Online Appendix to
"On the Permanent Nature of Affirmative Action Policies"

Philippe Jehiel*
Paris School of Economics
& University College London

Matthew V. Leduc†
Paris School of Economics
& Université Paris 1

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Abstract

In Section 1 of this online appendix, we present proofs of the three lemmas of the main paper. In Section 2, we present a generalized model in which we allow for strategic behavior by workers (Section 2.1), for affirmative action causing a labor market congestion externality (Section 2.2) and we microfound the wage-setting behavior of firms with Bertrand competition (Section 2.3). Section 2.4 includes the proofs of these additional results.

*Email: philippe.jehiel@gmail.com

†(Corresponding author) Email: mattvleduc@gmail.com. Mailing address: Paris Jourdan Sciences Economiques, 48 boulevard Jourdan, 75014 Paris, France. Tel.: (+33) 01 80 52 16 60

1 Proofs omitted from the main paper

We now provide proofs of Lemmas 1, 2 and 3, which we left out of the main paper due to space limitations.

Proof of Lemma 1.

Note that $\omega_t^*(\bar{c})$ is the conditional expectation of the worker's actual performance level at time t when declaring a curriculum vitae of quality $\bar{c}$ and when the equilibrium policy is σ^*. Thus,

$$\begin{aligned}
\omega_t^*(\bar{c}) &= \mathbb{E}_t[c|\bar{c},\sigma^*] \\
&= \mathbb{P}_t^*(\{aa\}|\bar{c}) \cdot g^{-1}(\bar{c}) + \big(1 - \mathbb{P}_t^*(\{aa\}|\bar{c})\big) \cdot \bar{c}
\end{aligned}$$

Now to express $\mathbb{P}_t^*(\{aa\}|\bar{c})$, we first express $\mathbb{P}_t^*(\{aa\}|\tilde{c} \in N(\bar{c},\epsilon))$, where $N(\bar{c},\epsilon)$ is an ϵ-neighborhood of $\bar{c}$:

$$\begin{aligned}
\mathbb{P}_t^*(\{aa\}|\tilde{c} \in N(\bar{c},\epsilon)) &= \frac{\mathbb{P}_t^*(\{\tilde{c} \in N(\bar{c},\epsilon)\} \bigcap \{aa\})}{\mathbb{P}_t^*(\tilde{c} \in N(\bar{c},\epsilon))} \\
&= \frac{\mathbb{P}_t^*(\tilde{c} \in N(\bar{c},\epsilon) \bigcap B) \cdot \sigma_t^*}{\mathbb{P}_t^*(\tilde{c} \in N(\bar{c},\epsilon))} \\
&= \frac{\mathbb{P}_t^*(\tilde{c} \in N(\bar{c},\epsilon)|B) \cdot \mathbb{P}(B) \cdot \sigma_t^*}{\mathbb{P}_t^*(\tilde{c} \in N(\bar{c},\epsilon))}
\end{aligned}$$

$$= \frac{\sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\bar{c}),\epsilon/g'^{-1}(\bar{c}))} f_{B,n_t}(g^{-1}(\tilde{c}))d\tilde{c}\frac{|B|}{|A|+|B|}}{\int_{\tilde{c} \in N(\bar{c},\epsilon)} f_A(\tilde{c})d\tilde{c}\frac{|A|}{|A|+|B|} + (1-\sigma_t^*)\int_{\tilde{c} \in N(\bar{c},\epsilon)} f_{B,n_t}(\tilde{c})d\tilde{c}\frac{|B|}{|A|+|B|} + \sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\bar{c}),\epsilon/g'^{-1}(\bar{c}))} f_{B,n_t}(g^{-1}(\tilde{c}))d\tilde{c}\frac{|B|}{|A|+|B|}}$$

$$= \frac{|B|\sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\bar{c}),\epsilon/g'^{-1}(\bar{c}))} f_{B,n_t}(g^{-1}(\tilde{c}))d\tilde{c}}{|A|\int_{\tilde{c} \in N(\bar{c},\epsilon)} f_A(\tilde{c})d\tilde{c} + |B|(1-\sigma_t^*)\int_{\tilde{c} \in N(\bar{c},\epsilon)} f_{B,n_t}(\tilde{c})d\tilde{c} + |B|\sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\bar{c}),\epsilon/g'^{-1}(\bar{c}))} f_{B,n_t}(g^{-1}(\tilde{c}))d\tilde{c}}$$

Then, we take the limit as $\epsilon \to 0$:

$$\begin{aligned}
\mathbb{P}_t^*(\{aa\}|\tilde{c}) &= \lim_{\epsilon \to 0} \mathbb{P}_t^*(\{aa\}|\tilde{c} \in N(\bar{c},\epsilon)) \\
&= \lim_{\epsilon \to 0} \frac{|B|\sigma_t^* f_{B,n_t}(g^{-1}(\bar{c}))2\epsilon/g'^{-1}(\bar{c})}{|A|f_A(\bar{c})2\epsilon + |B|(1-\sigma_t^*)f_{B,n_t}(\bar{c})2\epsilon + |B|\sigma_t^* f_{B,n_t}(g^{-1}(\bar{c}))2\epsilon/g'^{-1}(\bar{c})} \\
&= \frac{|B|\sigma_t^* f_{B,n_t}(g^{-1}(\bar{c}))/g'^{-1}(\bar{c})}{|A|f_A(\bar{c}) + |B|(1-\sigma_t^*)f_{B,n_t}(\bar{c}) + |B|\sigma_t^* f_{B,n_t}(g^{-1}(\bar{c}))/g'^{-1}(\bar{c})}
\end{aligned}$$

■

Proof of Lemma 2.

From Lemma 1 we know that

$$\omega_t^*(\bar{c}) = \mathbb{P}_t^*(\{aa\}|\bar{c}) \cdot g^{-1}(\bar{c}) + \big(1 - \mathbb{P}_t^*(\{aa\}|\bar{c})\big) \cdot \bar{c}$$

Part (i): When $\sigma^* = 1$, then $\mathbb{P}_t^*(\{aa\}|\bar{c}) > 0$. Since $g^{-1}(\bar{c}) < \bar{c}$, it follows immediately that $g^{-1}(\bar{c}) < \omega_t^*(\bar{c}) < \bar{c}$. Thus, if the worker does not benefit from affirmative action (i.e. $c = \bar{c}$), then $\omega_t^*(\bar{c}) < c$ and he gets a wage lower than his performance level. On the other hand, if the worker benefits from affirmative action (i.e. $c = g^{-1}(\bar{c})$), then $c < \omega_t^*(\bar{c})$ and he gets a wage higher than his performance level.

Part (ii): When $\sigma^* = 0$, then $\mathbb{P}_t^*(\{aa\}|\bar{c}) = 0$. Thus, $\omega_t^*(\bar{c}) = \bar{c}$ and $\bar{c} = c$ since no one benefits from affirmative action. ■

Proof of Lemma 3.

First note that for any group $G \in \{A, B\}$,

$$\sum_{t=1}^{\infty} \delta^t W_{G,t} = \sum_{t=1}^{\tau-1} \delta^t W_{G,t} + \delta^{\tau} W_{G,\tau} + \delta^{\tau+1} W_{G,\tau+1} + \sum_{t=\tau+2}^{\infty} \delta^t W_{G,t},$$

where only the terms $\delta^{\tau} W_{G,\tau}$ and $\delta^{\tau+1} W_{G,\tau+1}$ are different under policies σ versus σ'. We thus only need to compare these two terms under the two policies.

Suppose for now that $\delta = 1$.

For group A, the sum $\delta^{\tau} W_{A,\tau} + \delta^{\tau+1} W_{A,\tau+1}$ is the same under policies σ and σ'.

For group B, on the other hand, $\delta^{\tau} W_{B,\tau} + \delta^{\tau+1} W_{B,\tau+1}$ is strictly greater under plan σ than under σ'. To see this, note that under σ

$$\delta^{\tau} W_{B,\tau} + \delta^{\tau+1} W_{B,\tau+1} = \delta^{\tau} |B| \int_0^1 \omega_{\tau}^*(g(c)) f_{B,n_{\tau}}(c) dc + \delta^{\tau+1} |B| \int_0^1 \omega_{\tau+1}^*(c) f_{B,n_{\tau+1}}(c) dc$$

while under σ'

$$\delta^{\tau} W'_{B,\tau} + \delta^{\tau+1} W'_{B,\tau+1} = \delta^{\tau} |B| \int_0^1 \omega_{\tau}'^*(c) f_{B,n'_{\tau}}(c) dc + \delta^{\tau+1} |B| \int_0^1 \omega_{\tau+1}'^*(g(c)) f_{B,n'_{\tau+1}}(c) dc.$$

The fact that $\delta^{\tau} W_{B,\tau} + \delta^{\tau+1} W_{B,\tau+1} > \delta^{\tau} W'_{B,\tau} + \delta^{\tau+1} W'_{B,\tau+1}$, when $\delta = 1$, follows from the facts that $\omega_{\tau+1}^*(c) = \omega_{\tau}'^*(c) = c$, that $\omega_{\tau}^*(g(c)) = \omega_{\tau+1}'^*(g(c))$, that $f_{B,n_{\tau}}(c) = f_{B,n'_{\tau+1}}(c)$ and that $f_{B,n_{\tau+1}}(c) \succ f_{B,n'_{\tau}}(c)$.

By continuity, it then follows that there exists $\bar{\delta} \in (0,1)$ such that for all $\delta \in (\bar{\delta}, 1)$, the total welfare is also higher under plan σ than under σ'. ■

2 A more general model

We present here a more general model. The model presented in the main part of the paper is a particular case of that model.

Each agent lives for only one period, but at each time t, with probability $\delta \in (0,1)$, a mass $|A|$ and a mass $|B|$ of new agents from groups A and B respectively are born, with performance levels drawn according to $f_A(c)$ and $f_{B,n_t}(c)$. We therefore call δ the population's survival probability for one period. Note that δ serves also as discount factor in the government's objective function[1].

We will suppose that employers do not observe the time t, but they have a belief $\mathbb{P}(t)$ about t. It can be interpreted as an employer not knowing under which government the workers obtained their qualifications. This assumption allows employers to set a time-independent wage function. The case presented in the main paper corresponds to the particular case when t is observed and thus $\mathbb{P}(t) = 1$ for t.

[1] One might consider additional factors (such as impatience) affecting the discount rate. To the extent that the employers and the governments use the same discount rate, our analysis and main insights remain unaffected.

2.1 Allowing for strategic behavior by workers

We allow here agents to choose the curriculum vitae quality that they present to employers. This allows us to treat the more general case where the conditional expectation $\mathbb{E}[c|\hat{c}, \mu^*, \sigma^*]$ may not be monotone. We illustrate that the results presented in the main paper still hold, since they are just a particular case of this more general setting (i.e. the case when agents truthfully declare their curriculum vitae quality).

