Partnership Dissolution with Interdependent Values*

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Abstract

We study the issue of partnership dissolution when the parties' valuations are interdependent and only one of them is informed about the valuations. In contrast with the case of private values (Cramton, Gibbons, and Klemperer 1987), in which a fully efficient outcome can be achieved if and only if the initial property rights structure is mixed, in our setup there exists a wide class of situations in which a fully efficient trade cannot be reached. Moreover, in these cases: (1) The subsidy required to restore the efficient allocation is minimal when the entire ownership is initially allocated to one of the parties. (2) If there are no external subsidies and the second-best optimal dissolution mechanism is employed, total welfare is maximized when one of the parties initially has full ownership. Consequently, an early round of trade (before the informed party learns the state of the world) – in which one party obtains full ownership – can be welfare improving. This is consistent with Akerlof's Lemons model, and stands in contrast to the case of private value in which an early round of trade can be counter productive.

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1 Introduction

Consider a situation in which one asset belongs to two partners, and in which there is a common interest in dissolving the partnership. Such an interest could come from a difference in the valuations of the asset for the two partners, or from increasing returns in the ownership share. An efficient dissolution involves eliciting information from the parties so as to decide which party should obtain full ownership, and for which monetary compensation.

In a seminal paper, Cramton, Gibbons, and Klemperer [1987, henceforth CGK] studied the problem of partnership dissolution in a *private values* setting, in which the parties' valuations of full ownership are independent. In this paper we address the case of *interdependent values with one-sided information.* Here, a deterministic function relates the valuations of the two partners and only one party has information regarding the valuations.[1] Moreover, while CGK assume that there are constant returns in the ownership share (i.e., a partner's payoff is proportional to the share of the partnership she holds), we allow for situations with increasing returns in the ownership share (i.e., the valuation of a partial share $0 < \beta < 1$ may be less than β times the valuation of full ownership). The extension of the partnership dissolution problem to possibly increasing returns in the ownership share is desirable for applications. For example, increasing returns could arise if a partner who owns a considerable share is better motivated to invest in the partnership, or if she is able to direct the firm in a way that better exploits complementarities with her other assets.[2]

Our setup fits a number of important applications. For concreteness, consider a start-up company whose shares are owned by an inventor and a venture-capital fund. Following the first phase of development (in which it was beneficial that both parties hold positive shares), one party might now value full ownership more than the other. For instance, the second phase of development might require extensive marketing, which can best be done by the fund's professionals; or it might require integration with the product of another firm, in which case the inventor is more suitable for the job. Thus, at this stage it is preferable that the party with the higher valuation acquire full control in exchange for an appropriate monetary transfer.

The value of full ownership – whether to the inventor or to the fund – depends on the state of the project. Given that the inventor was more active in the first stage of development, she might be better

[1] The assumption of one-sided information, while restrictive, fits in well with a number of important applications. The assumption of a deterministic relation between the valuations is without loss of generality and made for simplicity.

[2] Indeed, constant returns arise naturally if full ownership can be allocated by lottery and if the ownership share is interpreted as the probability of obtaining full ownership. However, this option is often infeasible (for legal or fairness reasons).

informed about the project and its valuation at future stages. Effective dissolution of the partnership requires that the information held by the informed party be revealed, as this may determine which party would value full ownership more. However, in absence of adequate monetary compensations, the informed party's incentives might prevent her from fully revealing her information. If she knows that the project is successful, she will prefer obtaining full ownership, even if her valuation is lower than that of the fund; If the project is failure, she will prefer that the fund buy*s* her shares even if the fund's valuation is lower than hers.

In this paper we study the optimal mechanism that governs the transfer of ownership and the corresponding monetary compensation. We focus on how the total welfare of both parties (given that the optimal mechanism is applied) is affected by the initial ownership structure, i.e., the share owned by each party before the partnership is dissolved.

We find that there exists a wide class of situations in which a fully efficient trade cannot be achieved. Moreover, in these cases: (1) The subsidy required to restore the efficient allocation is minimal when the entire ownership is initially allocated to one of the parties. (2) If there are no external subsidies and the second-best optimal dissolution mechanism is employed, total welfare is maximized when one of the parties initially has full ownership.

These results stand in sharp contrast to those prevailing in the private values setting: CGK's main conclusion was that a fully efficient trade in property rights can be achieved if both parties initially hold significant shares in the partnership; inefficiency arises if one party initially owns the entirety of the shares (as shown by Myerson and Satterthwaite [1983]).

The main advantage of mixed ownership in CGK's setup is that it alleviates the participation constraints of extreme types, and the most severe participation constraints are then binding for intermediate values of the type realizations. This is helpful because it allows the designer to reduce the informational rent to be given to the parties – this insight is similar to the one prevailing in the countervailing incentive literature, see in particular Lewis-Sappington [1989a].[3]

In our setup, a mixed ownership does not alter the type(s) whose participation is hardest to obtain (while trying to implement the ex-post efficient allocation). As a result, mixed ownership loses its main advantage and an extreme allocation of property rights is preferable in a number of cases. More precisely, when there are constant returns in the ownership share (as in CGK) the subsidy required to achieve the first best varies linearly with the share β of the informed party (because the binding

[3] In the setup studied by CGK, in which partners are ex-ante symmetric, ex-post efficiency can be achieved for an ex-ante equal splitting of the ownership.

participation constraint is independent of the ex-ante share). It is thus minimized for either $\beta = 0$ or $\beta = 1$ (depending on further specifications). A similar conclusion arises when the returns in the ownership share are not too increasing. (This intuition also leads to the superiority of extreme allocations when there are no subsidies and the second-best mechanism is implemented.)

It should be noted that both the fact that only one party has private information and the interdependence of valuations play a role in the argument. (1) If both parties had private information, the types for whom the participation constraint is binding would be affected by the ex-ante share. (2) If valuations were not interdependent (and the valuation of one partner were known – so that only one partner would have private information), then *any* allocation of property rights would result in a fully efficient allocation.[4]

Our result has also important consequences in terms of the best *timing* of trade. In the private values case, an early round of trade in property rights – in which the agent with the highest ex-ante valuation obtains full ownership – might be counter-productive, since the subsequent trade (after the information is privately revealed to the agents) might not reach the efficient allocation. For example, CGK describe a proposal of the FCC to allocate spectrum bands by lottery, in expectation that subsequent trade would reach efficiency. Their result shows that this proposal would have led to an inferior final allocation, as compared to selling the licences to a cartel of the buyers in which each buyer owns some share (again letting subsequent trade yield the final allocation). In contrast, in our interdependent values setting, an early trading round in which full ownership is given to one of the parties (not necessarily the party whose expected partnership's valuation is highest) can actually lead to a better reallocation once the information is acquired and the optimal trading mechanism is applied. We also observe that even a naive dissolution procedure in which full ownership is initially given to a single party according to a probability proportional to her shares can perform as well as a mixed ownership structure.

The debate regarding the welfare effect of the initial property rights' allocation started with Coase [1960]. He suggested that in a zero transaction cost world with transferable utilities, the initial allocation of property rights is immaterial. Since then, it has been argued that one important context in which the allocation of property rights may affect efficiency is one with asymmetric information.[5] Myerson and Satterthwaite [1983] presented conditions under which efficiency could not be achieved in a private-values setting. Akerlof [1970] "Lemons" model is a celebrated example in which, because of interdependent

[4] A selling procedure of shares at a price equal to the valuation of the uninformed partner would achieve the first-best.

[5] The lack of full commitment may also be responsible for the failure of the Coase theorem (see Jehiel and Moldovanu (1999)).

valuations, efficiency could not be achieved despite the fact that one party is known to value the good more.

Our paper generalizes Akerlof's in two ways. First, while Akerlof studies a case in which the uninformed party always values the object more than the informed one, we also allow for situations in which the informed party always has the higher valuation, and for situations in which the party with the higher valuation depends on the state of the world (which is known to the informed party). Second, while in Akerlof [1970] the informed party has full property rights, we also allow for both parties to hold positive shares. We show that partial ownership does not alleviate the problem captured in Akerlof's model. Moreover, as in Akerlof [1970], we find that an early trading round – before one party acquires information – can enhance welfare.

Other related papers that deal with the case of interdependent values include Samuelson [1984], Gresik [1991], and Fieseler, Kittsteiner, and Moldovanu [2000]. Samuelson [1984] analyzes a "Lemons" type of situation in which, as in our paper, only one party is informed, from a mechanism design point of view. He provides some characterizations of the second-best mechanism but does not allow for a mixed ownership structure. Our paper generalizes Samuelson's by considering mixed ownership structures and by making comparative statics with respect to the property-rights structure (which is our main contribution).

Both Gresik [1991] and Fieseler, Kittsteiner, and Moldovanu [2000] study situations in which agents' information is ex-ante symmetric (i.e. all parties have private information and each party's valuation depends on others' information in the same way). While Gresik [1991] studies the second-best allocation, he does not consider the role of a mixed ownership structure, which is the main subject of our paper. Fieseler et al. [2000] analyze the conditions under which ex-post efficiency can or cannot be achieved, and they allow for mixed ownership. However, when ex-post efficiency cannot be achieved they provide no characterization of the second best. Their main result is that when the valuations of partners are decreasing functions of the others' signals, ex-post efficiency can be achieved for balanced ownership structure, but not necessarily otherwise.[6] It should be mentioned that in the symmetric setup of Fieseler et al. [2000], even if ex-post efficiency cannot be achieved, an equal splitting of ownership between the parties would minimize the subsidy required to achieve the first-best.[7] Thus, the derivation of our main insight requires some asymmetry between the parties. It should also be mentioned that our paper is

[6] This insight is in line with our insight that when valuations vary co-monotonically the ex-post efficient allocation cannot be achieved (see Corollary 1).

[7] The latter point is an easy extension of Fieseler et al. [2000]'s analysis (even though it is not covered by their paper).

the first attempt to characterize the second best in a partnership with mixed ownership structure and interdependent values (in which the first best cannot be achieved).

