Expectation Formation, Local Sampling and Belief Traps: A new Perspective on Education Choices*

Simon GLEYZE† Philippe JEHIEL‡

Abstract

Lack of diversity in higher education is partly driven by long-run belief distortions about admission chances at elite colleges. We depart from the rational expectation framework and propose a concrete model of expectation formation in which students estimate their admission chances by sampling peers who previously applied to elite colleges but need not exactly share the same characteristics as themselves. Two types of inefficiencies arise in steady state: high-achieving disadvantaged students self-select out of elite colleges, and average students from advantaged families apply to elite colleges even though their true admission chances are null. We then explore the additonial inefficiencies induced by competition across neighborhoods with different wealth characteristics, and we investigate the efficiency of several policy instruments such as quotas, affirmative action or the mixing of neighborhoods.

*We thank Roland Bénabou, Francis Bloch, Gabriel Carroll, Gabrielle Fack, Renato Gomez, Julien Grenet, Marc Gurgand, Yinghua He, Ronny Razin, Al Roth, Olivier Tercieux as well as seminar participants at PSE for useful comments. Jehiel thanks the European Research Council for funding (grant n° 742816).

†Paris School of Economics and University Paris 1 Panthéon-Sorbonne. Email: gleyze.simon@gmail.com

‡Paris School of Economics and University College London. Email: jehiel@enpc.fr

1 INTRODUCTION

According to the rational expectations paradigm that is commonly used by economists, students when applying to elite colleges should form *correct* beliefs about their admission chances, in particular assessing correctly how the admission chances vary with observable characteristics such as the results obtained in ability tests. By contrast, sociologists argue that students are embedded in their social environment and obtain information by observing the decisions made by others, leading to mistakes and biases. We note that there is ample evidence that agents hold incorrect beliefs that are correlated across agents in the social network.[1]

Neither economists nor sociologists, however, propose a coherent framework for understanding the two-way interactions between expectation formation and the social environment. In the rest of the paper, we propose such a framework in the particular context of a career choice problem in which students have to decide whether or not to apply to elite colleges.

Specifically, we assume that students differ in two dimensions: their ability (accessible through standardized test, say), and their cost of being rejected from elite colleges (that can alternatively be thought of as an opportunity cost induced by rejection). Students strategically choose one out of two occupations: unqualified jobs on the labor market (or non-selective vocational training), and elite colleges. Elite colleges have limited seats and select only the best students up to their capacity. Importantly, we assume that students *do not form rational expectations regarding their admission chances*. We consider instead that they form their expectations by estimating the admission probability using a sample of past experiences from their peers. This estimation procedure is constrained in three ways: First, the sample is endogenous and consists only of students who applied in the past to elite colleges. Second, the sample must have a size no smaller than some threshold τ viewed as necessary to make the statistics derived from the sample sufficiently reliable.

[1]To name a few, Kapor et al. (2020) elicit students' subjective admission chances in a low-income district of Connecticut uncovering important departures from rational expectations. On the social network dimension, Altmejd et al. (2020) show that older sibling's enrollment in college increases a younger sibling's probability of enrolling in college at all.