In this general model, a wage function $\omega(\hat{c})$ set by employers is the wage the worker earns when declaring a curriculum vitae of quality $\hat{c} \in [0,1]$ to the employer. Here, we see that a worker can declare a curriculum vitae of quality not necessarily equal to his actual quality $\overline{c}$. This is formalized in the following definition.

Definition OA 1 *A wage function $\omega : [0,1] \to [0,1]$ determines the wage a worker earns when declaring a curriculum vitae of quality $\hat{c}$ to the employer.*

The utility of a type $(c, \overline{c}, G)$ worker, when presenting a curriculum vitae of quality $\hat{c} \in [0,1]$, is thus

$$u_G(\hat{c}, c) \quad = \quad \omega(\hat{c}) - \gamma_G \max\{c - \omega(\hat{c}), 0\} - \kappa \max\{\hat{c} - \overline{c}, 0\} \tag{1}$$

where $\kappa \max\{\hat{c} - \overline{c}, 0\}$, with $\kappa > 0$, is a penalty suffered for cheating (i.e. presenting a curriculum vitae quality higher than the actual one $\overline{c}$). Note that no penalty is suffered for presenting a curriculum vitae of lower quality than $\overline{c}$.

A worker thus chooses to present a curriculum vitae of quality $\hat{c}$ such that

$$\hat{c} \in \underset{\tilde{c} \in [0,1]}{\text{argmax}} \; u_G(\tilde{c}, c)$$

Definition OA 2 *Given a wage function $\omega : [0,1] \to [0,1]$, a curriculum vitae declaration function $\mu : [0,1] \to [0,1]$ assigns a* declared *curriculum vitae quality $\hat{c}$ to an actual curriculum vitae quality $\overline{c}$, that is $\hat{c} = \mu(\overline{c})$.*

Definition OA 3 *Given a government policy strategy σ^*, a labor market equilibrium (ω^*, μ^*) is a continuous wage function and a curriculum vitae declaration function such that*

$$\omega^*(\hat{c}) = \mathbb{E}[c|\hat{c}, \mu^*, \sigma^*]$$

and

$$\mu^*(\overline{c}) \in \underset{\tilde{c} \in [0,\overline{c}]}{\text{argmax}} \; u_G(\tilde{c}, c).$$

Recall from Eq.(1) that the utility $u_G(\hat{c}, c)$ depends on the wage $\omega^*(\hat{c})$.

If κ is high enough, a continuous wage function $\omega(\hat{c})$ will prevent cheating since the marginal penalty of presenting a curriculum vitae quality greater than $\overline{c}$ will exceed the marginal benefit in terms of increased wage. A sufficient condition for this to hold is that $\kappa > \frac{\omega(\hat{c}) - \omega(\overline{c})}{\hat{c} - \overline{c}}$ for any $\hat{c} > \overline{c}$.

We thus have the following lemma.

Lemma OA 1 *Suppose κ is high enough. Given a policy strategy σ^*, there exist intervals $\{(c_l^L, c_l^H)\}_{l=1}^{\bar{l}}$ with $\bar{l} \geq 0$, so that the (weakly) increasing wage function*

$$\omega^*(\hat{c}) = \begin{cases} \mathbb{E}[c|\bar{c} = \hat{c}, \sigma^*] & \text{if } \hat{c} \notin \bigcup_l (c_l^L, c_l^H) \\ \mathbb{E}[c|\bar{c} \in (c_l^L, c_l^H), \sigma^*] & \text{if } \hat{c} \in (c_l^L, c_l^H) \end{cases} \tag{2}$$

and the curriculum vitae declaration strategy

$$\mu^*(\bar{c}) = \begin{cases} \bar{c} & \text{if } \bar{c} \notin \bigcup_l (c_l^L, c_l^H) \\ c_l^L & \text{if } \bar{c} \in (c_l^L, c_l^H) \end{cases} \tag{3}$$

constitute a labor market equilibrium.

In the above,

$$\mathbb{E}[c|\bar{c} = \hat{c}, \sigma^*] = \sum_{t=1}^{\infty} \mathbb{P}(t) \left(\mathbb{P}_t^*(\{aa\}|\bar{c}) \cdot g^{-1}(\bar{c}) + \left(1 - \mathbb{P}_t^*(\{aa\}|\bar{c})\right) \cdot \bar{c} \right),$$

with

$$\mathbb{P}_t^*(\{aa\}|\bar{c}) = \frac{|B|\sigma_t^* f_{B,n_t}(g^{-1}(\bar{c}))/g'^{-1}(\bar{c})}{|A|f_A(\bar{c}) + |B|(1-\sigma_t^*)f_{B,n_t}(\bar{c}) + |B|\sigma_t^* f_{B,n_t}(g^{-1}(\bar{c}))/g'^{-1}(\bar{c})},$$

$\{aa\}$ being the event that a worker benefited from affirmative action and

$$\mathbb{P}(t) = \delta^{t-1}(1-\delta)$$

is the probability of being at time t, while

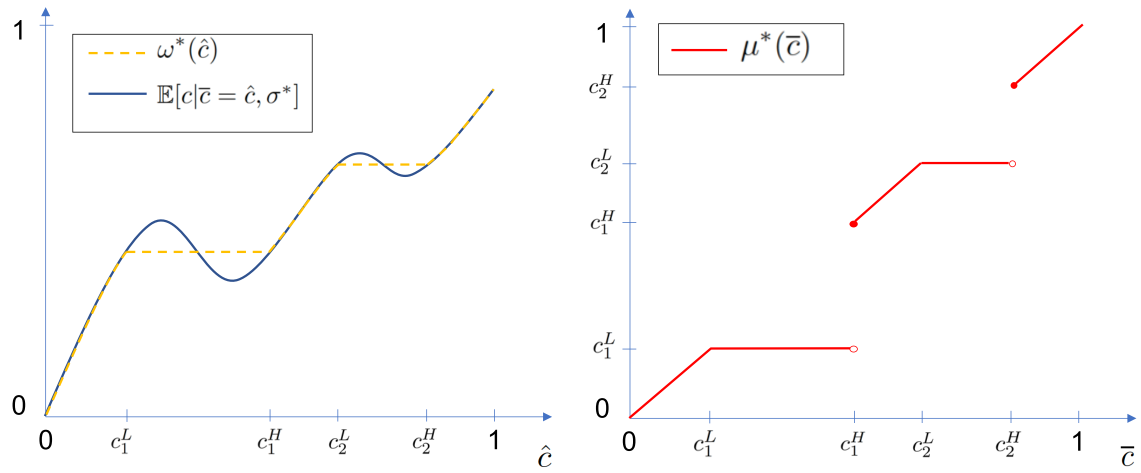
$$\mathbb{E}[c|\bar{c} \in (c_l^L, c_l^H), \sigma^*] = \sum_{t=1}^{\infty} \mathbb{P}(t) \int_{\bar{c}=c_l^L}^{c_l^H} \mathbb{E}[c|\bar{c} = \hat{c}, \sigma^*] f_t(\bar{c}) d\bar{c}$$

where

$$f_t(\bar{c}) = \frac{1}{|A| + |B|} \Big(|A|f_A(\bar{c}) + |B|\sigma_t^* f_{B,n_t}(g^{-1}(\bar{c}))/g'^{-1}(\bar{c}) + |B|(1-\sigma_t^*) f_{B,n_t}(\bar{c}) \Big)$$

is the overall population density for the curriculum vitae quality at time t.

The equilibrium wage function stated in Lemma OA 1 has the form described in Figure 1(a). We see that it is weakly increasing, but strictly increasing in certain sections. In the particular case when $\bar{l} = 0$, then it can be strictly increasing over the whole domain, as in the case presented earlier in the main part of the paper. The equilibrium curriculum vitae declaration function has the form described in Figure 1(b). It is such that a worker truthfully declares his curriculum vitae quality, i.e. $\hat{c} = \bar{c}$, when $\bar{c}$ is in an interval where the wage function is strictly increasing, since declaring anything lower would yield a lower salary. On the other hand, when $\bar{c}$ is in an interval where the wage function is flat, the worker declares the lowest curriculum vitae quality $\hat{c}$ on that flat interval, i.e. $\hat{c} = c_l^L$. Indeed, declaring such a curriculum vitae quality $\hat{c} \leq \bar{c}$ provides the worker with the same salary as he would get when declaring the actual one: $\omega(\hat{c}) = \omega(\bar{c})$. In the particular case where $\bar{l} = 0$ and the wage function is strictly increasing, then all workers would always declare their true curriculum vitae quality ($\mu^*(\bar{c}) = \bar{c}$, as in the case presented earlier in the main part of the



(a) (b)

Figure 1: Equilibrium wage function ω^* (panel (a)) and curriculum vitae declaration function μ^* (panel (b)).

paper). Note that, as required by the equilibrium definition, an employer correctly sets the wage equal to the conditional expectation of a worker's performance (i.e. $\omega^*(\hat{c}) = \mathbb{E}[c|\hat{c}, \mu^*, \sigma^*]$).

Lemma OA 2 *The equilibrium wage function $\omega^*(\hat{c})$ is weakly increasing, but strictly increasing at least on some regions of the support*[2] $[0, 1]$.

The next lemma is simply a more general version of Lemma 2 of the main paper, adapted to the labor market equilibrium concept defined in Definition OA 3.

Lemma OA 3 *If a worker benefits from affirmative action (i.e. $c = g^{-1}(\overline{c})$), then he gets a wage higher than his performance level (i.e. $c < \omega^*(\mu^*(\overline{c}))$). If a worker does not benefit from affirmative action (i.e. $c = \overline{c}$), then he gets a wage lower than his performance level (i.e. $\omega^*(\mu^*(\overline{c})) < c$).*

Using Lemma OA 3, we can make the same observations as in the main paper, namely that non-beneficiaries of affirmative action (of either group A or B) suffer a feeling of injustice, while beneficiaries do not.

Here we state a more general version of Proposition 1 of the main paper.

Proposition OA 1 (Equilibrium policy) *Given any $\lambda_B > 0$:*

(i) $\sigma_t^* = 1$ *for all t is an equilibrium.*

(ii) There exists $\overline{\gamma}_B$ such that for any $\gamma_B < \overline{\gamma}_B$, it is the unique equilibrium[3].

(iii) There exists $\overline{\beta}$ such that for any $\frac{|B|}{|A|+|B|} < \overline{\beta}$, it is the unique equilibrium.

[2]This is stronger than needed. $\omega^*(\hat{c})$ only needs to have these properties for the $\hat{c}$'s being played in equilibrium (i.e. $\hat{c} = \mu^*(\overline{c})$).