Papers that deal with the issue of property rights in the private values setting include, apart from CGK, Schweizer [1998] and Neeman [1999]. Schweizer [1998] shows that even when the symmetry assumption in CGK is relaxed, there is always an allocation of property rights that permits to reach the first best.[8] Neeman [1999] studies a public-goods setting with private values. He shows that the incentives for truthful revelation are satisfied only in an intermediate range of property rights allocations. The reason is that when the property rights allocation is extreme, each agent knows whether he is going to be a net seller or a net buyer. In the former case he can free-ride on the other members of his group by overstating his valuation, while in the latter case he will tend to understate it. When the ownership structure is mixed, the incentives to lie upward or downward cancel out since, a priory, he may be either a seller or a buyer.

Our paper has also close connections with the literature on countervailing incentives (see, e.g., Lewis and Sappington [1989], Maggi and Rodriguez-Clare [1995], Mezzetti [1997], Jullien [2000]), since the reservation utility of the informed party depends on his private information through his shares. Some features of the second best bear some resemblance with the insights developed in that literature. For example, the participation constraints are binding for intermediate values of the type realization, and for an interval of type realization, the second best requires pooling in the form of no trade.[9]

The remainder of the paper is organized as follows. In section 2 we develop the model and present the mechanism-design problem for trade in property rights at the stage after the informed party has acquired her information. Section 3 presents conditions under which the first-best solution can be achieved and studies the role of subsidies that permit efficient trade. In section 4 we characterize the second-best mechanism and study how the welfare it achieves depends on the initial ownership structure. Concluding remarks appear in Section 5.

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[8] This best allocation of property rights need not be the balanced structure. Yet, Myerson and Satterthwaite [1983]'s impossibility result implies that it is never an extreme allocation of property rights.

[9] See Maggi and Rodriguez-Clare [1995] for a thoughtful investigation of pooling in the countervailing-incentives literature. It should be emphasized though that in our case the no trade result appears because the first best would entail that only two possible outcomes be considered (either allocating full ownership to the informed party or to the uninformed party). In contrast, in the agency context considered by the contervailing-incentives literature, many more possibilities can arise.

2 Model and Preliminary Results

Two risk-neutral agents, I (informed) and U (uninformed) jointly own a partnership. Initially, agent I's share is β and agent U's share is $1-\beta$. Agent I learns the realization of the state w, which determines the value of the firm to agents I and U. Then, the agents can trade shares in exchange for monetary payments, reaching the final allocation $(\beta^F, 1-\beta^F)$. Agent I's value of owning a fraction β^F of the firm is $\phi(\beta^F)\cdot w$. Agent U's value of owning a fraction $1-\beta^F$ of the firm is $\phi(1-\beta^F)\cdot f(w)$. We assume that agent U does not acquire additional information before all monetary payments are performed (in particular, monetary transfers must be completed before U observes her payoff from her new share $1-\beta^F$).[10]

The function $f(\cdot)$ is assumed to be differentiable. The function $\phi(\cdot)$ is assumed to be increasing and convex, and w.l.o.g. satisfies $\phi(0)=0$, $\phi(1)=1$. The convexity of $\phi(\cdot)$ reflects increasing returns to scale with respect to the ownership share. When ϕ is more convex, the potential gains from the dissolution of the partnership are larger. (The extreme case in which $\phi(\beta)\equiv\beta$ is the one considered in CGK setting. In that case, an agent's share can be interpreted as the probability that she will receive the entirety of the firm.) Finally, state w is assumed to be drawn from the interval $[a,b]$ according to a probability distribution function $g(\cdot)$ and cumulative distribution $G(\cdot)$. Functions $f(\cdot)$, $\phi(\cdot)$ and $g(\cdot)$ are common knowledge.

We are interested in the analysis of the negotiation between agents I and U at the stage where only agent I knows w. In the tradition of Myerson and Satterthwaite (1983) and CGK, we model the negotiation as a mechanism designed to maximize the ex-ante welfare of agents I and U subject to incentive and participation constraints. By the revelation principle there is no loss of generality in restricting attention to direct truthful mechanisms.[11] We will thus consider mechanisms of the form $\{\gamma(w), T(w)\}$: agent I reveals her type w; following the announcement, $\gamma(w)$ is the net transfer of shares from U to I $(1-\beta\geq\gamma(w)\geq-\beta)$, and $T(w)$ is the monetary transfer from U to I. Voluntary participations and truth telling are part of an equilibrium if:

ICI: Agent I prefers to report her true type w (for any w):

$$\forall w, \widetilde{w}, \quad w\phi(\gamma(w)+\beta)+T(w)\geq w\phi(\gamma(\widetilde{w})+\beta)+T(\widetilde{w}).$$

[10] While this assumption is quite restrictive in many real-life contexts, it permits to apply standard mechanism-design tools and enables the comparison of our results to those of the mainstream adverse-selection literature. For a discussion of this issue, see the concluding section.

[11] This holds because we have assumed that agent U could does not obtain information related to w before monetary transfers ore implemented.

IRI: Agent I agrees to participate (for any w):

$$\forall w, \quad w\phi(\gamma(w)+\beta)+T(w)\geq w\phi(\beta).$$

IRU: Agent U agrees to participate:

$$E_w\left[f(w)[\phi(1-\beta-\gamma(w))-\phi(1-\beta)]-T(w)\right]\geq 0.$$

The optimal negotiation mechanism maximizes the expected welfare:

$$EW(\beta)=\int_{w=a}^{b}[\phi(\beta+\gamma(w))\,w+\phi(1-\beta-\gamma(w))\,f(w)]dG(w)$$

subject to ICI, IRI and IRU.

The following lemma follows from agent I's incentive-compatibility constraint:

Lemma 1

(i) $\gamma(w)$ is weakly increasing.

(ii) For any w, $\lim_{\varepsilon\to 0}T(w+\varepsilon)-T(w)=\lim_{\varepsilon\to 0}-w(\phi(\gamma(w+\varepsilon)+\beta)-\phi(\gamma(w)+\beta))$.

(iii) If γ is differentiable at w, then $T^{'}(w)\equiv -w\cdot\phi'(\gamma(w)+\beta)\cdot\gamma'(w)$.

(iv) There is a constant c such that for all w, $T(w)=\int_{x=a}^{w}\phi(\gamma(x)+\beta)dx-w\phi(\gamma(w)+\beta)+c$.

Proof. See Appendix.

Because of the convexity of $\phi(\cdot)$, full efficiency dictates allocating the full ownership to the agent who has the higher valuation. Denote:

$$\begin{aligned} W^I &= \{w\in[a,b]\mid w\geq f(w)\} \\ W^U &= \{w\in[a,b]\mid f(w)\geq w\} \end{aligned}$$

to be the sets of states in which agents I and U, respectively, value the partnership more. From now on we assume that $W^I\cap W^U$ has measure 0 (given $g(\cdot)$).

We are interested in how the optimal expected welfare is affected by the ex-ante ownership structure β. Our main focus is on whether $EW(\beta)$ is maximized at an extreme initial ownership structure, i.e. at $\beta=0$ or at $\beta=1$, or at an interior ownership structure (as in CGK's private values setting). We start by verifying whether the first-best outcome can be achieved, and if so, for which values of β.

3 Feasibility of the first-best outcome

In this section we study whether and for which initial ownership structures the first best can be achieved as the outcome of the negotiation. When the first best is not attainable, we verify whether it can be reached with the help of an external subsidy, and how the required subsidy depends on the ownership structure.

Because of the (weak) convexity of ϕ, it is optimal to give the full ownership to the agent who values the partnership more. Therefore, the first-best allocation rule is $\gamma(w) = 1 - \beta$ for w in W^I and $\gamma(w) = -\beta$ for w in W^U. Since agent I's incentive-compatibility constraints imply that the allocation rule $\gamma(w)$ must be non decreasing (see Lemma 1), a pre-condition for efficiency is that I's valuation be higher than U's when the state w is above some threshold w^{eff}. We will say that f satisfies the single-crossing (SC) condition if the set W^I is of the form $\left[w^{eff}, b\right]$ for some $w^{eff} \in [a, b]$. We thus have:[12]

Proposition 1 *If f does not satisfy single crossing, the first best cannot be achieved – with or without a subsidy – whatever the ownership structure β.*

When the single-crossing condition does hold, the first-best outcome is attainable, but sometimes only with the help of an external subsidy. The corresponding allocation rule is:

$$\gamma^{FB}(w) = \begin{cases} -\beta & \text{for } w < w^{eff} \\ 1 - \beta & \text{for } w > w^{eff} \end{cases}$$

By Lemma 1, the monetary transfer T must be constant when γ is. We will compute the *minimal* transfers that satisfy agent I's IR and IC constraints (recall our convention that the transfers are always from U to I). We will then check whether U's IR constraint is satisfied.

In the range $w < w^{eff}$, agent I is conceding his β share in exchange for a fixed sum of money. Thus, the condition IRI: $(w\phi(0) + T(w) \geq w\phi(\beta))$ is hardest to satisfy when he has the highest valuation. This yields the condition $T(w) \geq w^{eff}\phi(\beta)$. In the range $w > w^{eff}$, I receives U's $1 - \beta$ share. Here, the condition $(w\phi(1) + T(w) \geq w\phi(\beta))$ is hardest to satisfy when I has the lowest valuation, yielding

[12] Proposition 1 echoes on a result by now familiar in the literature on efficient mechanism design with interdependent valuations, see Maskin [1992], Dasgupta-Maskin [2000], Jehiel-Moldovanu [2001], Perry-Reny [2002], Ausubel [1999], Bergemann-Valimaki [2002] for more general expressions of it.

$T(w) \geq w^{eff}(\phi(\beta) - 1)$. Thus, the minimal transfers to I are:

$$\min T^{FB}(w) = \begin{cases} \phi(\beta)w^{eff} & \text{for } w < w^{eff} \\ (\phi(\beta) - 1)w^{eff} & \text{for } w > w^{eff} \end{cases}$$

(It can easily be verified that that the mechanism $(\gamma^{FB}(w),\ \min T^{FB}(w))$ satisfies ICI: if w is below w^{eff}, I will not report that it is above and vice versa.) Under this mechanism, U's expected payoff *loss* is:[13]

$$\begin{aligned} S(\beta) \ = \ & \int_{w=a}^{w^{eff}} [\phi(\beta)w^{eff} - (1 - \phi(1 - \beta))f(w)] \ dG(w) + \\ & \int_{w=w^{eff}}^{b} [(\phi(\beta) - 1)w^{eff} - (0 - \phi(1 - \beta))\, f(w)] \ dG(w) \end{aligned} \tag{1}$$

We thus have:

Proposition 2 *Suppose that f satisfies single crossing. Then:*

(i) The first best can be achieved if $S(\beta) \leq 0$.