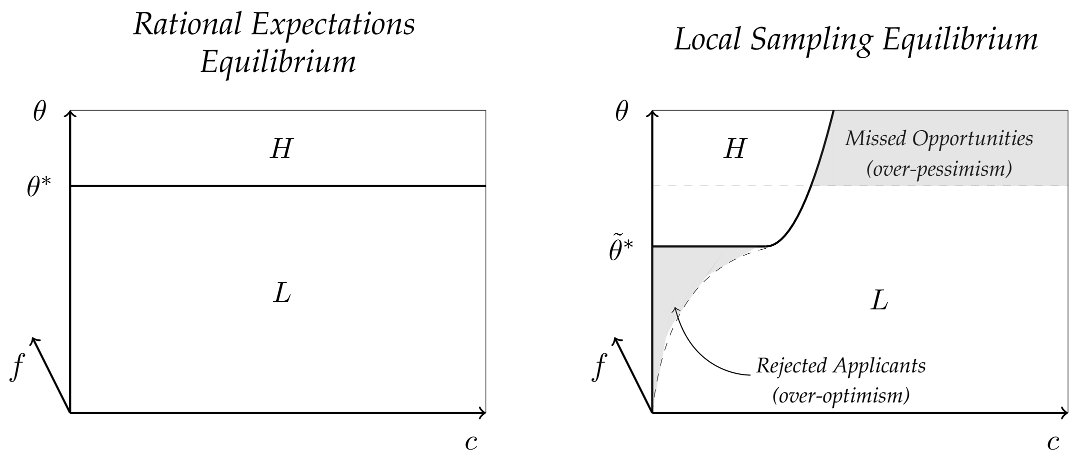

Figure 1: *The x-axis represents students' cost, the y-axis represents ability, and the z-axis is the population density. There are two occupations: H are elite colleges that have limited capacity, and L are jobs with no qualifications. (Left) Allocation of students to occupations in a rational expectations equilibrium. (Right) Allocation of students to occupations in a local sampling equilibrium. The shaded areas represent students who are mismatched: the top-right square corresponds to high-achieving disadvantaged students who self-select in non-selective colleges; the bottom-left triangle corresponds to average-achieving advantaged students who apply to elite colleges but are rejected.*

Third, students ask in priority peers with similar ability.[2] We introduce the "local sampling equilibrium" representing steady states of a process in which students best respond to their subjective beliefs viewed as the empirical frequencies of admissions in the samples, and subjective beliefs are consistent with the above sequential sampling estimation procedure.

In our model with a continuum of students and no aggregate uncertainty, under rational expectations, students perfectly sort in each occupation based on their ability and the equilibrium is efficient. Things are different in the

[2]The rationale for the first constraint is that information about rejection rates is typically confined in practice to the pool of applicants as for non-applicants this would rely on counterfactuals which are rarely accessible. The rationale for the second constraint can be thought of in relation to the bias-variance trade-off, which is a core idea in statistics. The rationale for the third constraint is that it seems plausible that students would know that the admission probabilities depend on ability, therefore leading them to ask first peers with similar ability. We briefly discuss in appendix the case in which students would ask according to the proximity in cost and observe it induces more biases.

sampling equilibrium. Indeed, if students were to follow the same strategy as in the rational expectations equilibrium, no student who apply would get rejected. Thus according to the sampling procedure, every student would apply believing that the acceptance probability is 1 whatever the type of student. Of course, when all students apply, some are rejected given the capacity constraint, thereby yielding a contradiction. Our first general result is that in a local sampling equilibrium two types of inefficiencies arise: First, some high-achieving disadvantaged students self-select out of elite colleges. Second, some average-ability advantaged students apply to elite colleges but are rejected. This equilibrium mismatch is due to the fact that average (or above) students who apply and get rejected induce a *strategic externality* on high-achieving students by distorting their perceived admission chances downward, and average (or below) students who apply and get accepted induce a strategic externality on low-achieving students by distorting their beliefs upward. These strategic externalities arise because of the combination of rationing at elite colleges and the non-rational character of expectations, which leads both high and low ability students to rely on average-ability peers with different admission results to compute their admission chances. By contrast, there is no rationing on the labor market, hence there are no payoff-relevant distortions for students in the assessment of this alternative. See Figure 1 for a graphical representation of the rational expectations equilibrium and the local sampling equilibrium. We observe that the local sampling equilibrium moves gradually toward the rational expectations equilibrium, as one reduces the size τ of the samples. At the other extreme, when τ is large, all students have the same expectation about the admission chances irrespective of their type. The sampling procedure can be seen as inducing in students' minds a kind of regression to the mean when assessing the link between ability and the admission chance, where the mean admission rate is endogenously determined by the application strategy of students.