[3]It is interesting to note that it is enough that such a parameter γ_B, capturing the feeling of injustice felt by members of group B not benefiting from affirmative action, corresponds to one chosen by the government and it need not be the actual one felt in population B. Indeed, recall that the equilibrium wage ω^* actually does not depend on γ_B. Only the welfare $W_{B,s}$ of group B does.

The intuition behind Proposition OA 1(ii) is that a deviation from a putative equilibrium in which $\sigma_t^* = 0$ to $\sigma_t = 1$ would increase the average performance of future cohorts of B workers (and thus the average wage they receive), but could also potentially increase the average feeling of injustice felt by B workers not benefiting from affirmative action in future periods. Indeed, the feeling of injustice could worsen following an increase in the performance level, if the latter increases faster than the wage received at a higher performance level (recall that the feeling of injustice is $\gamma_B \max\{c - \omega^*(\hat{c}), 0\}$). A sufficient condition for the postive effect to dominate the negative one is that the parameter γ_B be small enough.

Finally, the intuition behind Proposition OA 1(iii) is very simple. As the fraction of group B workers in the population decreases, the probability that a worker is a beneficiary of affirmative action also decreases. The conditional expectation of a worker's performance $\mathbb{E}[c|\hat{c}, \mu^*, \sigma^*]$ (and thus the wage function $\omega^*(\hat{c})$) therefore converges to the actual worker's performance c, and the feeling of injustice of group B workers, $\gamma_B \max\{c - \omega^*(\hat{c}), 0\}$, necessarily decreases. By the same argument as in Part (ii), this is thus a sufficient condition for any deviation from a putative equilibrium where $\sigma_t^* = 0$ to $\sigma_t = 1$ to improve the welfare.

Proposition 2 of the main paper carries through unchanged in this more general model, although its proof is slightly more intricate.

Proposition OA 2 (First-best policy) *Suppose that at time $t = 0$, a single government announces (and commits to) the policy plan $\hat{\sigma} = \{\hat{\sigma}_t\}_{t=1}^{\infty}$ that maximizes the welfare function $\sum_{t=1}^{\infty} \delta^t (W_{A,t} + \lambda_B W_{B,t})$, and assume $\gamma_A \neq 0$. Then for any $\lambda_B \in [0, 1]$, there exists $\overline{\delta} \in (0, 1)$ such that for all $\delta \in (\overline{\delta}, 1)$, $\{\hat{\sigma}_t\}_{t=1}^{\infty}$ has a threshold form $\hat{\sigma}_t = 1$ for $t < \overline{T}$ and $\hat{\sigma}_t = 0$ for $t \geq \overline{T}$, for some (finite) $\overline{T} \in \mathbb{N}$.*

2.2 Other extension: Adding a labor market congestion externality

In our model, non-beneficiaries of affirmative action suffer from receiving a wage that is lower than their actual performance level, while beneficiaries of affirmative action receive a wage that is higher than their actual performance level. Therefore, a transfer of utility between groups arises through the wage channel.

From another perspective, affirmative action is often thought of as an allocation problem, e.g. allocating a finite number of jobs between two groups, which would result in extra transfers between beneficiaries and non-beneficiaries of affirmative action in addition to the wage effect considered in our main model. While modeling a full-scale matching process is beyond the scope of this paper, our model can be extended in such a direction by adding a labor market congestion externality. This will be represented by a positive term in the utility function of a beneficiary and a negative term in the utility function of a non-beneficiary.

In order not to obscure the exposition, let us here take $\mu^*(\overline{c}) = \overline{c}$ (workers truthfully declare their curriculum vitae) in the model of Section 2.1, while keeping t unobservable by employers. This does not alter the conclusions of our analysis.

Thus, the utility of an A worker will take the form

$$u_A(\overline{c}, c) = \omega^*(c) - \gamma_A(c - \omega^*(c)) - \frac{\eta}{|A|}\sigma_s$$

while that of a B worker benefiting from affirmative action will take the form

$$u_B(\bar{c}, c|\{aa\}) = \omega^*(\bar{c}) + \frac{\eta}{|B|}$$

where $\eta > 0$ is a parameter measuring the magnitude of the congestion externality. It is easy to see that the aggregate transfer of utility from group A to group B due to labor market congestion, in a period when an affirmative action policy is implemented (i.e. when $\sigma_s = 1$), is simply η.

A simple calculation shows that a time-t government's objective function (evaluated at the equilibrium σ^*) can now be written as

$$\sum_{s=t}^{\infty}(W_{A,s} + \lambda_B W_{B,s} + \sigma_s^* \eta(\lambda_B - 1))\delta^{s-t}$$

where $W_{A,s}$ and $W_{B,s}$ are the aggregate welfare of groups A and B at time s, *absent* the congestion externality, while the additional term $\sigma_s^*\eta(\lambda_B - 1)$ represents the welfare associated to the transfer of utility from group A to group B (in equilibrium) due to labor market congestion. This term will be negative if the weight λ_B assigned by the government to group B is smaller than 1.

We therefore have the following analogue of Proposition OA 1.

Observation OA 1 (Equilibrium policy with congestion)

(i) If $\lambda_B = 1$, then Proposition OA 1 follows[4].

(ii) If $\lambda_B \in [0, 1)$, there exists $\underline{\eta}$ such that $\forall\eta > \underline{\eta}$, then σ^* with a threshold form $\sigma_t^* = 1$ for $t < T_{eq}$ and $\sigma_t^* = 0$ for $t \geq T_{eq}$, for some (finite) $T_{eq} \in \mathbb{N}$ is an equilibrium.

The intuition behind part (i) is that, when $\lambda_B = 1$, then labor market congestion simply causes a welfare transfer between groups A and B since the government cares equally about the two groups and thus it has no impact on the aggregate welfare.

The intuition behind part (ii) is that the benefits of choosing $\sigma_t = 1$ when sufficiently many governments have already done it before become marginal (due to the convergence of the performance distribution $f_{B,n_t}(c)$ to $\overline{f}_B(c)$). When $\lambda_B < 1$ and η is large enough, the negative term $\sigma_t\eta(\lambda_B - 1)$ in the welfare function then means that choosing $\sigma_t = 1$ will decrease the welfare more than it could increase due to a boosted beneficiary's curriculum vitae (i.e. $\bar{c} = g(c)$) and an improvement in $f_{B,n_t}(c)$. Naturally, the threshold T_{eq} is weakly decreasing in the magnitude of the congestion factor η.

We also have the following analogue of Proposition OA 2, in which we namely see that affirmative action will still last longer in equilibrium than a first-best policy would prescribe.

Observation OA 2 (First-best policy with congestion) *For any $\lambda_B \in [0, 1]$, Proposition OA 2 follows*[5]*. Moreover, the first-best threshold in the presence of congestion is weakly lower than the*

[4]Note that if λ_B were to be greater than 1, then a time-t government's decision to choose $\sigma_t = 1$ would be even further rewarded by the welfare transfer from group A to group B. Indeed, the only difference compared to Proposition OA 1 is that there would be an additional non-negative term $\sigma_t\eta(\lambda_B - 1)$ in the welfare function being maximized. Proposition OA 1 would thus also apply without change since whenever $\sigma_t = 1$ would have been chosen in the original setting, it would also be chosen in the presence of congestion.

[5]If λ_B were to be strictly greater than 1 (i.e. when the government cares relatively more about group B than group A), then we might need the feeling of injustice parameter γ_A to be sufficiently positive in order to justify stopping affirmative action even earlier. In such a case, $\gamma_A \geq \underline{\gamma}_A$ for some $\underline{\gamma}_A > 0$ would be a sufficient (but not always necessary) condition for Observation OA 2 to hold.

first-best threshold in the absence of congestion, i.e. $\overline{T}_{con} \leq \overline{T}$*, and it is also weakly lower than the one of the equilibrium policy with congestion, i.e.* $\overline{T}_{con} \leq T_{eq}$.

The intuition behind the fact that the first-best threshold with congestion $\overline{T}_{con}$ is weakly lower than the first-best threshold without congestion $\overline{T}$ is that implementing an affirmative action policy ($\sigma_t = 1$) at any given time t has an additional penalty $\sigma_s^* \eta(\lambda_B - 1)$ when $\lambda_B < 1$. Thus, the benefits of improving the performance distribution of group B will be cancelled more quickly in the presence of congestion than in the case without congestion.

Similarly, the intuition behind that fact that the first-best threshold with congestion $\overline{T}_{con}$ is weakly lower than the equilibrium threshold with congestion T_{eq} is that in the first-best case, the single $t = 0$ government takes the depressing wage effects into account (in addition to the penalties $\sigma_s^* \eta(\lambda_B - 1)$) when choosing a multi-period affirmative action policy plan $\hat{\sigma}$. The threshold $\overline{T}_{con}$ at which affirmative action stops is thus necessarily weakly lower than in the equilibrium case.

2.3 Micro-foundations: Wage setting with Bertrand competition

In order not to obscure the exposition, here we will take $\mu^*(\bar{c}) = \bar{c}$ (workers truthfully declare their curriculum vitae) in the model of Section 2.1, while keeping t unobservable by employers. This does not alter the conclusions of our analysis.

We suppose that each firm produces a numeraire good of price equal to 1 with a constant return to scale technology and using labor as the input. The quantity of the numeraire good produced by a unit mass of workers of performance level c is thus simply c. The profit generated by a unit mass of workers of performance level c, when they are paid a wage $\omega(\bar{c})$, is thus

$$\pi = c - \omega(\bar{c}).$$

Since a firm only observes the curriculum vitae quality $\bar{c}$ of a worker it hires, the expected profit generated by a unit mass of workers with such curriculum vitae is then

$$\mathbb{E}[\pi|\bar{c}] = \mathbb{E}[c|\bar{c}, \sigma^*] - \omega(\bar{c})$$

where, as we know, $\mathbb{E}[c|\bar{c}, \sigma^*]$ is the expected performance level of a worker presenting a curriculum vitae $\bar{c}$ when the equilibrium government affirmative action strategy is σ^*.

If the firm hires a mass q of workers with curriculum vitae qualities having a density function $f(\bar{c})$, then its expected profit is

$$\begin{aligned} \Pi &= q\mathbb{E}[\pi] \\ &= q \int_{\bar{c}} \mathbb{E}[\pi|\bar{c}] f(\bar{c}) d\bar{c} \\ &= q \int_{\bar{c}} \big(\mathbb{E}[c|\bar{c}, \sigma^*] - \omega(\bar{c})\big) f(\bar{c}) d\bar{c} \qquad (4) \end{aligned}$$

where Π is also the realized profit, since each worker has zero measure.