(ii) If $S(\beta) > 0$, the first best can be achieved iff there is a subsidy $S \geq S(\beta)$.

Remark: The mechanism $(\gamma^{FB}(w),\ \min T^{FB}(w))$ can be interpreted as giving agent I a menu of two options: sell shares at a price of $p^{sell} = \frac{\phi(\beta)}{\beta}w^{eff}$ $(\leq w^{eff})$ per share, or buy shares at a price of $p^{buy} = \frac{1-\phi(\beta)}{1-\beta}w^{eff}$ $(\geq w^{eff})$ per share. Given this menu, I chooses to sell all his shares if w is below w^{eff} and to buy all of U's shares if it is above. (In the case of a linear ϕ, the buy and sell prices are the same: w^{eff}.) Note that this mechanism has the worst possible prices for I. In the case where $S(\beta)$ is strictly negative, U will agree to trade under prices that are better for I (higher p^{sell} and lower p^{buy}).[14]

The last proposition can be used to characterize the range of ownership structures β for which the first best is attainable (without a subsidy). We first note that if f is non-increasing, it satisfies single crossing and $S(\beta)$ must be negative.[15] Thus:

Corollary 1 *If f is non-increasing, the first best can be achieved whatever the ownership structure β.*

We continue with the case in which f is increasing. One can verify that $S''(\beta) > 0$. Thus, $S(\beta) \leq 0$ in an interval of $\beta \in [c, d] \subseteq [0, 1]$ (the interval might be a singleton or empty). We can rewrite $S(\beta)$

[13] Recall that U's share grows from $1 - \beta$ to 1 below w^{eff}, and shrinks to 0 above.

[14] We thank one of the referees for suggesting this interpretation.

[15] This follows from (1), because $\phi(\beta) + \phi(1 - \beta) \leq 1$ by the convexity of ϕ, and because $f(w) \geq w^{eff}$ iff $w \leq w^{eff}$ by the monotonicity of f.

as:

$$S(\beta) = \phi(\beta)X + \phi(1-\beta)Y - Z$$

where

$$\begin{aligned} X &= \int_{w=a}^{b} w^{eff} dG(w), \\ Y &= \int_{w=a}^{b} f(w) dG(w), \\ Z &= \int_{w=a}^{w^{eff}} f(w) dG(w) + \int_{w=w^{eff}}^{b} w^{eff} dG(w). \end{aligned}$$

Consider first the case in which $a < w^{eff} < b$. It is easy to see that $Z < X, Y$ (since $f(w^{eff}) = w^{eff}$ and f is increasing). Thus, $S(\beta)$ can be non-positive only if $\phi(\beta) + \phi(1-\beta)$ is sufficiently below 1. This can happen only for $\beta \neq 0, 1$, and only if ϕ is sufficiently convex. Next, consider the case in which $w^{eff} = a$ (i.e., I's valuation is always higher). In this case we have $Z = X$ and thus $S(1) = 0$. Consequently, $S(\beta)$ is negative in some interval $[c, 1]$. (Note that when $\beta = 1$, the initial allocation is already the first best and no trade is needed.) Similarly, in the case of $w^{eff} = b$ (U's valuation is always higher), the first best is feasible for β in an interval $[0, d]$. These observations are summarized in the next two corollaries, which characterize the initial ownership structures for which the first-best allocation can be achieved without subsidies:

Corollary 2 *Suppose that f is increasing and satisfies single crossing.*

(i) If $w^{eff} = a$, the first best can be achieved in an interval $\beta \in [c, 1]$ where $0 \leq c \leq 1$.

(ii) If $w^{eff} = b$, the first best can be achieved in an interval $\beta \in [0, d]$ where $0 \leq d \leq 1$.

(iii) If $a < w^{eff} < b$, the first best might not be feasible. If it is, it can be achieved in an interval $\beta \in [c, d]$ where $0 < c \leq d < 1$.

Note that when $\phi(\beta) \equiv \beta$, $S(\beta)$ is linear in β. We thus have:

Corollary 3 *Suppose that f is increasing and satisfies single crossing, and assume that there are constant returns to scale ($\phi(\beta) \equiv \beta$). Then:*

(i) If $w^{eff} = a$, the first best can be achieved for any β if $Ef(w) \leq a$; otherwise it cannot be achieved for any $\beta < 1$.

(ii) If $w^{eff} = b$, the first best can be achieved for any β if $Ef(w) \geq b$; otherwise it cannot be achieved for any $\beta > 0$.

(iii) If $a < w^{eff} < b$, the first best cannot be achieved. In this case, the subsidy required to restore the first best is minimized when agent I has full ownership ($\beta = 1$) – if $Ef(w) \geq w^{eff}$, or when agent U has full ownership ($\beta = 0$) – if $Ef(w) \leq w^{eff}$.

This last result stands in sharp contrast with CGK's result for the case of private values. Note that the incentive to dissolve the partnership comes from two sources. The first is to give as large a share as possible to the agent who values the partnership more. The second is to exploit increasing returns to scale. When we eliminate the second incentive (as in CGK), then the first best is infeasible whatever the ownership structure (unless one party is known to always value the partnership more). Moreover, the subsidy that is required to restore the first best is minimized at an extreme initial property-rights allocation. In CGK the opposite conclusion holds: the first best is always feasible for some property-rights structure, and moreover, the range of property rights at which the first best is feasible is centered around the equal-shares allocation.

Our result that the minimal subsidy is obtained at an extreme property-rights allocation holds also for a strictly convex ϕ (i.e., when $\phi'(0)/\phi'(1)$ is strictly less than 1), if $Ef(w)$ and w^{eff} are far enough from each other:

Corollary 4 *Suppose that f is strictly increasing and satisfies single crossing.*

1. $S(1) = Min_\beta S(\beta)$ *whenever* $\frac{w^{eff}}{Ef(w)} < \frac{\phi'(0)}{\phi'(1)}$.
2. $S(0) = Min_\beta S(\beta)$ *whenever* $\frac{w^{eff}}{Ef(w)} > \frac{\phi'(1)}{\phi'(0)}$.

If neither condition holds, it may well be that $S(\beta)$ is minimized for an interior ex-ante share. In other words, corollary 4 says that minimal subsidy required to restore the first best is achieved at an extreme value of β if $\frac{w^{eff}}{Ef(w)}$ falls outside the range $[\frac{\phi'(0)}{\phi'(1)}, \frac{\phi'(1)}{\phi'(0)}]$. If ϕ is not very convex, that range is small, and thus it is likely that $S(\beta)$ is minimized at an extreme initial allocation. This is always the case if ϕ is linear (Corollary 3).

Finally, we verify what happens if there are stronger returns to scale, so that the incentive to dissolve the partnership is increased. Note that if $\widehat{\phi}$ is more convex than ϕ (i.e., $\widehat{\phi}(\beta) = h(\phi(\beta))$ for a strictly convex function h satisfying $h(0) = 0$, $h(1) = 1$), then the corresponding function $\widehat{S}(\beta)$ is strictly below $S(\beta)$ for any $\beta \neq 0, 1$. We thus have:

Corollary 5 *Suppose that f is increasing and satisfies single crossing, and assume that $\widehat{\phi}$ is more convex than ϕ. If the interval in which the first best can be attained under ϕ is non-empty, then the interval in which it can be attained under $\widehat{\phi}$ is strictly larger. Moreover, for any β for which first best is not feasible without a subsidy, the subsidy that is required to restore the first best is lower under $\widehat{\phi}$.*

4 The Second Best

In this section we study the optimal allocation rule when the first best is not attainable and no external subsidy is available. In contrast to the previous section, in which our task was to check whether the known allocation rule $\gamma^{FB}(w)$ (giving full ownership to the agent who values the partnership more) can be accompanied by monetary transfers that make it satisfy IR and IC constraints, now we need to find the allocation rule itself. We look for the rule that generates the highest surplus, subject to these constraints. Once we identify this second-best allocation rule, we will make welfare comparisons between the outcomes for different initial ownership structures β.

4.1 Characterization of the (second-best) optimal allocation rule

Denote the agents' "subjective" changes in ownership when the share γ is transferred from U to I (and I's initial share is β) by:

$$\begin{aligned}\Delta_I^{\beta}(\gamma) &= \phi(\beta+\gamma)-\phi(\beta), \text{ and}\\ \Delta_U^{\beta}(\gamma) &= \phi(1-\beta-\gamma)-\phi(1-\beta).\end{aligned}$$

The following proposition characterizes the second-best solution:

Proposition 3 *The highest (second-best) expected welfare $EW(\beta)$ is the solution to the program:*

$$EW(\beta) = \underset{\gamma(\cdot)}{Max} \int_{w=a}^{b} \left[\phi(\beta+\gamma(w))w + \phi(1-\beta-\gamma(w))f(w)\right] dG(w)$$

subject to $\gamma(\cdot)$ weakly increasing and satisfying

$$\begin{aligned}&\int_{w=a}^{w^*}\left[\Delta_I^{\beta}(\gamma(w))\left(w+\frac{G(w)}{g(w)}\right)+\Delta_U^{\beta}(\gamma(w))f(w)\right]\gamma(w)dG(w)+\\ &\int_{w=w^{**}}^{b}\left[\Delta_I^{\beta}(\gamma(w))\left(w-\frac{1-G(w)}{g(w)}\right)+\Delta_U^{\beta}(\gamma(w))f(w)\right]\gamma(w)dG(w)\geq 0\end{aligned}$$

where[16]

$$\begin{aligned}w^* &= \sup\{w:\gamma(w)<0\}\\ w^{**} &= \inf\{w:\gamma(w)>0\}\end{aligned}$$

Proof. See Appendix.

[16] *If $\gamma(w)\geq 0$ for all w, let $w^*=a$. If $\gamma(w)\leq 0$ for all w, let $w^{**}=b$.*

The intuition for this proposition is as follows. The types for whom the participation constraint is binding lie in the interval (w^*, w^{**}) in which there is no trade. The virtual valuation (i.e. accounting for the informational rent to be given for truthful revelation) for a type $w < w^*$ who is a net seller is $w + \frac{G(w)}{g(w)}$. The virtual valuation for a type $w > w^{**}$ who is a net buyer is $w - \frac{1-G(w)}{g(w)}$. The expression for the constraint in the above programme follows.