Our second main investigation concerns the study of competition across several neighborhoods where students across all neighborhoods compete for the same seats, but sampling takes place locally, separately in each neighborhood. Our main question of interest concerns how asymmetries across neighborhoods in the sampling size and/or in the cost distributions affect the welfare in the various neighborhoods as well as the average quality of admitted students. Simple results showing the comparative advantage of neighborhoods using smaller sampling size and/or having smaller costs are established in extreme cases, and simulations pointing to non-trivial comparative statics in general are provided. We also study the effect on redistribution and welfare of standard policy instruments such as quotas, affirmative action (based on neighborhood) and the mixing of neighborhoods. While we establish the redistributive role such policies may have, we provide several results suggesting that the effects of these on total welfare may be ambiguous or even positive.[3]

Related Literature At a methodological level, our model of belief formation can be viewed as offering a balance between strategic sophistication as usually considered in economics—which is empirically supported by some studies (Agarwal and Somaini, 2018)—and the embeddedness of students' beliefs as usually considered in sociology. This is to be contrasted with the "undersocialized" view of an atomic agent that forms correct beliefs independently from her environment as well as the "oversocialized" accounts of expectation formation in which students mechanically inherits the beliefs of their parents or have social capital fully account for educational choices (the sociology literature has departed from this Bourdieusian view in the last 20 years, see for instance Aschaffenburg and Maas (1997)).[4]

There is a growing empirical literature on expectation formation in education, broadly divided between beliefs on the returns to schooling and subjective admission chances.

Very few papers investigate subjective admission chances, which is the focus of our paper. Most notably, Hastings and Weinstein (2008) show that providing information about school quality and odds of admission to low-

[3]While more research would be needed in light of the hot debates surrounding the assessments of such policies, our results may be viewed as providing a tentative explanation as to why researchers have found little empirical basis for the mismatch hypothesis which asserts that affirmative action necessarily results in boosted students being admitted to colleges for which they are otherwise unqualified and reduce welfare (see Alon and Tienda (2005)).

[4]The distinction between undersocialized and oversocialized explanations is due to Granovetter (1985).

income families with high-achieving students increases application to good schools. It is unclear, however, if the effect is driven by growing awareness about these schools or changing expectations. Kapor et al. (2020) directly elicit admission probabilities of students facing a centralized school choice mechanism that rewards strategic behavior. They find that households play strategically, but do so with miscalibrated beliefs. Belief errors, however, do not seem to correlate with observable characteristics such as race or economic status. Finally, Altmejd et al. (2020) show that older sibling's enrollment in a better college increases a younger sibling's probability of enrolling in college at all, especially for families with low predicted probabilities of enrollment.

The empirical literature on the perceived returns to schooling that is less directly related to our model is mixed. In Wisconsin, Dominitz and Manski (1994) find that the perceived returns from a Bachelor's degree compared to a high school diploma are positive. In Chile, Hastings et al. (2015) show that low-achieving disadvantaged students who apply to low-earning college degree programs overestimate earnings for past graduates by over 100%, while beliefs for high-achieving students are correctly centered. Conversely in the Dominican Republic, Jensen (2010) find that the perceived returns to secondary school are extremely low, despite high measured returns.

The first theoretical model of expectation formation on the returns to schooling is due to Manski (1993). He postulates an additive log-income equation, and he assumes that students infer the returns to schooling by taking the conditional expectation of log-income. If students omit to condition on ability—e.g., because they do not observe the ability of their peers—he shows that more low-ability and less high-ability students enroll to college. To some extent, our model can be viewed as enriching that kind of approach to expectation formation about admission chances. We also add to Manski the idea of local sampling and also of selection neglect to the extent that our students do not take into account that in their sampling they only see students applying to elite colleges (see also Esponda (2008) and Jehiel (2018) for the modelling of selection neglect in other environments).