A firm will thus maximize this profit by choosing an optimal wage function ω^*. Note that the profit in Eq. (4) is additively separable across $\bar{c}$. A firm thus chooses, for each curriculum vitae quality $\overline{c}$, the wage $\omega^*(\overline{c})$ that maximizes

$$\mathbb{E}[\pi|\overline{c}] = \mathbb{E}[c|\overline{c}, \sigma^*] - \omega(\overline{c}).$$

Since we consider a perfectly competitive Bertrand setting, it follows that the optimal wage will be equal to a worker's expected performance level, i.e. $\omega^*(\overline{c}) = \mathbb{E}[c|\overline{c}, \sigma^*]$, which is the worker's marginal productivity. Indeed, giving a wage higher than $\mathbb{E}[c|\overline{c}, \sigma^*]$ would result in a negative profit from hiring workers of that curriculum vitae quality, while giving a wage lower than $\mathbb{E}[c|\overline{c}, \sigma^*]$ would result in another employer hiring the workers away with a slightly higher wage. It also follows that a firm's profit is zero, i.e. $\Pi = 0$, on the equilibrium path. Thus, even if the governments care about the firms' welfare, the latter will not appear in their objective function (i.e. in Eq. (2) of the main paper) along the equilibrium path. Including profits in the government's objective function would affect the assessment of deviations, but the qualitative insights presented throughout the paper would be unaffected.

2.4 Proofs of results in Section 2

Proof of Lemma OA 1.

Throughout this proof, we suppose κ is high enough to prevent cheating. A sufficient condition for this to hold is that $\kappa > \frac{\omega(\hat{c})-\omega(\overline{c})}{\hat{c}-\overline{c}}$ for any $\hat{c} > \overline{c}$. In such a case, the marginal penalty of presenting a curriculum vitae quality greater than $\overline{c}$ will exceed the marginal benefit in terms of increased wage.

Step I: Compute the wage $\tilde{\omega}$ assuming truthful declaration of $\overline{c}$.

Suppose first that workers truthfully declare their curriculum vitae quality, i.e. $\hat{c} = \mu(\overline{c}) = \overline{c}$. Under such a declaration function μ, call $\tilde{\omega}(\hat{c}) = \mathbb{E}[c|\hat{c}, \mu, \sigma^*]$ the conditional expectation of the actual performance level when declaring a curriculum vitae of quality $\hat{c}$. Then,

$$\begin{aligned}
\tilde{\omega}(\hat{c}) &= \mathbb{E}[c|\hat{c}, \mu, \sigma^*] \\
&= \mathbb{E}[c|\hat{c} = \overline{c}, \sigma^*] \\
&= \mathbb{E}\big[\mathbb{E}[c|\hat{c} = \overline{c}, \sigma^*, t]|\hat{c} = \overline{c}, \sigma^*\big] \\
&= \sum_{t=1}^{\infty} \mathbb{P}(t)\mathbb{E}[c|\hat{c} = \overline{c}, \sigma^*, t] \\
&= \sum_{t=1}^{\infty} \mathbb{P}(t)\left(\mathbb{P}_t^*(\{aa\}|\overline{c}) \cdot g^{-1}(\overline{c}) + \big(1 - \mathbb{P}_t^*(\{aa\}|\overline{c})\big) \cdot \overline{c}\right)
\end{aligned}$$

where $\mathbb{P}(t)$ is the probability of being at time t.

Now to express $\mathbb{P}_t^*(\{aa\}|\overline{c})$, we first express $\mathbb{P}_t^*(\{aa\}|\tilde{c} \in N(\overline{c}, \epsilon))$, where $N(\overline{c}, \epsilon)$ is an ϵ-neighborhood of $\overline{c}$:

$$\begin{aligned}
\mathbb{P}_t^*(\{aa\}|\tilde{c} \in N(\overline{c},\epsilon)) &= \frac{\mathbb{P}_t^*(\{\tilde{c} \in N(\overline{c},\epsilon)\} \bigcap \{aa\})}{\mathbb{P}_t^*(\tilde{c} \in N(\overline{c},\epsilon))} \\
&= \frac{\mathbb{P}_t^*(\tilde{c} \in N(\overline{c},\epsilon) \bigcap B) \cdot \sigma_t^*}{\mathbb{P}_t^*(\tilde{c} \in N(\overline{c},\epsilon))} \\
&= \frac{\mathbb{P}_t^*(\tilde{c} \in N(\overline{c},\epsilon)|B) \cdot \mathbb{P}(B) \cdot \sigma_t^*}{\mathbb{P}_t^*(\tilde{c} \in N(\overline{c},\epsilon))}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\overline{c}),\epsilon/g'^{-1}(\overline{c}))} f_{B,n_t}(g^{-1}(\tilde{c})) d\tilde{c} \frac{|B|}{|A|+|B|}}{\int_{\tilde{c} \in N(\overline{c},\epsilon)} f_A(\tilde{c}) d\tilde{c} \frac{|A|}{|A|+|B|} + (1-\sigma_t^*) \int_{\tilde{c} \in N(\overline{c},\epsilon)} f_{B,n_t}(\tilde{c}) d\tilde{c} \frac{|B|}{|A|+|B|} + \sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\overline{c}),\epsilon/g'^{-1}(\overline{c}))} f_{B,n_t}(g^{-1}(\tilde{c})) d\tilde{c} \frac{|B|}{|A|+|B|}} \\
&= \frac{|B|\sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\overline{c}),\epsilon/g'^{-1}(\overline{c}))} f_{B,n_t}(g^{-1}(\tilde{c})) d\tilde{c}}{|A| \int_{\tilde{c} \in N(\overline{c},\epsilon)} f_A(\tilde{c}) d\tilde{c} + |B|(1-\sigma_t^*) \int_{\tilde{c} \in N(\overline{c},\epsilon)} f_{B,n_t}(\tilde{c}) d\tilde{c} + |B|\sigma_t^* \int_{\tilde{c} \in N(g^{-1}(\overline{c}),\epsilon/g'^{-1}(\overline{c}))} f_{B,n_t}(g^{-1}(\tilde{c})) d\tilde{c}}
\end{aligned}$$

Then, we take the limit as $\epsilon \to 0$:

$$\begin{aligned}
\mathbb{P}_t^*(\{aa\}|\tilde{c}) &= \lim_{\epsilon \to 0} \mathbb{P}_t^*(\{aa\}|\tilde{c} \in N(\overline{c},\epsilon)) \\
&= \lim_{\epsilon \to 0} \frac{|B|\sigma_t^* f_{B,n_t}(g^{-1}(\overline{c})) 2\epsilon / g'^{-1}(\overline{c})}{|A| f_A(\overline{c}) 2\epsilon + |B|(1-\sigma_t^*) f_{B,n_t}(\overline{c}) 2\epsilon + |B|\sigma_t^* f_{B,n_t}(g^{-1}(\overline{c})) 2\epsilon / g'^{-1}(\overline{c})} \\
&= \frac{|B|\sigma_t^* f_{B,n_t}(g^{-1}(\overline{c})) / g'^{-1}(\overline{c})}{|A| f_A(\overline{c}) + |B|(1-\sigma_t^*) f_{B,n_t}(\overline{c}) + |B|\sigma_t^* f_{B,n_t}(g^{-1}(\overline{c})) / g'^{-1}(\overline{c})}
\end{aligned}$$

We finally show that

$$\mathbb{P}(t) = \delta^{t-1}(1-\delta)$$

is the probability of being at time t.

Let $\tau \geq 1$ be the extinction time of the population and let $p(\tau) = \delta^{\tau-1}(1-\delta)$ be the probability that the population goes extinct at time τ. Then, being at time $t = 1$ is compatible with extinction times $\tau \geq 2$, while being at time $t = 2$ is compatible with extinction times $\tau \geq 3$, etc. In summary, being at time t is compatible with extinction times $\tau \geq t + 1$.

Thus, the probability of passing through time t is the probability that $\tau \geq t + 1$:

$$\begin{aligned}
\tilde{\mathbb{P}}(t) &= \sum_{\tau=t+1}^{\infty} p(\tau) \\
&= \sum_{\tau=t+1}^{\infty} \delta^{\tau-1}(1-\delta) \\
&= (1-\delta) \sum_{\tau=t+1}^{\infty} \delta^{\tau-1} \\
&= (1-\delta) \Big(\sum_{\tau=0}^{\infty} \delta^{\tau} - \sum_{\tau=0}^{t-1} \delta^{\tau} \Big) \\
&= (1-\delta) \Big(\frac{1}{1-\delta} - \frac{1-\delta^t}{1-\delta} \Big) \\
&= \delta^t
\end{aligned}$$

Employers form a belief about being at time t conditional on being at an information set that contains all decision nodes, i.e. $\{t = 1, t = 2, t = 3, ...\}$. We use the notion of consistency proposed by Piccione and Rubinstein (1997) that can receive a frequentist interpretation. That is, letting $\tilde{\mathbb{P}} = \sum_{t=1}^{\infty} \tilde{\mathbb{P}}(t)$, the belief attached by an employer being at t is defined by $\mathbb{P}(t) = \tilde{\mathbb{P}}(t)/\tilde{\mathbb{P}}$ where $\mathbb{P}(t)$ represents the frequency with which an employer is at t (in the unique information set $\{t = 1, t = 2, ...\}$). Formally,

$$\begin{aligned}\sum_{t=1}^{\infty} \tilde{\mathbb{P}}(t) &= \sum_{t=1}^{\infty} \delta^t \\ &= \frac{1}{1-\delta} - 1 \\ &= \frac{\delta}{1-\delta}\end{aligned}$$

and we therefore obtain $\mathbb{P}(t) = \tilde{\mathbb{P}}(t)\frac{1-\delta}{\delta} = \delta^{t-1}(1-\delta)$.

Step II: Such a wage function $\tilde{\omega}$ cannot in general be part of an equilibrium.

Suppose that $\tilde{\omega}$ is increasing for $c \in [0, c_1]$ and decreasing over some interval $[c_1, c_1']$. If the wage function is $\tilde{\omega}$, then a worker with an actual curriculum vitae quality $\overline{c} \in (c_1, c_1']$ will choose to declare a curriculum vitae quality $\hat{c} < \overline{c}$ since he can obtain a higher wage $\tilde{\omega}(\hat{c}) > \tilde{\omega}(\overline{c})$ by doing so. It follows that $\mu(\overline{c}) = \overline{c}$ cannot be part of an equilibrium since $\mu(\overline{c}) \notin \underset{\tilde{c}\in[0,\overline{c}]}{\operatorname{argmax}}\, u_G(\tilde{c}, c)$ for such $\overline{c}$.