4.2 Bang-Bang allocation rule

In Section 3 we saw that in those cases in which the first-best solution is infeasible, the maximal payments that U is willing to make are lower than the minimal payments demanded by some of I's types. More precisely, when I's valuation is slightly below w^{eff}, he demands a too high price for his shares, and when his valuation is slightly above w^{eff} he is willing to pay too little for U's shares.

While the types of I whose constraints are binding are the types around w^{eff}, it makes sense to assume that the surplus that is created when they trade is smallest. (Recall that at w^{eff}, both parties have the same valuation.) This is captured by Assumption 1, below, which implies that the difference between U and I's valuations is increasing as we move away from w^{eff}. We will show that the optimal allocation rule is to give up the trades when w is close to w^{eff}, while giving full ownership to the party with higher valuation when w is sufficiently far away from w^{eff} By doing that we relax I's most severe IR constraints, while minimizing the lost surplus. We will call such an allocation rule "Bang-Bang": either full ownership is given to one of the parties, or no trade takes place at all. Formally, we make the following assumptions:

Assumption 1: $f(\cdot)$ is strictly increasing and satisfies $f'(w) < \frac{\phi'(0)}{\phi'(1)}$ for all w.[17]

Assumption 2: $G(w)/g(w)$ is increasing and $(1 - G(w))/g(w)$ is decreasing.

Assumption 1 is a strengthening of the single-crossing condition. It is somewhat stronger than required, but it allows us to simplify our analysis. It says that the rate at which U's valuation increases with I's signal w is positive but bounded from above by the constant $\frac{\phi'(0)}{\phi'(1)}$ (which is less than 1). This may, for example, reflect the idea that I's information is a better proxy for her own valuation.[18]

[17] Due to the convexity of ϕ, this implies (by the intermediate value theorem) that $f'(w) < \frac{\phi(\beta)}{1-\phi(1-\beta)}$ for all β. Since $\frac{\phi(\beta)}{1-\phi(1-\beta)} < \frac{1-\phi(\beta)}{\phi(1-\beta)}$, we have $f'(w) < \frac{\phi(\beta)}{1-\phi(1-\beta)}, \frac{1-\phi(\beta)}{\phi(1-\beta)}$ for all β, which is really what is needed for our argument. Note that if we were to consider only discrete values of final shares β, then this condition need apply only for these values of β.

[18] To see this more formally, consider the case in which each party's valuation is the sum of a commom-value component, v_c, and a private-value component, v_I or v_U. While I observes $w = v_c + v_I$, U observes nothing. If v_c, v_I and v_U are independent, the correlation between w and $f(w) = v_c + v_U$ is less than 1. Thus, for an increase of ε in w, the expected increase in $f(w)$ is less than ε.

Assumption 2 is a standard technical assumption in implementation theory.

4.2.1 When one party's valuation is always higher

We start with the simpler case in which one party (either U or I) values the partnership more for all w (Akerlof's Lemon's model falls in this category). Consider first the case in which agent I values the partnership more for all w. The first-best allocation rule dictates the transfer of all of U's $1-\beta$ shares to I for any w. Under this rule, the binding IR constraint of I pertains to lowest type, $w=a$, who is willing to pay $(1-\phi(\beta))a$ for U's $1-\beta$ shares. The first best cannot be attained if this is less than U's valuation of the shares he gives up, $\phi(1-\beta)Ef(w)$, i.e., if $\frac{a}{Ef(w)} < \frac{\phi(1-\beta)}{1-\phi(\beta)}(<1)$.

The way to satisfy agent U's IR constraint is to raise the share price from $\frac{1-\phi(\beta)}{1-\beta}a$ to $\frac{1-\phi(\beta)}{1-\beta}\underline{w}$ for some sufficiently high $\underline{w}$. The drawback is that I's types below $\underline{w}$ do not agree to buy U's shares, and some surplus is lost. The second-best solution is achieved when we choose the lowest $\underline{w}$ for which I's valuation is at least as high as U's ex-ante valuation:[19]

$$w_I = \min\left\{\underline{w} \text{ } s.t. \text{ } (1-\phi(\beta))\cdot \underline{w} \geq \phi(1-\beta)\cdot E[f(w)|w \geq \underline{w}]\right\}.$$

In the opposite case in which U's valuation is always higher, a symmetric argument applies. (Note that since now I sells shares in some range $[a, \overline{w}]$, the binding IR constraint pertains to his *highest* type $\overline{w}$ who values them by $\phi(\beta)\overline{w}$; U's valuation for these shares is $(1-\phi(1-\beta))\, E[f(w)|w \leq \overline{w}]$.)

This intuition leads to the following proposition:

Proposition 4 *Let $f(\cdot)$ and $g(\cdot)$ satisfy assumptions 1 and 2.*

(i) If $w \geq f(w)$ for all w, the optimal assignment function is:

$$\gamma(w) = \begin{cases} 0 \quad for \quad w < w_I \\ 1-\beta \quad for \quad w \geq w_I \end{cases}$$

where w_I satisfies

$$w_I = \min\left\{\underline{w} \text{ } s.t. \text{ } \underline{w} \geq \frac{\phi(1-\beta)}{1-\phi(\beta)} E[f(w)|w \geq \underline{w}]\right\}$$

[19] It should be noted that when we increase $\underline{w}$, U's expected valuation of the shares he gives up is increased. This is because, conditional on the sale taking place, w is in the range $[\underline{w}, b]$. However, since $f' < 2$ (this is implied by assumption 1), the expected valuation increases in $\underline{w}$ at a rate of less than 1. Thus, the difference between I's valuation ($\underline{w}$) and U's ($E[f(w)|w \geq \underline{w}]$) shrinks in $\underline{w}$. This shows that the solution, if it exists, is unique. To see why existence holds, note that at $\underline{w} = b$ the inequality is satisfied: the LHS is at least 1 ($b > f(b)$) and the RHS is at most 1.

(ii) If $f(w) \geq w$ for all w, the optimal assignment function is:

$$\gamma(w) = \begin{cases} -\beta & \textit{for} \;\; w < w_u \\ 0 & \textit{for} \;\; w \geq w_u \end{cases}$$

where w_U satisfies

$$w_U = \max\left\{\overline{w} \;\, s.t. \;\; \overline{w} \leq \frac{1-\phi(1-\beta)}{\phi(\beta)} E[f(w)|w \leq \overline{w}]\right\}.$$

Proof. Same as for the general case (see below). ■

Remark 1: Part (ii) of the proposition deals with a Lemons-type situation (Akerlof [1970]): The potential buyer (who is uninformed about the quality of the good) always has a higher valuation than the seller (who is informed).[20] While in Akerlof's model the seller initially owns the entire good ($\beta = 1$), the proposition also admits situations of mixed initial ownership ($\beta < 1$).

Remark 2: When the first-best cannot be achieved (i.e., when $w_I \neq a$ or $w_U \neq b$ in the above expressions) then the mixed ownership structure is strictly dominated by the extreme ownership structure which gives all shares to the party who values the good most. In the case of constant returns to ownership ($\phi(\beta) \equiv \beta$) if the first-best cannot be achieved for *some* mixed ownership structure then it can never be achieved for *any* mixed ownership structure,[21] and the only optimal ownership structure is the one that allocates all shares to the party who values the partnership more.

4.2.2 The general case

We now turn to the general case in which, depending on the realization of w, either agent I or agent U may have a higher valuation of the good. The following proposition characterizes the optimal second-best solution when the first best cannot be achieved:

Proposition 5 *Let $f(\cdot)$ and $g(\cdot)$ satisfy Assumptions 1 and 2. The second-best assignment function is*

$$\gamma(w) = \begin{cases} -\beta & \textit{for} \quad w < w^* \\ 0 & \textit{for} \quad w^* < w < w^{**} \\ 1-\beta & \textit{for} \quad w > w^{**} \end{cases} \tag{2}$$

[20] While many variations of the Lemon's problem fit our model, Akerlof's original formulation violates Assumption 1 (since $f'(w) = 1.5 > 1$). Nonetheless, one can verify that the proposition holds also in this case.

[21] When ϕ is linear $\frac{\phi(1-\beta)}{1-\phi(\beta)} \equiv \frac{1-\phi(1-\beta)}{\phi(\beta)} \equiv 1$ and w_I, w_U are independent of β.

where (w^*, w^{**}) *is the smallest interval*[22] *s.t.* $a < w^* < w^{eff}$ *and* $b > w^{**} > w^{eff}$ *satisfy:*

$$\int_a^{w^*} [(1-\phi(1-\beta))\, f(w) - \phi(\beta) w^*] dG(w) + \int_{w^{**}}^{b} [(1-\phi(\beta))\, w^{**} - \phi(1-\beta) f(w)] dG(w) = 0 \qquad (3)$$

and

$$\left(\frac{1-\phi(1-\beta)}{\phi(\beta)} f(w^*) - w^*\right) \frac{g(w^*)}{G(w^*)} = \left(w^{**} - \frac{\phi(1-\beta)}{1-\phi(\beta)} f(w^{**})\right) \frac{g(w^{**})}{1-G(w^{**})} \qquad (4)$$

Proof. See Appendix.

The intuition resembles that of Proposition 4. In some range around w^{eff} there is no trade, so that I's IR constraints are relaxed. Condition 3 corresponds to the conditions in Proposition 4. In the range $[a, w^*]$, agent U buys I's β share at the minimal amount that I's type w^* would accept: $\phi(\beta)w^*$. This share is worth $(1-\phi(1-\beta))\, f(w)$ to him. In the range $[w^*, b]$, agent U sell his $1-\beta$ share to I for the maximal amount that I's type w^{**} is willing to pay: $(1-\phi(\beta))\, w^{**}$. This share is worth $\phi(1-\beta) f(w^{**})$ to him. The condition is that the expected net gain from trade to agent U be zero. Condition 4 trades off the two options for relaxing I's IR constraints – increasing w^{**} or decreasing w^*. At the margin, the welfare loss in both options (corrected by the rate at which the constraint is relaxed) should be the same.