There is a vast literature on social learning illustrating that past cohorts' behavior influences the expectations of current cohorts (Banerjee, 1992; Bikhchandani et al., 1992; Ellison and Fudenberg, 1995). These papers, however, typically assume that agents have enough prior information to infer the outcome of counterfactual actions using Bayes' rule. Manski (2004) relaxes this assumption by considering a social learning environment in which students have no prior belief on the distribution of outcomes conditional on actions—as in our model. Hence students cannot infer anything on counterfactual actions. Only assuming the stationarity of the outcome distribution—as we do in this paper[5]—he shows that learning induces a process of sequential reduction in ambiguity. Though similar in motivation, our papers differ with the social learning literature because we account for strategic interactions among students which are instrumental to produce belief distortion.

Finally, several papers in behavioral game theory have introduced various departures from the rational expectation hypothesis. These include among others the cursed equilibrium Eyster and Rabin (2005), the analogy-based expectation equilibrium (Jehiel, 2005) or the Berk-Nash equilibrium Esponda and Pouzo (2016). The spirit of our approach is maybe closest in spirit to Jehiel (2005) who introduces a model of coarse expectations in which players bundle actions into classes. In equilibrium, players best-respond to their analogy-based expectations, and expectations correctly represent the average behavior in every class. Our paper is based on a different learning rule where students average the outcome of an endogenously chosen group of players and do not bundle actions, whereas in Jehiel (2005) players average the outcome of an exogenously given bundle of actions using past observations from an exogenously given group of players (see however Jehiel and Mohlin (2022) for a model that endogenizes the analogy classes of Jehiel (2005) based on a similar bias-variance trade-off as the one used to motivate our sampling heuristic, see also Mohlin (2014) on the bias-variance trade-off).

2 Setup

We introduce a stylized model of career choice with strategic students and rationing at elite colleges. There is a unit mass of students indexed by their ability $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}_+$, and by their cost $c \in [\underline{c}, \overline{c}] \subseteq \mathbb{R}_+$. There is a probability distribution F on $N \equiv [\underline{\theta}, \overline{\theta}] \times [\underline{c}, \overline{c}]$ with continuous density f that has full

[5] Meaning that colleges never modify their admission criteria.

support. In the main text, we view the cost as an opportunity cost from being rejected from elite colleges.[6] In the Appendix, we discuss richer formulations for the opportunity cost and briefly develop the analysis for the application cost formulation.

Students choose among two occupations: going directly on the labor market (or a non-selective vocational training) L, or applying to selective colleges H. Without loss of generality, the utility of attending an elite college is $U^H(\theta) = \theta$, whereas for simplicity we assume that the utility of going directly on the labor market is $U^L(\theta) = 0$ for all θ.

Students can apply to only one occupation: the action space is then $A = \{L, H\}$. There is no rationing for going on the labor market. Elite colleges, however, have a limited number of seats and they select students with the highest ability (among the pool of applicants) up to their capacity $q \ll 1$.[7] The payoffs are as follows:

– If student (θ, c) goes on the labor market L her utility is 0.

– If student (θ, c) applies to H and obtains a seat, her utility is θ.

– If student (θ, c) applies to H but does not get a seat, she goes on the labor market and her utility is $-c$.

A natural interpretation of our model is that if a student applies to H and gets rejected, he loses some time before entering the job market resulting in a loss of c. A more elaborate interpretation would allow for three levels H, M and L of applications where L represents the labor marker and H and M represent high and medium ranked colleges respectively. The initial choice is whether to apply to H or M and in case of rejection only L would be left. Our model corresponds to a stylized version of this, assuming there is limited capacity constraint on M.

Getting back to our model, a strategy profile $\sigma : N \longrightarrow \Delta A$ is a (mesurable) function from the population of students to mixed actions. This is a binary

[6] We have in mind that poorer students generally have a higher rejection cost since conditional on being rejected at an elite college, poorer students would have benefited more from going directly on the labor market.

[7] Our results are unchanged if colleges only receive a noisy signal about students' ability.

action game, hence we let $\sigma(\theta, c) \in [0, 1]$ simply denote the probability that student (θ, c) applies to H.