Since $\mu(\overline{c}) = \overline{c}$ is not part of an equilibrium, it follows that $\tilde{\omega}(\hat{c}) = \mathbb{E}[c|\overline{c} = \hat{c}, \sigma^*]$ is not equal to the correct conditional expectation $\mathbb{E}[c|\hat{c}, \mu^*, \sigma^*]$ where μ^* is an equilibrium declaration function. Thus, $\tilde{\omega}(\hat{c})$ cannot in general be the equilibrium wage function.

Step III: Building a weakly increasing wage function $\omega^*(\hat{c})$ using $\tilde{\omega}(\hat{c})$.

On the other hand, there exist $c_1^L < c_1$ and $c_1^H \geq c_1'$ such that a wage

$$\omega^*(\hat{c}) = \begin{cases}\tilde{\omega}(\hat{c}), & \text{if } \hat{c} \in [0, c_1^L] \\ \tilde{\omega}(c_1^L) & \text{when } \hat{c} \in (c_1^L, c_1^H]\end{cases} \tag{5}$$

corresponds to $\mathbb{E}[c|\hat{c}, \mu^*, \sigma^*]$, where μ^* is as in the statement of the lemma. Such a pair $\{c_1^L, c_1^H\}$ satisfies

$$\tilde{\omega}(c_l^L) = \sum_{t=1}^{\infty} \mathbb{P}(t) \int_{\overline{c}=c_l^L}^{c_l^H} \tilde{\omega}(\overline{c}) f_t(\overline{c}) d\overline{c} \tag{6}$$

$$\tilde{\omega}(c_l^H) = \sum_{t=1}^{\infty} \mathbb{P}(t) \int_{\overline{c}=c_l^L}^{c_l^H} \tilde{\omega}(\overline{c}) f_t(\overline{c}) d\overline{c} \tag{7}$$

and

$$\sum_{t=1}^{\infty} \mathbb{P}(t) \int_{\overline{c}=c_l^H}^{1} \tilde{\omega}(\overline{c}) f_t(\overline{c}) d\overline{c} > \tilde{\omega}(c_l^H). \tag{8}$$

where

$$f_t(\overline{c}) = \frac{1}{|A| + |B|}\Big(|A| f_A(\overline{c}) + |B|\sigma_t^* f_{B,n_t}(g^{-1}(\overline{c}))/g'^{-1}(\overline{c}) + |B|(1-\sigma_t^*) f_{B,n_t}(\overline{c})\Big)$$

is simply the overall population density for the curriculum vitae quality $\overline{c}$ at time t.

By construction, $\omega^*(\hat{c})$ is strictly increasing for $\hat{c} \in [0, c_1^L]$ and flat for $\hat{c} \in (c_1^L, c_1^H]$. This is pictured in Figure 1(a). We will generalize this in Step V below.

Step IV: Verifying that (ω^*, μ^*) is a labor market equilibrium for $\overline{c} \in [0, c_1^H]$.

For any worker with an actual curriculum vitae quality $\overline{c} \in [0, c_1^L]$, the best response to such a wage function is $\mu^*(\overline{c}) = \overline{c} = \underset{\tilde{c} \in [0,\overline{c}]}{\text{argmax}}\ u_G(\tilde{c}, c)$ since $\omega^*(\hat{c})$ is strictly increasing over that range and thus the worker chooses to declare $\hat{c} = \overline{c}$ to maximize his wage. Therefore, $\omega^*(\hat{c}) = \mathbb{E}[c|\hat{c}, \mu^*, \sigma^*] = \mathbb{E}[c|\hat{c} = \overline{c}, \sigma^*] = \tilde{\omega}(\hat{c})$ for $\overline{c} \in [0, c_1^L]$. It follows that ω^* and μ^* satisfy the labor market equilibrium condition for $\overline{c} \in [0, c_1^L]$.

Moreover, for any worker with an actual curriculum vitae quality $\overline{c} \in (c_1^L, c_1^H]$, the best response set to a such a wage function is $[c_1^L, \overline{c}] = \underset{\tilde{c} \in [0,\overline{c}]}{\text{argmax}}\ u_G(\tilde{c}, c)$. A worker is indeed indifferent about declaring any $\hat{c} \in [c_1^L, \overline{c}]$, since it yields a salary $\omega^*(\hat{c}) = \tilde{\omega}(c_1^L)$, which is the maximum the worker can obtain. It follows that $\mu^*(\overline{c}) = c_1^L \in \underset{\tilde{c} \in [0,\overline{c}]}{\text{argmax}}\ u_G(\tilde{c}, c)$. Since, $\omega^*(c_1^L) = \mathbb{E}[c|\hat{c}, \mu^*, \sigma^*] = \mathbb{E}[c|\hat{c} = c_1^L, \mu^*, \sigma^*] = \mathbb{E}[c|\overline{c} \in [c_1^L, c_1^H], \sigma^*] = \tilde{\omega}(c_1^L)$, it follows that ω^* and μ^* satisfies the labor market equilibrium condition for $\overline{c} \in (c_1^L, c_1^H]$.

Step V: Generalizing to $\overline{c} \in [0, 1]$.

If $c_1^H < 1$ and $\tilde{\omega}(\overline{c})$ is decreasing over some range(s) in $[c_1^H, 1]$, then an iterative application of conditions (6), (7) and (8) allows to find other pairs $\{c_l^L, c_l^H\}$ such that

$$\omega^*(\hat{c}) = \begin{cases} \mathbb{E}[c|\overline{c} = \hat{c}, \sigma^*] & \text{if } \hat{c} \notin \bigcup_l (c_l^L, c_l^H) \\ \mathbb{E}[c|\overline{c} \in (c_l^L, c_l^H), \sigma^*] & \text{if } \hat{c} \in (c_l^L, c_l^H) \end{cases}$$

and the analysis of Steps II, III and IV generalizes to the rest of the support.

■

Proof of Lemma OA 2.

This is a corollary of Lemma OA 1.

Lemma OA 1 states that $\omega^*(\hat{c}) = \mathbb{E}[c|\overline{c} \in (c_l^L, c_l^H), \sigma^*]$ for any $\hat{c} \in (c_l^L, c_l^H)$, implying that $\omega^*(\hat{c})$ is flat for such $\hat{c}$ (since $\mathbb{E}[c|\overline{c} \in (c_l^L, c_l^H), \sigma^*]$ is a constant).

On the other hand, Lemma OA 1 states that $\omega^*(\hat{c}) = \mathbb{E}[c|\overline{c} = \hat{c}, \sigma^*]$ when $\hat{c} \notin \bigcup_l (c_l^L, c_l^H)$ and Steps III and V of the proof of Lemma OA 1 show that $\omega^*(\hat{c})$ is constructed so as to be strictly increasing over such intervals.

■

Proof of Lemma OA 3.

When $\overline{c} \notin \bigcup_l (c_l^L, c_l^H)$, then from Lemma OA 1 we know that a worker truthfully declares a curriculum vitae quality $\hat{c} = \overline{c}$ and gets a wage

$$\omega^*(\overline{c}) = \sum_{t=1}^{\infty} \mathbb{P}(t) \left(\mathbb{P}_t^*(\{aa\}|\overline{c}) \cdot g^{-1}(\overline{c}) + \left(1 - \mathbb{P}_t^*(\{aa\}|\overline{c})\right) \cdot \overline{c} \right)$$

Since $g^{-1}(\overline{c}) < \overline{c}$, it follows immediately that $g^{-1}(\overline{c}) < \omega^*(\overline{c}) < \overline{c}$.

Thus, if the worker does not benefit from affirmative action (i.e. $c = \overline{c}$), then $\omega^*(\overline{c}) < c$ and he gets a wage lower than his performance level. On the other hand, if the worker benefits from affirmative action (i.e. $c = g^{-1}(\overline{c})$), then $c < \omega^*(\overline{c})$ and he gets a wage higher than his performance level.

We now show that this is also true when $\bar{c} \in \bigcup_l (c_l^L, c_l^H)$.

Recall from Lemma OA 1 that the wage function is flat over $[c_l^L, c_l^H]$ and equal to $\omega^*(c_l^L)$. Thus, a worker of performance level $c_l'^L$ who does not benefit from affirmative action gets a wage $\omega^*(c_l^L)$ with $\omega^*(c_l^L) < c'$ and a worker of performance level $c_l''^H$ who does not benefit from affirmative action also gets a wage $\omega^*(c_l^L)$ and $\omega^*(c_l^L) < c''$. Consider now a worker who does not benefit from affirmative action and $\bar{c} \in (c_l^L, c_l^H)$. Then, $c = \bar{c}$ with $c' < c < c''$ and the worker gets a wage $\omega^*(c_l^L)$. It follows that $\omega^*(c_l^L) < c$ and he gets a wage lower than his performance level. This applies to any $\bar{c} \in \bigcup_l (c_l^L, c_l^H)$.

Now again, recall from Lemma OA 1 that the wage function is flat over $[c_l^L, c_l^H]$ and equal to $\omega^*(c_l^L)$. Thus, a worker of performance level $g^{-1}(c_l^L)$ who benefits from affirmative action gets a wage $\omega^*(c_l^L)$ with $g^{-1}(c_l^L) < \omega^*(c_l^L)$ and a worker of performance level $g^{-1}(c_l^H)$ who benefits from affirmative action also gets a wage $\omega^*(c_l^L)$ and $g^{-1}(c_l^H) < \omega^*(c_l^L)$. Consider now a worker who benefits from affirmative action and $\bar{c} \in (c_l^L, c_l^H)$. Then, $c = g^{-1}(\bar{c})$ with $g^{-1}(c_l^L) < g^{-1}(\bar{c}) < g^{-1}(c_l^H)$ and the worker gets a wage $\omega^*(c_l^L)$. It follows that $c = g^{-1}(\bar{c}) < \omega^*(c_l^L)$ and he gets a wage higher than his performance level. This applies to any $\bar{c} \in \bigcup_l (c_l^L, c_l^H)$. ■

We now prove Proposition OA 1 for a general shape of $\mathbb{E}[c|\bar{c} = \hat{c}, \sigma^*]$ and using the labor market equilibrium concept of Definition OA 3 and Lemma OA 1. The particular case presented in the main part of the paper simply corresponds to a truthful curriculum vitae declaration, i.e. $\mu^*(\bar{c}) = \bar{c}$, and the time t being observed by employers (i.e. $\mathbb{P}(t) = 1$ for t).

Proof of Proposition OA 1.

We first have the following simple lemma.

Lemma OA 4 *Let $h(c)$ be any weakly increasing function that is strictly increasing at least on some opened subinterval of its support $[0, 1]$ and is differentiable almost everywhere. If $f \succ \tilde{f}$, where f and $\tilde{f}$ are probability density functions on $[0, 1]$ and $\succ$ indicates strict first-order stochastic dominance, then $\int_0^1 h(c)f(c)dc > \int_0^1 h(c)\tilde{f}(c)dc$.*

Proof of Lemma OA 4.