Remark: The second best allocation rule can be interpreted as giving agent I the choice among the following three alternatives: don't trade, buy all U's shares at price $P^{buy} = (1-\phi(\beta))w^{**}$ or sell all I's shares at (the lower) price $P^{sell} = \phi(\beta)w^*$. I's optimal choice will then be: sell all his shares if $w < w^*$, buy all of U's shares if $w > w^{**}$, and not trade if w is in between. (Note that in the case where the first best was attainable, we had $w^* = w^{**} = w^{eff}$.)

4.3 Efficiency and the initial share

When a partnership is formed, there are various factors that may affect the desired distribution of shares. One example, which has been given considerable attention in the literature, is moral hazard. Consider again the start-up company example. If the first stage of development requires considerable effort on the part of the inventor, moral-hazard considerations would push towards giving him a large

[22] *Observing that when w^* gets close to a, the corresponding w^{**} satisfying (4) must get close to b, it is readily verified from the theorem of intermediate values that there must exist w^*, w^{**} satisfying (3) and (4). Taking the smallest such interval defines it uniquely.*

initial share. Our results call for another important consideration: adverse-selection effects in future trade.

In devising the initial shares, β and $1-\beta$, the parties should take into account their effect on future trade in ownership. That is, knowing that adverse-selection problems will govern future trade (due to the fact that one party is about to receive better information than the other regarding the value of the partnership), the parties should try to minimize the expected welfare loss from adverse selection. In this paper we abstract from all other aspects, and focus on the adverse-selection effect. That is, we investigate how the initial shares β and $1-\beta$ affect the total welfare that is reached after the optimal negotiation mechanism is applied.

Recall that the incentive to dissolve the partnership comes from two sources. The first is to allocate the shares to the party who values the partnership more. That is, party I for $w > w^{eff}$ and party U for $w < w^{eff}$. The second is the increasing returns in the share of ownership: if ϕ is strictly convex, then for $0 < \beta < 1$ we have $\phi(\beta) + \phi(1-\beta) < 1 = \phi(1)$. Obviously, if ϕ is extremely convex, an intermediate initial ownership structure will oblige agents to agree to trade. (When $\phi(\beta)$ and $\phi(1-\beta)$ are very low, both agents' IR constraints are easy to satisfy.) Thus, in such a case, an intermediate initial ownership structure is preferred to an extreme one. However, as we saw in Section 3, when ϕ is less convex the first best may no longer be attainable even for intermediate values of β. The question then arises as to which value of β maximizes $EW(\beta)$. Our main insight will be to identify a class of situations in which $EW(\beta)$ is maximal for an extreme allocation of shares, i.e, for $\beta = 0$ or $\beta = 1$.

The class of situations in which we obtain that the best allocation of shares is extreme assumes that there are nearly constant returns in the ownership share, i.e. $\phi(\cdot)$ is almost linear. To simplify the exposition, we will present our results for the linear case, i.e. $\phi(\beta) \equiv \beta$ for all β. But it should be understood that our results (in particular Proposition 7) hold true when $\phi(\cdot)$ is not exactly linear, but close to it.[23]

As we saw in Proposition 5, for w below w^* or above w^{**} the best allocation is reached: the party who values the partnership more obtains full ownership. In the middle range, $[w^*, w^{**}]$ no trade takes place and the original shares remain. When we change the initial shares, there are two effects. The first is that the range of inefficiency $[w^*, w^{**}]$ changes. The second is that within that range, the final allocation is changed (because it is the same as that dictated by the initial shares).

We first study the effect of the initial shares on the range of inefficiency. Let $w^*(\beta)$ and $w^{**}(\beta)$ be

[23] The maximization problem of Proposition 3 is continuous with respect to $\phi(\cdot)$.

the thresholds defined in Proposition 5 when the initial share of I is β. We first observe that these thresholds vary monotonically either in the direction of w^{eff} or in the opposite direction as β varies from 0 to 1:

Proposition 6 *Let $f(\cdot)$ and $g(\cdot)$ satisfy Assumptions 1 and 2, and suppose that $\phi(\beta) \equiv \beta$. The functions $w^*(\beta)$ and $w^{**}(\beta)$ are both monotonic. They move in opposite directions (i.e. if one is increasing, the other is decreasing). Moreover, $w^*(\beta)$ is increasing (and $w^{**}(\beta)$ decreasing) whenever:*

$$\int_a^{w^*(0)} [w^*(0) - f(w)]dG(w) < 0. \tag{5}$$

or equivalently, whenever:

$$\int_{w^{**}(1)}^b [w^{**}(1) - f(w)]dG(w) > 0. \tag{6}$$

Proof. See Appendix.

Proposition 6 shows that efficient trade occurs with largest probability for an extreme initial property-rights allocation, either $\beta = 0$ or 1, depending on the parameter configuration. However, the welfare conditional on no trade (in the range $(w^*(\beta), w^{**}(\beta))$) depends on β, since the parties maintain their initial shares. Conditional on having no trade, the corresponding conditional expected welfare for an initial property-rights allocation β is given by

$$\frac{1}{G(w^{**}) - G(w^*)} \int_{w^*}^{w^{**}} [\beta w + (1-\beta)f(w)]dG(w).$$

This conditional welfare is also maximized for an extreme β, but not necessarily the same as the one that maximizes the probability of trade. If it is the same, we can conclude that an extreme property-rights structure is best:

Proposition 7 *Let $f(\cdot)$ and $g(\cdot)$ satisfy Assumptions 1 and 2, and suppose that $\phi(\beta) \equiv \beta$. If*

$$\int_a^{w^*(0)} [w^*(0) - f(w)]dG(w) > 0 \tag{7}$$

and

$$\int_{w^*(0)}^{w^{**}(0)} [f(w) - w]dG(w) > 0. \tag{8}$$

Then

$$EW(0) > EW(\beta) \text{ for all } \beta > 0. \tag{9}$$

Similarly, if

$$\int_{w^{**}(1)}^{b} [w^{**}(1) - f(w)]dG(w) > 0 \tag{10}$$

and

$$\int_{w^{*}(1)}^{w^{**}(1)} [w - f(w)]dG(w) > 0. \tag{11}$$

Then

$$EW(1) > EW(\beta) \text{ for all } \beta < 1. \tag{12}$$

Proof. See Appendix.

4.4 Naive resolution

In this section we study the welfare properties of *naive resolution* of the partnership: An early round of trade (prior to information acquisition by agent I) in which full ownership is allocated by lottery to one of the parties, with probabilities that equal the initial shares.[24] While the expected welfare when the initial shares are β and $1-\beta$ is $EW(\beta)$, the expected welfare due to naive resolution is $\beta EW(1) + (1-\beta)EW(0)$. The example, described by CGK, of the FCC's proposal to allocate spectrum bands by lottery (rather than selling the licences to a cartel of the buyers in which each owns some share), is in fact a proposal for naive resolution. CGK showed that in their symmetric private values case, naive resolution can be counter productive. They showed that for all β in a neighborhood of $1/2$, the first-best can be achieved, while for extreme initial shares it cannot. This implies that for such β, $EW(\beta) > \beta EW(1) + (1-\beta)EW(0)$.

We now verify how the two possibilities compare in our interdependent values setting. We start with the case of constant returns in ownership share ($\phi(\cdot)$ linear). In contrast with CGK's private-values case, there are cases where naive dissolution does as well as the initial mixed ownership for any initial share β (although it can never strictly dominate the initial ownership). We then move to the case of increasing returns ($\phi(\cdot)$ convex). Here, it may well be that maintaining a mixed ownership till the negotiation phase is strictly dominated by the naive dissolution scenario.

The constant returns case:

With linear $\phi(\cdot)$, a naive dissolution does not outperform the initial mixed ownership structure because the mixed ownership problem can at worst be decomposed into a convex combination of extreme

[24] A negotiation governed by Proposition 4 would follow the initial allocation of shares.

ownership problems. (With convex $\phi(\cdot)$, the decomposition would induce inefficiencies, see below). Formally,

Proposition 8 *Let $f(\cdot)$ and $g(\cdot)$ satisfy Assumptions 1 and 2, and suppose that $\phi(\beta) \equiv \beta$. Then:*

$$EW(\beta) \geq \beta EW(1) + (1-\beta)EW(0).$$

Proof: Let $\widehat{EW}(\beta)$ be the expected welfare when I's share is 0 for $w \leq w_U$, β for $w_U < w < w_I$, and 1 for $w > w_I$ where $w_U = w^*(1)$ and $w_I = w^{**}(0)$ (see Proposition 5).[25] Clearly, $\widehat{EW}(\beta) = \beta EW(1) + (1-\beta)EW(0)$ (because the no-trade range does not vary with β). Furthermore, observe that $\widehat{EW}(\beta)$ can be viewed as the expected welfare obtained when party I can sell her shares at a per share price of w_U or buy U's shares at a per share price of w_I, and party U can accept or reject the deal *after* observing the selling/purchasing decision of party I. (In equilibrium, party I will sell all his shares when $w < w_U$ and buy all U' shares when $w > w_I$, and whatever the deal chosen by party I, party U will be indifferent as to whether to accept or reject the deal, see the definitions of w_I and w_U). Since $\widehat{EW}(\beta)$ can be achieved through some mechanism, we may conclude that $EW(\beta) \geq \widehat{EW}(\beta)$ and thus $EW(\beta) \geq \beta EW(1) + (1-\beta)EW(0)$. ■

Remark 1: The above observation implies that $\widehat{EW}(\beta)$ is the solution to the same program as in Proposition 5, but with a more stringent constraint: that each of the two summands in condition (3) equal 0. This condition means that agent U must be indifferent when he buys I's shares (segment $[a, w^*]$) at the lowest price in which I's type w^* is willing to sell, and must also be indifferent when he sells his shares (segment $[w^{**}, b]$) at the highest price in which I's type w^{**} is willing to buy. In contrast, when we compute $EW(\beta)$, condition (3) in Proposition 5 only requires that agent U be indifferent on average under these two possibilities; cross subsidy between the two types of trade – positive or negative – is allowed (and condition (4) means that all gains from cross subsidy are exploited). Thus, for $\beta \neq 0, 1$, $EW(\beta)$ is strictly higher than $\widehat{EW}(\beta)$ as soon as $w_U \neq w^*(\beta)$ (or equivalently $w_I \neq w^{**}(\beta)$). If $w_U = w^*(\beta)$, then $EW(\beta) = \widehat{EW}(\beta)$. (Thus, if w_U and w_I satisfy condition (4) in Proposition 5, then $EW(\beta) = \widehat{EW}(\beta)$. This is the case in our linear-uniform example below.)