A key object that drives the choice of student (θ, c) is the subjective probability this student (subjectively) assigns to obtaining a seat at an elite college conditional on applying to H. In both the rational case and our approach, this subjective probability turns out to depend only on θ and we denote it by $p(\theta)$ accordingly. Based on $p(\theta)$, student (θ, c) applies to H whenever

$$p(\theta)\theta - (1 - p(\theta))c \geq 0$$

This leads to the following definition of an optimal strategy profile.[8]

DEFINITION 1. *σ is optimal given subjective beliefs $p(\cdot)$ if*

$$\sigma(\theta, c) = \begin{cases} 1 & \textit{when} \quad c \leq \frac{p(\theta)}{1-p(\theta)}\theta \\ 0 & \textit{when} \quad c > \frac{p(\theta)}{1-p(\theta)}\theta \end{cases}$$

For any strategy profile, let $\theta(\sigma)$ denote the cutoff at H such that any student with ability $\theta > \theta(\sigma)$ who applies to H is admitted. It is defined as follows: $\theta(\sigma) = \underline{\theta}$ when

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{c}}^{\overline{c}} \sigma(\theta, c) f(\theta, c) \,\mathrm{d}c \,\mathrm{d}\theta < q$$

Otherwise, $\theta(\sigma)$ is uniquely defined as the largest θ^* such that

$$\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{\overline{c}} \sigma(\theta, c) f(\theta, c) \,\mathrm{d}c \,\mathrm{d}\theta = q$$

Subjective beliefs are rational when they are consistent with the admission cutoff, given a strategy profile.

[8] For completeness, we assume that the student applies to H when indifferent, but how indifferences are resolved plays no role in the analysis.

Definition 2. *$p^R(\cdot)$ is rationally consistent with σ if*

$$p^R(\theta) = \begin{cases} 1 & when \quad \theta \geq \theta(\sigma) \\ 0 & when \quad \theta < \theta(\sigma) \end{cases}$$

Therefore, the rational expectations equilibrium is defined as follows.

Definition 3 (Rational Expectations Equilibrium)**.** *σ^R is a rational expectations equilibrium if there exist subjective beliefs p^R such that σ^R is optimal given p^R and p^R is rationally consistent with σ^R.*

Let us now characterize the unique rational expectations equilibrium—thus proving existence. Given the strategy profile σ and the consistency of beliefs, it is optimal to apply to H for all students with ability $\theta > \theta^*$ where $\theta^* = \theta(\sigma)$ as defined above. It follows that in a rational expectations equilibrium, the admission cutoff θ^* solves

$$\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{\overline{c}} f(c,\theta)\, \mathrm{d}c\, \mathrm{d}\theta = q \iff \theta^* = H^{-1}(1-q)$$

where H denoted the cdf of the marginal distribution of θ. In other words, the equilibrium allocation of students to occupations can be described by the cutoff strategy consisting in applying to H whenever θ is higher than $H^{-1}(1-q)$, thereby yielding:

Proposition 1 (Equilibrium Characterization)**.** *In the unique rational expectations equilibrium, students $N^H = \{(\theta, c) : \theta > H^{-1}(1-q)\}$ obtain a seat at elite colleges, and $N^L = N \setminus N^H$ go on the labor market.*

(All formal proofs appear in the Appendix). The rational expectations equilibrium induces perfect assortative matching as students sort across occupations based on their ability. Namely, high-achieving students go to elite colleges, and average- or low-ability students go on the labor market. No student applying to H gets rejected. See Figure 1 (Left) above for a graphical illustration of the equilibrium.

Define welfare as

$$W(\sigma) = \int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta,p(\theta))} \theta f(\theta,c)\,\mathrm{d}c\,\mathrm{d}\theta - \int_{\underline{\theta}}^{\theta^*} \int_{\underline{c}}^{c^H(\theta,p(\theta))} c f(\theta,c)\,\mathrm{d}c\,\mathrm{d}\theta$$

where $c^H(\theta, p(\theta))$ is the cost below which student (θ, c) applies to H conditional on admission chances $p(\theta)$. In the rational expectation equilibrium, $c^H(\theta, p(\theta)) = \overline{c}$ for all $\theta \geq H^{-1}(1-q)$ and $c^H(\theta, p(\theta)) = \underline{c}$ for all $\theta < H^{-1}(1-q)$. Rational expectations induce perfect sorting, which is welfare maximizing.