The inequality rewrites

$$\int_0^1 h(c)\left[f(c) - \tilde{f}(c)\right] dc > 0.$$

After integrating by parts, this can be written as

$$\left[h(c)\left[F(c) - \tilde{F}(c)\right]\right] |_0^1 - \int_0^1 h'(c)\left[F(c) - \tilde{F}(c)\right] dc$$

where F and $\tilde{F}$ are the CDFs associated with the PDFs f and $\tilde{f}$. The first term is equal to 0 since $F(0) = \tilde{F}(0) = 0$ and $F(1) = \tilde{F}(1) = 1$. Moreover, since $h'(c) \geq 0$ almost everywhere with $h'(c) > 0$ on non-trivial parts of the support, the last term is strictly greater than 0 if $F(c) < \tilde{F}(c)$ for all $c \in (0, 1)$, i.e. if $f \succ \tilde{f}$. ■

Given some equilibrium decision profile $\sigma^* = \{\sigma_s^*\}_{s=1}^{\infty}$, any deviation σ_t' at some time t has no impact on the wages at any future time since this deviation is unobserved by employers. Indeed, for any $\tau \geq t$, employers form the wage $\omega^*(\hat{c})$ based on the conditional expectation $\mathbb{E}[c|\hat{c}, \mu^*, \sigma^*]$, which depends on the equilibrium decision profile $\sigma^* = \{\sigma_s^*\}_{s=1}^{\infty}$, none of these decisions being actually observed. Therefore, $\sum_{s=t}^{\infty} W_{A,s}\delta^{s-t}$, the discounted future welfare of group A, is completely unaffected by an unobserved deviation to σ_t'. Indeed, recall that $c = \overline{c}$ and thus $\mu^*(\overline{c}) = \mu^*(c)$. Therefore, $W_{A,s} = |A| \int_0^1 u_A(\mu^*(c), c) f_A(c) dc$, where the density function $f_A(c)$ is constant through time and thus not impacted by $\{\sigma_s\}_{s=1}^{\infty}$, while $u_A(\mu^*(c), c) = \omega^*(\mu^*(c)) - \gamma_A(c - \omega^*(\mu^*(c)))$ and the wage $\omega^*(\mu^*(c))$ is unaffected by an unobserved deviation to σ_t'.

Part (i):

We now show that $\{\sigma_s^*\}_{s=1}^{\infty} = \{1\}_{s=1}^{\infty}$ is an equilibrium. Given an equilibrium decision profile $\{\sigma_s^*\}_{s=1}^{\infty} = \{1\}_{s=1}^{\infty}$, $\sum_{s=t}^{\infty} \lambda_B W_{B,s} \delta^{s-t}$ is strictly lower following an unobserved deviation from $\sigma_t^* = 1$ to $\sigma_t' = 0$.

Indeed, at time $s = t$,

$$\begin{aligned}
W_{B,t|\sigma_t'=0} &= |B| \int_0^1 u_B(\mu^*(c), c) f_{B,n_t}(c) dc \\
&= |B| \int_0^1 \omega^*(\mu^*(c)) - \gamma_B(c - \omega^*(\mu^*(c))) f_{B,n_t}(c) dc \\
&< |B| \int_0^1 \omega^*(\mu^*(g(c))) f_{B,n_t}(c) dc \\
&= |B| \int_0^1 u_B(\mu^*(g(c)), c) f_{B,n_t}(c) dc \\
&= W_{B,t|\sigma_t^*=1}
\end{aligned}$$

where we have used the fact that the wage is not affected by an unobserved deviation and the facts that $\omega^*(\mu^*(c)) \leq \omega^*(\mu^*(g(c)))$ and that $c > \omega^*(\mu^*(c))$ by Lemma OA 3.

Moreover, at times $s > t$, $f_{B,n_s|\sigma_t'=0}(c) \prec f_{B,n_s|\sigma_t^*=1}(c)$ since a deviation to $\sigma_t' = 0$ has the effect of not changing the distribution of performance at time $t+1$ compared to the previous period t. Thus, using the fact that the wage is not affected by an unobserved deviation, then for all $s > t$,

$$\begin{aligned}
W_{B,s|\sigma_t'=0} &= |B| \int_0^1 u_B(\mu^*(g(c)), c) f_{B,n_s|\sigma_t'=0}(c) dc \\
&= |B| \int_0^1 \omega^*(\mu^*(g(c))) f_{B,n_s|\sigma_t'=0}(c) dc \\
&< |B| \int_0^1 \omega^*(\mu^*(g(c))) f_{B,n_s|\sigma_t^*=1}(c) dc \\
&= |B| \int_0^1 u_B(\mu^*(g(c)), c) f_{B,n_s|\sigma_t^*=1}(c) dc \\
&= W_{B,s|\sigma_t^*=1}
\end{aligned}$$

where the inequality follows from Lemma OA 4. Indeed, from Lemma OA 2, $\omega^*(\mu^*(g(c)))$ is strictly increasing in c on parts of its support (and weakly increasing overall).

It follows that as long as $\lambda_B > 0$, then $\sigma_t^* = 1$ for all t will be an equilibrium.

Part (ii):

To show that this is the unique equilibrium, we now have to show that a deviation from $\sigma_t^* = 0$ to $\sigma_t' = 1$ is always desirable for a time-t government. For that purpose, suppose that $\sigma_t^* = 0$ for some t. Then, we must show that $\sum_{s=t}^{\infty} \lambda_B W_{B,s} \delta^{s-t}$ is strictly higher following an unobserved deviation from $\sigma_t^* = 0$ to $\sigma_t' = 1$.

Consider first the effect of this deviation on the welfare at time t of members of group B. The same argument as in Part (i) can be used to show that $W_{B,t|\sigma'_t=1} > W_{B,t|\sigma^*_t=0}$.

Consider now the effect of this deviation on the welfare, at any future time $s > t$, of members of group B who benefit from affirmative action. We know that $f_{B,n_s|\sigma^*_t=0}(c) \prec f_{B,n_s|\sigma'_t=1}(c)$ since a deviation to $\sigma'_t = 1$ has the effect of shifting (in a strict first-order stochastic dominance sense) the distribution of performance from time t to $t+1$. Moreover, since $u_B(\mu^*(g(c)), c) = \omega^*(\mu^*(g(c)))$, and the wage is not affected by an unobserved deviation, then for all $s > t$,

$$\begin{aligned} W_{B,s|\{aa\},\sigma^*_t=0} &= |B| \int_0^1 \omega^*(\mu^*(g(c))) f_{B,n_s|\sigma^*_t=0}(c)dc \\ &< |B| \int_0^1 \omega^*(\mu^*(g(c))) f_{B,n_s|\sigma'_t=1}(c)dc \\ &= W_{B,n_s|\{aa\},\sigma'_t=1} \end{aligned}$$

where we made use of Lemma OA 4, since $\omega^*(\mu^*(g(c)))$ is strictly increasing in c on parts of its support (and weakly increasing overall from Lemma OA 2) and $f_{B,n_s|\sigma^*_t=0}(c) \prec f_{B,n_s|\sigma'_t=1}(c)$ for all $s > t$.

Consider finally the effect of this deviation on the welfare, at any future time $s > t$, of members of group B who do *not* benefit from affirmative action. Using Lemma OA 3,

$$\begin{aligned} W_{B,s|\{aa\}^c} &= |B| \int_0^1 \big(\omega^*(\mu^*(c)) - \gamma_B(c - \omega^*(\mu^*(c)))\big) f_{B,n_s}(c)dc \\ &= |B| \int_0^1 \big((1+\gamma_B)\omega^*(\mu^*(c)) - \gamma_B c\big) f_{B,n_s}(c)dc \end{aligned} \tag{9}$$

Moreover, since $f_{B,n_s|\sigma'_t=1}(c) \succ f_{B,n_s|\sigma^*_t=0}(c)$ for all $s > t$ and making use of Lemma OA 4, note that

$$\int_0^1 \big((1+\gamma_B)\omega^*(\mu^*(c))\big) f_{B,n_s|\sigma'_t=1}(c)dc > \int_0^1 \big((1+\gamma_B)\omega^*(\mu^*(c))\big) f_{B,n_s|\sigma^*_t=0}(c)dc$$

for all $\gamma_B \geq 0$. It thus follows that there exists $\overline{\gamma}_B$ such that for $\gamma_B < \overline{\gamma}_B$,

$$\begin{aligned} &\int_0^1 \big((1+\gamma_B)\omega^*(\mu^*(c))\big) f_{B,n_s|\sigma'_t=1}(c)dc - \int_0^1 \big((1+\gamma_B)\omega^*(\mu^*(c))\big) f_{B,n_s|\sigma^*_t=0}(c)dc \\ &\qquad > \int_0^1 \gamma_B c f_{B,n_s|\sigma'_t=1}(c)dc - \int_0^1 \gamma_B c f_{B,n_s|\sigma^*_t=0}(c)dc \end{aligned} \tag{10}$$

and thus that $W_{B,n_s|\{aa\}^c,\sigma'_t=1} > W_{B,n_s|\{aa\}^c,\sigma^*_t=0}$. We can then conclude that the future welfare of all members of group B (benefitting from affirmative action or not) increases following this unobserved deviation from $\sigma^*_t = 0$ to $\sigma'_t = 1$ and thus that $\sum_{s=t}^{\infty} \lambda_B W_{B,s} \delta^{s-t}$ is indeed strictly higher. Therefore, when the feeling of injustice parameter is small enough, $\gamma_B < \overline{\gamma}_B$, an equilibrium decision profile $\{\sigma^*_s\}_{s=1}^{\infty}$ in which $\sigma^*_t = 0$ for some t cannot exist.

Part (iii):

We will verify that the proof of Part (ii) also holds for any $\gamma_B \geq 0$ as long as $\beta = \frac{|B|}{|A|+|B|}$ is small enough.

First note that $\omega^*(\mu^*(\overline{c}))$ converges to c as $\beta \to 0$. Indeed, as the relative size of the targeted group decreases, the probability that a worker benefited from affirmative action (and thus an inflated curriculum vitae quality) decreases as well and thus the wage becomes equal to the actual performance level c in the limit as β goes to 0.

Thus, for any $\epsilon > 0$, there exists $\overline{\beta}$ such that for all $\beta < \overline{\beta}$, we have $| \int_0^1 \omega^*(\mu^*(\overline{c})) f_{B,n_s}(c)dc - \int_0^1 c f_{B,n_s}(c)dc| < \epsilon$.