Remark 2: Even though not stated in CGK, the conclusion that naive dissolution cannot strictly dominate would also arise in the private value setup whether or not the first-best can be achieved with

[25] That is, $w_I = \min\{\underline{w} \text{ s.t. } \underline{w} \geq E[f(w)|w \geq \underline{w}]\}$ and $w_U = \max\{\overline{w} \text{ s.t. } \overline{w} \leq E[f(w)|w \leq \overline{w}]\}$.

mixed ownership. This is because the decomposition idea works, as long as there are constant returns to ownership.

We now observe that in contrast with CGK, there is a class of cases in which naive resolution performs equally well for all β. (Another class of cases with this property appears in section 4.5.)

Claim 1: Let $f(\cdot)$ and $g(\cdot)$ satisfy Assumptions 1 and 2, and suppose that $\phi(\beta) \equiv \beta$. If $f(w) > w$ for all w, or if $w > f(w)$ for all w, then

$$EW(\beta) = \beta EW(1) + (1-\beta)EW(0).$$

Proof: Because $\phi(\cdot)$ is linear, $\frac{\phi(1-\beta)}{1-\phi(\beta)} \equiv \frac{1-\phi(1-\beta)}{\phi(\beta)} \equiv 1$. Thus, the range of states w in which the seller's share is transferred to the buyer is independent of β (see the expressions of w_I and w_U in Proposition 4). This implies that $EW(\beta) = \beta EW(1) + (1-\beta)EW(0)$.[26] ■

The increasing returns case:

We now turn to analyze the effect of increasing returns in the ownership share. Starting from a linear $\phi(\cdot)$ and moving to a slightly strictly convex $\phi(\cdot)$, the welfare from naive resolution is unchanged. However, (for an interior initial allocation of shares β) two opposing effects arise: on the one hand, there is a wider range of w for which there is efficient trade (this is because the individual rationality constraints become less stringent); on the other hand, in the range of w in which there is no trade, the outcome is relatively worse in the mixed ownership case than in the random extreme ownership structure. From these two effects, it is not difficult to see that if we start from a situation where with constant returns $EW(\beta)$ is equal (or close) to $\beta EW(1)+(1-\beta)EW(0)$, then when we move to a slightly strictly convex $\phi(\cdot)$ the comparison can go either way depending on the parameters of the problem.[27] This discussion is summarized in the following claim:

Claim 2: Let $f(\cdot)$ and $g(\cdot)$ satisfy Assumptions 1 and 2, and suppose that $\phi(\beta)$ is convex. Then it may be that:

$$EW(\beta) < \beta EW(1) + (1-\beta)EW(0).$$

[26] Observe that it need not be known which party will be the one who values the partnership more for (**??**) to hold. What is required is that at the time of the naive dissolution it is known that when the negotiation comes about parties know who values the partnership more.

[27] For example, consider the situation in which agent I values the partnership more and consider a slight convexification of ϕ starting from the linear specification. If $g(\cdot)$ puts sufficient weight on (a, w_I) maintaining the mixed ownership would be dominated by the naive dissolution scenario.

4.5 A linear, uniform example:

Finally, we work out a simple example with a uniform distribution of w on $[0,1]$, a linear valuation function $f(w) = \mu w + \nu$, and with constant returns in the ownership share. We wish to verify the effect of the initial ownership structure on the final expected welfare.

Proposition 9 *Let w be uniformly distributed on $[0,1]$ and let $f(w) = \mu w + \nu$ where $0 < \mu, \nu < 1$, $\mu + \nu < 1$. Assume that $\phi(\beta) = \beta$. The functions $w^*(\beta)$ and $w^{**}(\beta)$ are constant, where:*

$$\begin{aligned} w^*(\beta) &= w^* = \frac{\nu}{1-(\mu/2)} \\ w^{**}(\beta) &= w^{**} = \frac{(\mu/2)+\nu}{1-(\mu/2)} \end{aligned}$$

$EW(\beta)$ is maximized at $\beta = 1$ (full ownership of the informed party) whenever $E(w - f(w))$ is positive. $EW(\beta)$ is maximized at $\beta = 0$ (full ownership of the uninformed party) whenever $E(w - f(w))$ is negative. This is also the ownership structure for which the subsidy required to attain the first best is minimal. Besides,

$$EW(\beta) = \beta EW(1) + (1-\beta)EW(0).$$

Proof. See Appendix.

The linear, uniform case has special properties that one should not expect in general. For example, there is no reason to expect in general that the best allocation coincide with the best ex-ante allocation when no trade is allowed.[28] Similarly, there is no reason in general to expect that the best allocation is also such that the subsidy required to achieve the first-best is minimal.[29] However, the observation that the best allocation structure is an extreme one will emerge in a number of scenarios as illustrated in Section4.3.[30]

[28] To see this, consider the case where $f(\cdot)$ is replaced by a new function that is the same over $[a, w^*]$ and $[w^{**}, b]$, but different over (w^*, w^{**}). This would not affect the conditions characterizing (w^*, w^{**}) and thus we would still have $EW(0) < EW(1)$ whenever $E[w - f(w) \mid w \in (w^*, w^{**})] > 0$. However, in this modified case, one may well have $E[w - f(w) \mid w \in (w^*, w^{**})] > 0$ and $E(w - f(w)) < 0$.

[29] $S(1) < S(0)$ whenever $E(f(w) - w^{eff}) > 0$, and this is compatible with $E[w - f(w) \mid w \in (w^*, w^{**})] < 0$ when $f(\cdot)$ is modified in the range (w^*, w^{**}).

[30] By perturbing slightly the example we could have a situation in which naive dissolution is dominated (see Claim 1) and yet the best initial share is either 0 or 1.

5 Concluding Remarks

We have studied the case of partnership dissolution when the valuations are interdependent and only one party is (fully) informed about the valuations. This model generalizes Akerlof's "Lemons" model by allowing for general property-rights structures and by allowing situations in which each agent may have a higher valuation at some states.

In contrast with the private-values case studied by CGK and Neeman [1999], we saw that an extreme initial property-rights structure can be preferred to mixed ownership. More precisely, in our setup a mixed ownership structure can only be desirable if there are sufficient increasing returns in ownership (i.e., $\phi(\cdot)$ sufficiently convex). Otherwise, when there are no or little increasing returns in ownership (so that the incentive to dissolve the partnership comes only from the attempt to give full ownership to the agent who values the partnership more) the first best cannot be attained if a-priori each party might have a higher valuation. In this case: (1) the subsidy required to reach the first-best allocation is minimized at one of the extreme initial allocations, and (2) the second-best allocation yields the highest total surplus at one of the extreme initial allocations. This means that an early round of trade (before the informed party learns the state of the world), in which one party obtains full ownership, can be welfare improving. This message is consistent with that of Akerlof's model, and stands in contrast to insight from the case of private values, in which an early round of trade can be counter productive.

In our analysis, we have pursued a mechanism-design approach. However, the second-best outcome of Proposition 3 can be achieved with a decentralized procedure – rather than by using a mechanism. Specifically, in our model, the trading mechanism takes a very simple form: there are two prices, P^{buy} and P^{sell}, and the informed party has to decide (according to the realization of w) whether she prefers to sell all her shares to the uninformed party for a price P^{sell}, or buy all the shares of the uninformed parties for a price P^{buy}, or not trade at all. When the first best is not feasible these two prices must differ. It is worth observing that if party I sets the prices P^{buy} and P^{sell} *before* she learns the realization w of the state of the world, then she will pick up the optimal prices as determined in Section 4, thus providing a simple decentralized procedure that implements the second best.[31]

In this paper we have assumed that agent U does not receive information regarding the state of the world w before the allocation and monetary transfers are finalized. There are cases in which it is

[31] The reason is that the second-best mechanism is such that (i) the ex-ante welfare of the two parties is maximized (in particular over all possible choices of P^{buy} and P^{sell}) and (ii) the participation constraint of the uninformed party U is binding. These two conditions ensure that party I will choose P^{buy} and P^{sell} exactly as in the optimal mechanism-design programme. Of course, such a scheme heavily relies on the feature that only one party receives private information. The issue of decentralization in the more general two-sided case is left for future research.

realistic to assume that some information about w may become available to agent U, for example, due to an increase in his share. Our analysis would not be the same in such an environment. Mezzetti [2004] showed that in a general social-choice problem with interdependent values (but with no participation constraints), if agents can observe their payoffs after the choice of alternative has been implemented, a two-stage mechanism, in which the allocation is decided first and monetary transfers are made only after agent observe their payoffs, can always implement the first-best outcome – unlike one-stage mechanisms. One may conjecture that such a two-stage mechanism might lead to better outcomes also in our framework if we assume that payoff information can be obtained before monetary transfers are finalized. (Such reasoning could potentially explain some real-life contracts that delay part of the monetary transfers - e.g., through clauses such as royalty payments).

We chose to abstract from this issue and focus on the extreme case in which no additional information arrives. In doing so, our results can more easily be put in the perspective of the mainstream literature on adverse selection – in particular Akerlof [1970]. An interesting topic for future research could be to analyze the case in which agent U receives additional information when his share varies (and check whether the above conjecture holds). However, one should note that there is a large gap between Mezzetti's framework and ours. Apart from the issue of participation constraints, one must address the question of how the information that U obtains after the trade in shares is related to w. One possibility is that an increase in the share of U results in a better signal of the true w. A reasonable model of the relationship between the level of ownership and the quality of the signal is needed. Another possibility is that after some time (long time?) U learns his payoff from his share (whatever this share is). But, it is important to note that payoff information might only be a noisy indicator of I's information w. For example, if w represents I's expectations of future profits, their realization (say, after U acquired full ownership) would only be a noisy signal of w. If the additional payoff information received by U is a poor indicator of I's original information w, a two-stage mechanism is presumably of little help. (Note that Mezzetti's assumption that all relevant information is spanned by the players' types is violated in such a stochastic environment.)[32]

In this paper we have emphasized the idea that the allocation of shares between two differently informed partners may affect the efficiency of the partnership dissolution due to subsequent adverse-selection considerations. The corporate-finance literature offers a different perspective, on the effect of letting the partners or managers own some shares of the company they belong to. In a moral-hazard

[32] We thank an anonymous referee for pointing out the relevance and importance of Mezzetti's [2004] insight.

setup in which the manager must make some effort or investment, that literature suggests that letting the manager owns more shares is an instrument to align the manager's interest with that of the firm. It would clearly be interesting to mix the adverse-selection considerations developed in this paper with the moral-hazard considerations developed in the corporate-finance literature.[33]

[33] For a review of the corporate finance literature see, for example, Tirole (1998).

