac{\alpha}{2-\alpha}\bar{z},\underline{z})$ and the corresponding welfare

[17] If $\underline{x} > \alpha E(x \mid x > \underline{x})$ set $\underline{x} = x^b(\alpha)$.

$E(z \mid z > z^b(\alpha))$ is depicted in the following fi



The welfare function $v^b(\alpha, 1)$

The contrast between Propositions 2 and 3 is striking. The collective search model explains why unanimity is undesirable in large committees, and the collective bargaining model does not.

5 Conclusion

This paper has provided a very simple model that accounts for the widely spread intuition that as committees get large, (well chosen) majority rules are preferable to unanimity. Unlike the well developed models of collective bargaining (with transferable utility) which would unambiguously favor unanimity, our model assumes that members do not control the proposal put to a vote. The main drawback of unanimity in such collective search settings is that it makes it too difficult to find a proposal acceptable by all, which in turn induces extra delay costs in comparison with majority rules. The majority rule should not be too low though, as it would result in the acceptance of too inefficient proposals. The best majority rule is the one that solves best this trade-off.[18]

[18] A very different argument in favor of majority as opposed to unanimity follows the line of the Condorcet jury theorem by suggesting that majority rules may better aggregate

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information (this has been formalized recently by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996)).

Our setup offers a different perspective by emphasizing the delay costs attached to unanimity (and by removing the common value uncertainty present in such models - in our setting unlike in those settings, every member knows how much he values the proposal put to a vote, hence our setting falls in the category of private value uncertainty).

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Appendix

Proof of Proposition 1. For any w we define $\pi(w) \equiv \Pr(x > -\theta^*(\alpha) + \delta w)$, and $u(w) = E[x \mid x > -\theta^*(\alpha) + \delta w]$. Also define w^* as the value satisfies:

$$w = \frac{\pi(w)}{1 - \delta + \delta\pi(w)} u(w) \tag{9}$$

Note that by construction, we have $w^* = v^*(x^*, \delta)$ and $x^* = -\theta^*(\alpha) + \delta w^*$.

We show below that the equilibrium value must be close to w^* when n gets large. The argument makes use of the following lemma (some form of the law of large numbers). Define the events $B_\varepsilon, C_\varepsilon, D_\varepsilon$ as:

$$\begin{aligned} B_\varepsilon &= \{\#\{i,\ \theta_i > \theta^*(\alpha) + \varepsilon\}/n > \alpha\} \\ C_\varepsilon &= \{\#\{i,\ \theta_i < \theta^*(\alpha) - \varepsilon\}/n > 1 - \alpha\} \qquad (10) \\ D_\varepsilon &= \{\frac{1}{n}\sum_i \theta_i \notin [-\varepsilon, +\varepsilon]\} \qquad (11) \end{aligned}$$

and let E_ε denote the event complement to $B_\varepsilon \cup C_\varepsilon \cup D_\varepsilon$.

Lemma. $\forall \varepsilon, \exists \overline{n}$ such that for all $n > \overline{n}, \Pr\{E_\varepsilon\} > 1 - \varepsilon$.

Now choose ε small, and n large enough so that the inequality of the Lemma holds ($n > \bar{n}$).

Assume now that the equilibrium value is w. We are going to establish bounds on w. To this end, it is convenient to denote by A the event where the current proposal passes. It is also convenient to denote by $F^+_{w,\varepsilon}$ the event $\{x > -\theta^*(\alpha) + \delta w + \varepsilon\}$, by $F^-_{w,\varepsilon}$ the event $\{x < -\theta^*(\alpha) + \delta w - \varepsilon\}$, by $F^0_{w,\varepsilon}$ the event complement to $F^+_{w,\varepsilon} \cup F^-_{w,\varepsilon}$. Note that since the distribution over proposals has a continuously differentiable density, there exists h such that $\Pr(F^+_{w,\varepsilon}) < h\varepsilon$

Observe that under $F^+_{w,\varepsilon}$, member i accepts x if $\theta_i + x > \delta w$, hence a fortiori if $\theta_i > \theta^*(\alpha) - \varepsilon$. Under event E_ε, there is a fraction α of members for which this is true, hence such proposals x must pass. Similarly, under event $E_\varepsilon \cap F^-_{w,\varepsilon}$, proposals cannot pass. It follows that in any period, a proposal passes with probability at least

$$\pi^- \equiv \Pr(E_\varepsilon \cap F^+_{w,\varepsilon}) = (1-\varepsilon)\pi(w + \varepsilon/\delta)$$

and at most

$$\pi^+ \equiv 1 - \Pr(E_\varepsilon \cap F^-_{w,\varepsilon}) = 1 - (1-\varepsilon)(1 - \pi(w - \varepsilon/\delta)).$$

We now derive bounds on the expected payoff that any given member i obtains, conditional on the event A where the current proposal passes.

Observe that by symmetry, for all x,

$$E[\theta_i \mid A] = E[\frac{1}{n}\sum_i \theta_i \mid A_x]$$

which implies that

$$\mid E[\theta_i \mid A \cap E_\varepsilon] \mid < \varepsilon.$$

Finally we have seen that $E_\varepsilon \cap F^+_{w,\varepsilon} \subset A$ and that $E_\varepsilon \cap F^-_{w,\varepsilon} \cap A = \emptyset$, thus:

$$\Pr(E_\varepsilon \cap F^+_{w,\varepsilon} \mid A) > 1 - \Pr E^c_\varepsilon - \Pr F^0_{w,\varepsilon} > 1 - (1+h)\varepsilon$$

It follows that $E[x + \theta_i \mid A]$ is bounded above by

$$u^+ \equiv (1 - (1+h)\varepsilon)E[x \mid F^+_{w,\varepsilon}] + (1+h)\varepsilon\bar{u}$$

and bounded below by:

$$u^- \equiv (1 - (1+h)\varepsilon)E[x \mid F^+_{w,\varepsilon}] + (1+h)\varepsilon\underline{u}$$

where $\bar{u}$ and $\underline{u}$ are bounds on the payoff that any member may get. The equilibrium value must satisfy:

$$\frac{\pi^-}{1-\delta+\delta\pi^-}u^- < w < \frac{\pi^+}{1-\delta+\delta\pi^+}u^+$$

As ε get small, π^+ and π^- converge to $\pi(w)$, and u^+ and u^- converge to $u(w)$. Hence w must converge to the (unique) solution of (9), that is, w^*. **Q. E. D.**