Since for any $\gamma_B \geq 0$,

$$(1+\gamma_B)\Big(\int_0^1 c f_{B,n_s|\sigma'_t=1}(c)dc - \int_0^1 c f_{B,n_s|\sigma^*_t=0}(c)dc \Big) > \gamma_B \Big(\int_0^1 c f_{B,n_s|\sigma'_t=1}(c)dc - \int_0^1 c f_{B,n_s|\sigma^*_t=0}(c)dc \Big)$$

then we have that

$$\begin{aligned}(1+\gamma_B)\Big(\int_0^1 \omega^*(\mu^*(\overline{c})) f_{B,n_s|\sigma'_t=1}(c)dc - \int_0^1 \omega^*(\mu^*(\overline{c})) f_{B,n_s|\sigma^*_t=0}(c)dc \Big) &> \\ (1+\gamma_B)\Big(\int_0^1 c f_{B,n_s|\sigma'_t=1}(c)dc - \int_0^1 c f_{B,n_s|\sigma^*_t=0}(c)dc \Big) - 2\epsilon &> \\ \gamma_B \Big(\int_0^1 c f_{B,n_s|\sigma'_t=1}(c)dc - \int_0^1 c f_{B,n_s|\sigma^*_t=0}(c)dc \Big)&\end{aligned}$$

where the last inequality holds for small enough $\epsilon > 0$ and thus when $\beta < \overline{\beta}$.

It then follows that Eq. (10) in Part (ii) also holds for any $\gamma_B \geq 0$ as long as $\beta = \frac{|B|}{|A|+|B|} < \overline{\beta}$. ■

We now prove Proposition OA 2.

Proof of Proposition OA 2.

We start with the following lemma.

Lemma OA 5 *Let $\{\sigma_t\}_{t=1}^{\infty}$ be a policy plan with $\sigma_\tau = 0$ and $\sigma_{\tau+1} = 1$ for some τ. Let $\{\sigma'_t\}_{t=1}^{\infty}$ be another policy plan with $\sigma'_\tau = 1$, $\sigma'_{\tau+1} = 0$ and $\sigma'_t = \sigma_t$ for all other t. Then there exists $\overline{\delta} \geq 0$ such that for all $\delta \in (\overline{\delta}, 1)$, $\{\sigma'_t\}_{t=1}^{\infty}$ yields a strictly higher welfare than $\{\sigma_t\}_{t=1}^{\infty}$.*

Proof of Lemma OA 5. We will prove this lemma for a general shape of $\mathbb{E}[c|\overline{c} = \hat{c}, \sigma^*]$ and using the labor market equilibrium concept of Definition OA3 and Lemma OA 1.

Suppose first that $\delta = 1$.

Then $\{\sigma'_t\}_{t=1}^{\infty}$ and $\{\sigma_t\}_{t=1}^{\infty}$ yield the same wage function[6] $\omega^*(\hat{c})$ since $\mathbb{P}\{\tau\} = \mathbb{P}\{\tau+1\}$.

Therefore $\sum_{t=1}^{\infty} \delta^t W_{A,t} = \sum_{t=1}^{\infty} \delta^t |A| \int_0^1 \big((1+\gamma_A)\omega^*(\mu^*(c)) - \gamma_A c\big) f_A(c)dc$ is the same under plans $\{\sigma'_t\}_{t=1}^{\infty}$ and $\{\sigma_t\}_{t=1}^{\infty}$.

On the other hand, $\lambda_B \sum_{t=1}^{\infty} \delta^t W_{B,t}$ is strictly greater under plan $\{\sigma'_t\}_{t=1}^{\infty}$ than under $\{\sigma_t\}_{t=1}^{\infty}$. To see this, note that $\lambda_B \sum_{t=1}^{\tau-1} \delta^t W_{B,t}$ is the same under both policy plans, while $\lambda_B \sum_{t=\tau}^{\infty} \delta^t W_{B,t} = \lambda_B \sum_{t=\tau}^{\infty} \delta^t |B| \int_0^1 u_B(\mu^*(\overline{c}), c) f_{B,n_t}(c)dc$ is strictly greater under plan $\{\sigma'_t\}_{t=1}^{\infty}$ than under $\{\sigma_t\}_{t=1}^{\infty}$. Indeed, under $\{\sigma'_t\}_{t=1}^{\infty}$,

$$\sum_{t=\tau}^{\infty} \delta^t W'_{B,t} = \delta^\tau |B| \int_0^1 \omega^*(\mu^*(g(c))) f_{B,n'\tau}(c)dc + \delta^{\tau+1}|B| \int_0^1 \big((1+\gamma_B)\omega^*(\mu^*(c)) - \gamma_B c\big) f_{B,n'_{\tau+1}}(c)dc + \sum_{t=\tau+2}^{\infty} \delta^t W'_{B,t}$$

while under $\{\sigma_t\}_{t=1}^{\infty}$

[6] Formally, we would take a limit as $\delta \to 1$ and note that $\omega'^*(\hat{c}) \xrightarrow[\delta\to 1]{} \omega^*(\hat{c})$.

$$\sum_{t=\tau}^{\infty}\delta^t W_{B,t}=\delta^\tau|B|\int_0^1\big((1+\gamma_B)\omega^*(\mu^*(c))-\gamma_B c\big)f_{B,n\tau}(c)dc+\delta^{\tau+1}|B|\int_0^1\omega^*(\mu^*(g(c)))f_{B,n_{\tau+1}}(c)dc+\sum_{t=\tau+2}^{\infty}\delta^t W_{B,t}$$

Only the first two terms differ and their sum is greater under $\{\sigma_t'\}_{t=1}^{\infty}$ <u>when $\delta=1$</u>. Indeed, this follows from the facts that $\omega^*(\mu^*(g(c)))>\big((1+\gamma_B)\omega^*(\mu^*(c))-\gamma_B c\big)$, that $f_{B,n'_{\tau+1}}(c)\succ f_{B,n\tau}(c)$ and that $f_{B,n'\tau}(c)=f_{B,n_{\tau+1}}(c)$.

By continuity, it follows that there exists[7] $\overline{\delta}\in(0,1)$ such that for all $\delta\in(\overline{\delta},1)$, the total welfare is also higher under plan $\{\sigma_t'\}_{t=1}^{\infty}$ than under $\{\sigma_t\}_{t=1}^{\infty}$.

■

Therefore, when δ is high enough, it follows by iterative application of Lemma OA 5 that the optimal policy has a threshold form $\hat{\sigma}_t=1$ for $t<\overline{T}$ and $\hat{\sigma}_t=0$ for $t\geq\overline{T}$ for some $\overline{T}\in\mathbb{N}\bigcup\infty$.

We will now rule out the case where $\overline{T}$ could be infinite and thus show that $\overline{T}\in\mathbb{N}$.

To reduce the notational burden, we will prove this in the setting where $\mathbb{E}[c|\overline{c}=\hat{c},\sigma^*]$ in non-decreasing (and thus $\mu^*(\overline{c})=\overline{c}$), as in the main part of the paper. Naturally, the result still holds using the labor market equilibrium concept of Definition OA 3 and Lemma OA 1.

Let us thus compare the welfare of some (large) $\overline{T}<\infty$ to that of the case $\overline{T}'=\infty$. In what follows, the quantities with a prime ($'$) will be the ones associated to $\overline{T}'=\infty$.

We need to show that

$$\sum_{t=1}^{\infty}\delta^t(W_{A,t}+\lambda_B W_{B,t})>\sum_{t=1}^{\infty}\delta^t(W'_{A,t}+\lambda_B W'_{B,t}). \tag{11}$$

Equivalently, it will be convenient to multiply the welfare by the constant $\frac{(1-\delta)}{\delta(|A|+|B|)}$ and verify that

$$\frac{(1-\delta)}{\delta(|A|+|B|)}\Big(\sum_{t=1}^{\infty}\delta^t(W_{A,t}+\lambda_B W_{Bt})-\sum_{t=1}^{\infty}\delta^t(W'_{A,t}+\lambda_B W'_{B,t})\Big)>0$$

First recall that $\mathbb{P}(t)=\delta^{t-1}(1-\delta)$ and we can rewrite the left-hand side of this expression as

$$\begin{aligned}
\sum_{t=1}^{\infty}\frac{\delta^{t-1}(1-\delta)}{(|A|+|B|)}((W_{A,t}+\lambda_B W_{B,t})-(W'_{A,t}+\lambda_B W'_{B,t})) &= \sum_{t=1}^{\infty}\frac{\mathbb{P}(t)}{(|A|+|B|)}((W_{A,t}+\lambda_B W_{B,t})-(W'_{A,t}+\lambda_B W'_{B,t}))\\
&= \sum_{t=1}^{\infty}\mathbb{P}(t)\Big(\frac{|A|}{|A|+|B|}\int\omega^*(c)f_A(c)dc\\
&\quad+\frac{\lambda_B|B|}{|A|+|B|}\int[\sigma_t\omega^*(g(c))+(1-\sigma_t)\omega^*(c)]f_{B,n_t}(c)dc\\
&\quad-\frac{|A|}{|A|+|B|}\int\omega'^*(c)f_A(c)dc\\
&\quad-\frac{\lambda_B|B|}{|A|+|B|}\int\omega'^*(g(c))f_{B,n_t}(c)dc\Big)\\
&\quad+\sum_{t=1}^{\infty}\mathbb{P}(t)\frac{|A|}{|A|+|B|}\gamma_A\int(\omega^*(c)-\omega'^*(c))f_A(c)dc\\
&\quad-\sum_{t=\overline{T}}^{\infty}\mathbb{P}(t)\frac{\lambda_B|B|}{|A|+|B|}\gamma_B\int(c-\omega^*(c))f_{B,n_{\overline{T}}}(c)dc
\end{aligned} \tag{12}$$

[7]Note that even if $f_{B,n'_{\tau+1}}(c)$ is arbitrarily close to $f_{B,n_\tau}(c)$ (which will happen when n_τ is large), the argument goes through as long as $\omega^*(\mu^*(g(c)))>(1+\gamma_B)\omega^*(\mu^*(c))-\gamma_B c$ (which follows from Lemmas OA 2 and OA 3). This means δ is not required to get closer and closer to 1 as τ increases to infinity.

The case $\lambda_B = 1$ is interesting and worth examining first. In that case, note that the first two terms of the right-hand side of Eq. (12) rewrite as

$$\sum_{t=1}^{\infty}\mathbb{P}(t)\Big(\frac{|A|}{|A|+|B|}\int \omega^*(c)f_A(c)dc+\frac{|B|}{|A|+|B|}\int[\sigma_t\omega^*(g(c))+(1-\sigma_t)\omega^*(c)]f_{B,n_t}(c)dc\Big)=\mathbb{E}[c], \tag{13}$$

since the time-wise average wage under policy $\overline{T}<\infty$ is equal to the time-wise average performance level under policy $\overline{T}<\infty$ (here denoted by $\mathbb{E}[c]$).