Appendix

Proof of Lemma 1:

(i) By ICI, for any w_1, w_2 we must have $w_1(\phi(\gamma(w_1)+\beta)-\phi(\gamma(w_2)+\beta)) \geq T(w_2)-T(w_1)$ and $w_2(\phi(\gamma(w_1)+\beta)-\phi(\gamma(w_2)+\beta)) \leq T(w_2)-T(w_1)$. Hence, $w_1(\phi(\gamma(w_1)+\beta)-\phi(\gamma(w_2)+\beta)) \geq w_2(\phi(\gamma(w_1)+\beta)-\phi(\gamma(w_2)+\beta))$ or $(w_1-w_2)(\phi(\gamma(w_1)+\beta)-\phi(\gamma(w_2)+\beta)) \geq 0$. Since ϕ is strictly increasing, γ must be (weakly) increasing.

(ii) Setting $w_2 = w+\varepsilon$ and $w_1 = w$ in the above two inequalities for $T(w_2)-T(w_1)$ we obtain: $w(\phi(\gamma(w)+\beta)-\phi(\gamma(w+\varepsilon)+\beta)) \geq T(w+\varepsilon)-T(w) \geq (w+\varepsilon)(\phi(\gamma(w)+\beta)-\phi(\gamma(w+\varepsilon)+\beta))$.

(iii) When $\gamma(w)$ is continuous at w, we can divide the inequality in (ii) by ε. Taking the limit as $\varepsilon \to 0$ we obtain $T'(w) = \lim_{\varepsilon\to 0} \frac{T(w+\varepsilon)-T(w)}{\varepsilon} = \lim_{\varepsilon\to 0} w\frac{\phi(\gamma(w)+\beta)-\phi(\gamma(w+\varepsilon)+\beta)}{\varepsilon} = w\frac{\partial(\phi(\gamma(w)+\beta)-\phi(\gamma(w+\varepsilon)+\beta))}{\partial\varepsilon}$.

(iv) If there are points at which γ is not differentiable in the interval (a, w), denote them by $w_1 ... w_n$. Denote also $w_0 = a$ and $w_{n+1} = w$. For sufficiently small $\varepsilon > 0$, we can write:

$$\begin{aligned}
T(w) \;&=\; T(a)+T(a+\varepsilon)-T(a) \\
&\quad +\sum_{i=1}^{n+1}[T(w_i-\varepsilon)-T(w_{i-1}+\varepsilon)]+\sum_{i=1}^{n}[T(w_i+\varepsilon)-T(w_i-\varepsilon)] \\
&\quad +T(w)-T(w-\varepsilon) \\
&=\; T(a)+T(a+\varepsilon)-T(a) \\
&\quad +\sum_{i=1}^{n+1}\left[\int_{x=w_{i-1}+\varepsilon}^{w_i-\varepsilon} T'(x)dx\right]+\sum_{i=1}^{n}[T(w_i+\varepsilon)-T(w_i-\varepsilon)] \\
&\quad +T(w)-T(w-\varepsilon)
\end{aligned}$$

applying parts (ii) and (iii) of the lemma and taking the limit as $\varepsilon \to 0$, we obtain:

$$\begin{aligned}
T(w) \;&=\; T(a)-a[\phi(\gamma(a^+)+\beta)-\phi(\gamma(a)+\beta)] \\
&\quad +\sum_{i=1}^{n+1}\left[\int_{x=w_{i-1}}^{w_i} -x\cdot\phi'(\gamma(x)+\beta)\cdot\gamma'(x)dx\right]+\sum_{i=1}^{n}\left[-w_i[\phi(\gamma(w_i^+)+\beta)-\phi(\gamma(w_i^-)+\beta)]\right] \\
&\quad -w[\phi(\gamma(w)+\beta)-\phi(\gamma(w^-)+\beta)],
\end{aligned}$$

where, for any w, $\gamma(w^-)$ and $\gamma(w^+)$ denote $\lim_{v\to w, v<w}\gamma(v)$ and $\lim_{v\to w, v>w}\gamma(v)$, respectively. Integrating by parts, we obtain:

$$\begin{aligned}
T(w) &= T(a) - a\phi(\gamma(a^+)+\beta) + a\phi(\gamma(a)+\beta) \\
&\quad + \sum_{i=1}^{n+1}\left[\int_{x=w_{i-1}}^{w_i} \phi(\gamma(x)+\beta)dx - w_i \cdot \phi(\gamma(w_i^-)+\beta) + w_{i-1}\cdot\phi(\gamma(w_{i-1}^+)+\beta)\right] \\
&\quad - \sum_{i=1}^{n} w_i[\phi(\gamma(w_i^+)+\beta) - \phi(\gamma(w_i^-)+\beta)] \\
&\quad - w\phi(\gamma(w)+\beta) + w\phi(\gamma(w^-)+\beta) \\
&= T(a) + a\phi(\gamma(a)+\beta) - w\phi(\gamma(w)+\beta) + \sum_{i=1}^{n+1}\left[\int_{x=w_{i-1}}^{w_i} \phi(\gamma(x)+\beta)dx\right] \\
&= c - w\phi(\gamma(w)+\beta) + \int_{x=a}^{w} \phi(\gamma(x)+\beta)dx
\end{aligned}$$

where $c = T(a) + a\phi(\gamma(a)+\beta)$. ■

Proof of Proposition 3:

By Lemma 1 (iv), condition ICI holds if and only if:

$$T(w) = \int_{x=a}^{w} \phi(\gamma(x)+\beta)dx - w\phi(\gamma(w)+\beta) + c'$$

for some constant c' or equivalently if

$$T(w) = \int_{x=a}^{w} (\phi(\gamma(x)+\beta) - \phi(\beta))dx - w(\phi(\gamma(x)+\beta) - \phi(\beta)) + c$$

for some constant c.

Plugging this expression into IRI we obtain:

$$\forall w \quad \int_{x=a}^{w} (\phi(\gamma(x)+\beta) - \phi(\beta))dx + c \geq 0$$

The derivative of the expression with respect to w is $\Delta_I^\beta(\gamma(w)) = \phi(\gamma(w)+\beta) - \phi(\beta)$. This is negative if $\gamma(w) < 0$ and positive if $\gamma(w) > 0$. Since $\gamma(w)$ is non-decreasing (see Lemma 1), the derivative is negative below w^*, positive above w^{**} and 0 in between. Thus, the condition is satisfied for all w if and only if it is satisfied for w^*:

$$\text{I.} \quad \int_{x=a}^{w^*} \Delta_I^\beta(\gamma(x))dx + c \geq 0$$

Plugging the expression for $T(w)$ into IRU ($E_w[f(w)\Delta_U^\beta(\gamma(w)) - T(w)] \geq 0$), we obtain:

$$\text{II.} \quad \int_{w=a}^{b} \left[f(w)\Delta_U^\beta(\gamma(w)) + w\Delta_I^\beta(\gamma(w)) - \int_{x=a}^{w} \Delta_I^\beta(\gamma(x))dx \right] dG(w) - c \geq 0.$$

There is a constant c satisfying I and II iff:

$$\int_{w=a}^{b} \left[f(w)\Delta_U^\beta(\gamma(w)) + w\Delta_I^\beta(\gamma(w)) - \int_{x=a}^{w} \Delta_I^\beta(\gamma(x))dx \right] dG(w) + \int_{x=a}^{w^*} \Delta_I^\beta(\gamma(x))dx \geq 0 \tag{13}$$

Now, $\int_{w=a}^{b}\int_{x=a}^{w} \Delta_I^\beta(\gamma(x))dxdG(w) = \int_{x=a}^{b} \Delta_I^\beta(\gamma(x))(1-G(x))\,dx$.

Noting that $\Delta_U^\beta(\gamma(w)) = \Delta_I^\beta(\gamma(w)) = 0$ for w lying in (w^*, w^{**}), we get:

$$\begin{aligned} & -\int_{w=a}^{b}\int_{x=a}^{w} \Delta_I^\beta(\gamma(x))dxdG(w) + \int_{x=a}^{w^*} \Delta_I^\beta(\gamma(x))dx \qquad (14)\\ = & -\int_{x=a}^{w^*} \Delta_I^\beta(\gamma(x))G(x)dx + \int_{x=w^{**}}^{b} \Delta_I^\beta(\gamma(x))(1-G(x))dx \\ = & -\int_{x=a}^{w^*} \Delta_I^\beta(\gamma(x))\frac{G(x)}{g(x)}dG(x) + \int_{x=w^{**}}^{b} \Delta_I^\beta(\gamma(x))\frac{(1-G(x))}{g(x)}dG(x) \end{aligned}$$

and

$$\begin{aligned} & \int_{w=a}^{b} \left[f(w)\Delta_U^\beta(\gamma(w)) + w\Delta_I^\beta(\gamma(w)) \right] dG(w) \qquad (15)\\ = & \int_{w=a}^{w^*} \left[f(w)\Delta_U^\beta(\gamma(w)) + w\Delta_I^\beta(\gamma(w)) \right] dG(w) + \int_{w=w^{**}}^{b} \left[f(w)\Delta_U^\beta(\gamma(w)) + w\Delta_I^\beta(\gamma(w)) \right] dG(w) \end{aligned}$$

Adding up (14) and (15) to rewrite inequality (13) we obtain the desired result. ■

Proof of Proposition 5:

Denoting the Lagrange multiplier of the constraint in Proposition **3** by $\lambda \geq 0$, the Hamiltonian of the maximization program can be written as:

$$\begin{aligned} &(1+\lambda)\int_{w=a}^{w^*} \{\Delta_I^\beta(\gamma(w)) \left[w + \frac{\lambda}{(1+\lambda)}\frac{G(w)}{g(w)} \right] + \Delta_U^\beta(\gamma(w))f(w)\}dG(w) + \\ &(1+\lambda)\int_{w=w^{**}}^{b} \{\Delta_I^\beta(\gamma(w)) \left[w - \frac{\lambda}{(1+\lambda)}\frac{1-G(w)}{g(w)} \right] + \Delta_U^\beta(\gamma(w))f(w)\}dG(w) \end{aligned}$$

Let $\gamma(w)$ be the function that maximizes the Hamiltonian within the domain of non-decreasing functions (recall that $\gamma(w)$ must be non-decreasing – Lemma 1).

Consider the first range ($w < w^*$). Here, by definition of w^*, $\gamma(w) < 0$. We will show that for each w in this range $\gamma(w)$ must be at the lowest feasible value $-\beta$. For $w < w^*$ the integrand in the Hamiltonian is:

$$\Delta_I^\beta(\gamma)\left[w + \frac{\lambda}{(1+\lambda)}\frac{G(w)}{g(w)}\right] + \Delta_U^\beta(\gamma) f(w).$$

We can rewrite this expression as:

$$\Delta_I^\beta(\gamma) \cdot H(\gamma, w),$$

where

$$H(\gamma, w) \equiv w - \frac{\phi(1-\beta) - \phi(1-\beta-\gamma)}{\phi(\beta+\gamma) - \phi(\beta)} f(w) + \frac{\lambda}{(1+\lambda)}\frac{G(w)}{g(w)}.$$

Due to the convexity of ϕ, it is readily verified that $\frac{\phi(1-\beta)-\phi(1-\beta-\gamma)}{\phi(\beta+\gamma)-\phi(\beta)}$ is (weakly)decreasing in γ. Thus, H is increasing in γ for all w. H is also strictly increasing in w for all γ since $\frac{\phi(1-\beta)-\phi(1-\beta-\gamma)}{\phi(\beta+\gamma)-\phi(\beta)}$ is no more than $\frac{\phi'(1)}{\phi'(0)}$ and thus by assumption 1 the derivative of the second term with respect to w is less than 1, and since the third term is increasing by assumption 2. Note also that $\Delta_I^\beta(\gamma)$ is (weakly) increasing in γ and $\Delta_I^\beta(0) = 0$.

Suppose it was the case that for some $\widehat{w}$, $-\beta < \gamma(\widehat{w}) < 0$. If $H(\gamma(\widehat{w}), \widehat{w}) > 0$, then $\Delta_I^\beta(\gamma) \cdot H(\gamma, w)$ would be strictly negative for all $\widehat{w} \le w < w^*$. But then we could set $\gamma(w)$ to be 0 in that range; this will increase the Hamiltonian without violating the constraint that $\gamma(w)$ is non-decreasing – contradiction to the assumed optimality of $\gamma(w)$. If $H(\gamma(\widehat{w}), \widehat{w}) \le 0$, then both $\Delta_I^\beta(\gamma)$ and $H(\gamma, w)$ are negative for all $0 \le w \le \widehat{w}$. But then the Hamiltonian would increased if we set $\gamma(w)$ to be $-\beta$ in that range (again, keeping $\gamma(w)$ non-decreasing. Thus, it can only be the case that $\gamma(\widehat{w}) = -\beta$.

A very similar argument shows that in the range $w > w^{**}$, the only value of $0 < \gamma(w) \le 1 - \beta$ that can be optimal is $\gamma(w) = 1 - \beta$. Therefore, the optimal solution is of the form

$$\gamma(w) = \begin{cases} -\beta & \text{for} \quad w < w^* \\ 0 & \text{for} \quad w^* < w < w^{**} \\ 1-\beta & \text{for} \quad w > w^{**} \end{cases}$$

Now, the constraint of the maximization program must be binding and together with (2) yields after an integration by parts condition (3). Finally, assuming an interior solution, maximization with respect to w^* and w^{**} yields:

$$w^* - \frac{1-\phi(1-\beta)}{\phi(\beta)} f(w^*) + \frac{\lambda}{(1+\lambda)}\frac{G(w^*)}{g(w^*)} = 0 \tag{16}$$

$$w^{**} - \frac{\phi(1-\beta)}{1-\phi(\beta)} f(w^{**}) - \frac{\lambda}{(1+\lambda)} \frac{1-G(w^{**})}{g(w^{**})} = 0. \tag{17}$$

Conditions (16) and (17) together yield condition (4) which is always met for some $w^* < w^{eff} < w^{**}$. ■

Proof of Proposition 6:

a) When $\phi(\beta) \equiv \beta$, condition (4) from proposition 5 is:

$$(f(w^*) - w^*) \frac{g(w^*)}{G(w^*)} = (w^{**} - f(w^{**})) \frac{g(w^{**})}{1-G(w^{**})}. \tag{18}$$

Since $w^* \rightarrow [-w^* + f(w^*)] \frac{g(w^*)}{G(w^*)}$ is decreasing for $w^* \leq w^{eff}$ and $w^{**} \rightarrow [w^{**} - f(w^{**})] \frac{g(w^{**})}{1-G(w^{**})}$ is increasing for $w^{**} \geq w^{eff}$, we immediately obtain that if $w^*(\beta') > w^*(\beta)$, then $w^{**}(\beta') < w^{**}(\beta)$.

b) Consider β and its optimal solution $\gamma(\cdot)$, which is a solution to (rewrite Proposition 5 for the case $\phi(\beta) \equiv \beta$):

$$Max_{\gamma(\cdot)} \int_{w=a}^{b} (w - f(w)) \gamma(w) dG(w) \tag{19}$$

subject to

$$-\beta \int_{a}^{w^*} [w - f(w) + \frac{G(w)}{g(w)}] dG(w) + (1-\beta) \int_{w^{**}}^{b} [w - f(w) - \frac{1-G(w)}{g(w)}] \gamma(w) dG(w) \geq 0. \tag{20}$$

Consider $\beta' = \beta + \varepsilon > \beta$ and suppose that

$$-\int_{a}^{w^*(\beta)} [w - f(w) + \frac{G(w)}{g(w)}] dG(w) - \int_{w^{**}(\beta)}^{b} [w - f(w) - \frac{1-G(w)}{g(w)}] \gamma(w) dG(w) \geq 0. \tag{21}$$

The optimal allocation rule for β also satisfies constraint (20) of the program with β'. Using the inverse monotonicity relations observed in step a and the fact that condition (20) is binding, this allows us to conclude that:[34]

$$w^{**}(\beta) > w^{**}(\beta') > w^*(\beta') > w^*(\beta).$$

Note that condition (21) is equivalent to

$$\int_{a}^{w^*(\beta)} [w - f(w) + \frac{G(w)}{g(w)}] dG(w) < 0,$$

since at the optimum, constraint (20) is binding.

c) Suppose condition (5) holds. Step b above shows that $w^*(\beta)$ is increasing as long as $\int_a^{w^*(\beta)} [w -$

[34] With β' the range of w with efficient trade must be larger with β.

$f(w) + \frac{G(w)}{g(w)}]dG(w) < 0$ or given the monotonicity of $w - f(w) + \frac{G(w)}{g(w)}$ as long as $w^*(\beta) \leq w^*(1)$ (which is precisely defined by $\int_a^{w^*(1)}[w - f(w) + \frac{G(w)}{g(w)}]dG(w) = 0$).

Finally, observe that if $w^*(\beta) > w^*(1)$ for some β, then there must exist $0 < \beta' < \beta$ such that $w^*(\beta') > w^*(1)$ and $w^*(\cdot)$ is locally increasing at β'. But this would contradict step b, since $w^*(\beta') > w^*(1)$ implies $\int_a^{w^*(\beta')}[w - f(w) + \frac{G(w)}{g(w)}]dG(w) > 0$. ■

Proof of Proposition 7:

We prove here the first claim (the proof of the second is analogous). By Proposition 6, condition (7) guarantees that $w^*(\beta_1)$ and $w^{**}(\beta_1)$ are respectively decreasing and increasing functions of β_1. Thus, for all β_1

$$w^*(\beta_1) < w^*(0) < w^{eff} < w^{**}(0) < w^*(\beta_1).$$

Together with (8), this implies (9) (consider the intervals $(a, w^*(\beta_1))$, $(w^*(\beta_1), w^*(0))$, $(w^*(0), w^{**}(0))$, $(w^{**}(0), w^*(\beta_1))$, and $(w^*(\beta_1), b)$ separately). ■

Proof of Proposition 9:

Applying proposition 5, simple calculations show that $w^*(\beta)$ and $w^{**}(\beta)$ are respectively: $w^*(\beta) = w^* = \frac{\nu}{1-(\mu/2)}$ and $w^{**}(\beta) = w^{**} = \frac{(\mu/2)+\nu}{1-(\mu/2)}$. Thus, the final allocation of shares is independent of β for $w < w^*$ and $w > w^{**}$. In between for $w \in (w^*, w^{**})$ there is no trade whatever β, so the initial allocation remains. This in turn yields:

$$EW(\beta) = \beta EW(1) + (1 - \beta)EW(0).$$

Besides, in order to maximize $EW(\beta)$ it is optimal to allocate full ex ante ownership to the agent who values the partnership more in the range $w \in (w^*, w^{**})$. That is, agent I should get full ownership whenever $\int_{w^*}^{w^{**}} wdw > \int_{w^*}^{w^{**}} f(w)dw$ and agent U get full ownership in the opposite holds. One can verify that $\int_{w^*}^{w^{**}} [w - f(w)]\, dw = \frac{\mu^2}{8[1-(\mu/2)]}(1 - \mu - 2\nu)$. Thus, what matters is the sign of $1 - \mu - 2\nu$, and we get the desired result by observing that $E(w - f(w)) = \frac{1}{2}(1 - \mu - 2\nu)$. As for the minimal subsidy, recall (Corollary 3) that the minimal subsidy is lower at $\beta = 0$ if $E(f(w) - w^{eff})$ is positive and at $\beta = 1$ if it is negative. We obtain the same condition since $w^{eff} = \frac{\nu}{1-\mu}$ and thus $E(f(w) - w^{eff}) = \frac{\mu}{1-\mu}\frac{1}{2}(1 - \mu - 2\nu)$.

■

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