Likewise, the third and fourth terms rewrite as

$$-\sum_{t=1}^{\infty}\mathbb{P}(t)\Big(\frac{|A|}{|A|+|B|}\int \omega'^*(c)f_A(c)dc+\frac{|B|}{|A|+|B|}\int \omega'^*(g(c))f_{B,n_t}(c)dc\Big)=-\mathbb{E}'[c], \tag{14}$$

since the time-wise average wage under policy $\overline{T}'=\infty$ is equal to the time-wise average performance level under policy $\overline{T}'=\infty$ (here denoted by $\mathbb{E}'[c]$).

We then have that the right-hand side of Eq. (12) can be written as

$$\big(\mathbb{E}[c]-\mathbb{E}'[c]\big)+\frac{|A|}{|A|+|B|}\gamma_A\int(\omega^*(c)-\omega'^*(c))f_A(c)dc-\frac{|B|}{|A|+|B|}\gamma_B\int(c-\omega^*(c))f_{B,n_{\overline{T}}}(c)dc\sum_{t=\overline{T}}^{\infty}\mathbb{P}(t)$$

where we have used the fact that $\sum_{t=1}^{\infty}\mathbb{P}(t)=1$ in the fifth term of Eq. (12).

We must now verify if this is greater than 0. We first make the following observations:

- The first term, $\mathbb{E}[c]-\mathbb{E}'[c]<0$. Indeed, the time-wise average performance level is higher when affirmative action is implemented for more periods. Moreover, this term converges to 0 as $\overline{T}\to\infty$ since $\mathbb{E}[c]\to\mathbb{E}'[c]$, capturing the fact that the improvements in the performance distribution of group B become marginal after a while.
- The third term is smaller than 0 since $c-\omega^*(c)>0$ as the members of group B suffer a feeling of injustice at times greater or equal to $\overline{T}$, when affirmative action has been stopped. On the other hand, for any policy $\overline{T}<\infty$, this term also converges to 0 as $\delta\to 1$. Indeed when affirmative action is stopped after a finite number of periods, the probability that a worker presenting a curriculum vitae c benefited from affirmative action goes to 0 when the survival probability becomes large enough, i.e. $\mathbb{P}^*(\{aa\}|c)=\sum_{t=1}^{\infty}\mathbb{P}(t)\mathbb{P}_t^*(\{aa\}|c)\xrightarrow[\delta\to 1]{}0$. It then follows that in such a case, a worker is paid his actual performance level, i.e. $\omega^*(c)\xrightarrow[\delta\to 1]{}c$. Moreover, since $\sum_{s=\overline{T}}^{\infty}\mathbb{P}(t)\le 1$, it follows that the third term converges to 0 as $\delta\to 1$. The interpretation is that the feeling of injustice becomes negligible in such a case under policy $\overline{T}<\infty$.
- The second term is positive since $\omega^*(c)-\omega'^*(c)>0$. Moreover, under a policy of permanent affirmative action $\overline{T}'=\infty$,

$$\begin{aligned}\omega'^*(c) &= \sum_{t=1}^{\infty}\mathbb{P}(t)\big(\mathbb{P}_t'^*(\{aa\}|c)g^{-1}(c)+(1-\mathbb{P}_t'^*(\{aa\}|c))c\big)\\ &< c\end{aligned}$$

where the second equality comes from the fact that $\mathbb{P}_t'^*(\{aa\}|c)>0$ for all t. Since as seen previously, $\omega^*(c) \xrightarrow[\delta\to 1]{} c$ for any $\overline{T} < \infty$, then we conclude that $\omega^*(c) - \omega'^*(c) \xrightarrow[\delta\to 1]{} \Delta$ for some $\Delta > 0$ and thus the second term is bounded away from 0 in the limit. This captures the gain to group A of stopping affirmative action after a finite number of periods.

From the above three observations, we can formally state that $\forall\epsilon > 0$, there exists $\overline{T} < \infty$ large enough and $\overline{\delta}(\overline{T}) \in (0, 1)$ such that $\forall\delta \in (\overline{\delta}(\overline{T}), 1)$

$$|\mathbb{E}[c] - \mathbb{E}'[c]| < \epsilon,$$

$$\frac{|B|}{|A| + |B|}\gamma_B \int (c - \omega^*(c))f_{B,n_{\overline{T}}}(c)dc \sum_{s=\overline{T}}^{\infty} \mathbb{P}(t) < \epsilon$$

and

$$\frac{|A|}{|A| + |B|}\gamma_A \int (\omega^*(c) - \omega'^*(c))f_A(c)dc > 2\epsilon$$

from which it follows that the right-hand side of Eq. (12) is positive and thus that Eq. (11) is verified.

To complete the proof, we now turn to the case when $\lambda_B < 1$.

First note that, unsurprisingly, group A gains from stopping affirmative action, whereas group B loses (when δ is high enough). Thus, rearranging the left-hand side of Eq. (12) as follows

$$\sum_{t=1}^{\infty} \frac{\delta^{t-1}(1-\delta)}{(|A| + |B|)}\big((W_{A,t} - W'_{A,t}) + \lambda_B(W_{B,t}) - W'_{B,t})\big),$$

we notice that decreasing the weight λ_B placed on the welfare of group B to values strictly smaller than 1 keeps this quantity positive. We can thus conclude that it will still be worth stopping affirmative action after $\overline{T} < \infty$ periods as opposed to continuing it forever. The first-best optimal policy $\overline{T}_{\lambda_B}$ for some $\lambda_B < 1$ will thus be such that $\overline{T}_{\lambda_B} \leq \overline{T}_{\lambda_B=1} < \infty$.

Let us now verify that, group A gains from stopping affirmative action, whereas group B loses. From the decomposition in Eq. (12), we can write the welfare gain of group A from stopping affirmative action as

$$\sum_{t=1}^{\infty} \frac{\delta^{t-1}(1-\delta)}{(|A| + |B|)}\big(W_{A,t} - W'_{A,t}\big) = \sum_{t=1}^{\infty} \mathbb{P}(t)\frac{|A|}{|A| + |B|}(1 + \gamma_A)\int (\omega^*(c) - \omega'^*(c))f_A(c)dc > 0.$$

This is positive, even when $\gamma_A = 0$, since $\omega^*(c) > \omega'^*(c)$. We can also write the welfare gain associated with group B from stopping affirmative action as

$$\begin{aligned}\sum_{t=1}^{\infty} \frac{\delta^{t-1}(1-\delta)\lambda_B}{(|A| + |B|)}\big(W_{B,t} - W'_{B,t}\big) \quad &= \quad \sum_{t=1}^{\infty} \mathbb{P}(t)\frac{\lambda_B|B|}{|A| + |B|}\int [\sigma_t\omega^*(g(c)) + (1-\sigma_t)\omega^*(c)]f_{B,n_t}(c)dc \\ &\quad - \sum_{t=\overline{T}}^{\infty} \mathbb{P}(t)\frac{\lambda_B|B|}{|A| + |B|}\gamma_B \int (c - \omega^*(c))f_{B,n_{\overline{T}}}(c)dc \\ &\quad - \sum_{t=1}^{\infty} \mathbb{P}(t)\frac{\lambda_B|B|}{|A| + |B|}\int \omega'^*(g(c))f_{B,n_t}(c)dc. \qquad (15)\end{aligned}$$

The second term
$$-\sum_{t=\overline{T}}^{\infty}\mathbb{P}(t)\frac{\lambda_B|B|}{|A|+|B|}\gamma_B\int(c-\omega^*(c))f_{B,n_{\overline{T}}}(c)dc \xrightarrow[\delta\to 1]{} 0$$
since $\omega^*(c) \xrightarrow[\delta\to 1]{} c$ and is negative for any $\delta < 1$ since $c > \omega^*(c)$, i.e. members of group B suffer a feeling of injustice after affirmative action has been stopped.

The first term can be split into
$$\sum_{t=1}^{\infty}\mathbb{P}(t)\frac{\lambda_B|B|}{|A|+|B|}\int\sigma_t\omega^*(g(c))f_{B,n_t}(c)dc = \sum_{t=1}^{\overline{T}-1}\mathbb{P}(t)\frac{\lambda_B|B|}{|A|+|B|}\int\omega^*(g(c))f_{B,n_t}(c)dc \xrightarrow[\delta\to 1]{} 0$$
and
$$\begin{aligned}\sum_{t=1}^{\infty}\mathbb{P}(t)\frac{\lambda_B|B|}{|A|+|B|}\int(1-\sigma_t)\omega^*(c)f_{B,n_t}(c)dc \quad &= \quad \frac{\lambda_B|B|}{|A|+|B|}\int\omega^*(c)f_{B,n_{\overline{T}}}(c)dc\sum_{t=\overline{T}}^{\infty}\mathbb{P}(t)\\ &\xrightarrow[\delta\to 1]{} \quad \frac{\lambda_B|B|}{|A|+|B|}\int cf_{B,n_{\overline{T}}}(c)dc\sum_{t=\overline{T}}^{\infty}\mathbb{P}(t)\\ &> \quad 0\end{aligned}$$
since $\omega^*(c) \xrightarrow[\delta\to 1]{} c$.

Finally, the third term
$$-\sum_{t=1}^{\infty}\mathbb{P}(t)\frac{\lambda_B|B|}{|A|+|B|}\int\omega'^*(g(c))f_{B,n_t}(c)dc < 0$$
is bounded below 0.

Thus as δ approaches 1, the right hand side of Eq. (15) approaches
$$\frac{\lambda_B|B|}{|A|+|B|}\int cf_{B,n_{\overline{T}}}(c)dc\sum_{t=\overline{T}}^{\infty}\mathbb{P}(t)-\sum_{t=1}^{\infty}\mathbb{P}(t)\frac{\lambda_B|B|}{|A|+|B|}\int\omega'^*(g(c))f_{B,n_t}(c)dc < 0$$
which is indeed negative since $\omega'^*(g(c)) > c$ (i.e. beneficiaries of affirmative action do not suffer a feeling of injustice) and since f_{B,n_t} first-order stochastically dominates $f_{B,n_{\overline{T}}}$ for $t > \overline{T}$. Group B would therefore benefit from affirmative action continuing permanently.

Note that when $\lambda_B < 1$, the parameter γ_A governing the feeling of injustice of group A can be 0 and the first-best policy would still prescribe stopping affirmative action after a finite number of periods. On the other hand, if λ_B were to be strictly greater than 1 and thus if the government placed more weight on the welfare of group B than that of group A, then it might become necessary at some point for γ_A to be sufficiently positive in order to justify stopping affirmative action. ∎

References

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