3 Expectation Formation and Belief Traps

In this section we introduce a simple model of expectation formation based on extrapolations and sampling, and we show, among other things, how it leads to persistent belief distortions among high-achieving disadvantaged students—so-called "belief traps."

Students have no prior over the distribution of admissions conditional on applications. We assume that they non-parametrically estimate this distribution by averaging the outcome of their peers who are closest to them in terms of ability. Let $\mathcal{B}(N)$ denote the set of measurable subsets of N.

Definition 4. *The sample for action H of student (θ, c) conditional on a strategy profile σ (from the previous generation) is*

$$S(\theta, c \mid \sigma) = \underset{B \in \mathcal{B}(N)}{\arg\inf} \left\{ \int_B |\theta - \tilde{\theta}|\,\mathrm{d}F(\tilde{\theta}, \tilde{c}) : \int_B \sigma(\tilde{\theta}, \tilde{c})\,\mathrm{d}F(\tilde{\theta}, \tilde{c}) > \min(\tau, \tau(\sigma)) \right\}$$

where $\tau(\sigma) = \int_N \sigma(\tilde{\theta}, \tilde{c})\,\mathrm{d}F(\tilde{\theta}, \tilde{c})$ is the total mass of students applying to H.

In words, S is the set with mass τ of students applying to H and having ability closest to θ. There is a convex penalty of including students with dissimilar ability, hence the sample $S(\theta, c \mid \sigma)$ is rectangular and it can be described by a simple index:

$$b(\theta, \sigma) = \inf \left\{ b > 0 : \int_{\max\{\underline{\theta}, \theta - b\}}^{\min\{\overline{\theta}, \theta + b\}} \int_{\underline{c}}^{\overline{c}} \sigma(\tilde{\theta}, \tilde{c})\,\mathrm{d}F(\tilde{\theta}, \tilde{c}) > \tau \right\}.$$

This means that the sample for action H of student (θ, c) is obtained by taking all students with ability $\theta' \in [\theta - b(\theta,\sigma), \theta + b(\theta,\sigma)]$ regardless of their cost. See Figure 2 below for a graphical illustration.

We can now define subjective admission chances. As in the previous section, we denote by $\theta(\sigma)$ the admission cutoff at elite colleges given the strategy profile σ. The subjective admission chances at elite colleges H are obtained by averaging the experiences of the students in the sample.

DEFINITION 5. *Subjective admission chances at elite colleges p are τ-consistent with σ if*[9]

$$p(\theta) = \frac{1}{\min(\tau, \tau(\sigma))} \int_{S(\theta,c|\sigma)} \sigma(\tilde{\theta}, \tilde{c}) \mathbf{1}\{\tilde{\theta} > \theta(\sigma)\} \, \mathrm{d}F(\tilde{\theta}, \tilde{c}).$$

We now introduce our solution concept, the local sampling equilibrium, which requires optimality of actions and consistency of beliefs.

DEFINITION 6 (Local Sampling Equilibrium)**.** *σ is a local sampling equilibrium if there exists p such that σ is optimal given p and p is τ-consistent with σ.*

We interpret this solution concept as the stationary point of an intergenerational model of learning in which students of the current generation ask peers from the previous generation the outcome of their behavior. Therefore, this sample is completely endogenous as it depends on the strategy profile of the previous generation. Importantly, students know nothing ex-ante about the admission process: it could be either because schools do not disclose their admission criteria, or because students lack the ability to understand the admission process, or because they do not trust publicly disclosed information. Therefore, students entirely rely on the information provided by their social network. Of course, this is a stylized assumption and in practice we expect students to use a mix of information sources to form their expectations.

We made two assumptions on the learning process. First, students care about the precision of their estimate hence they must acquire a sufficient amount of data for each action. Formally, this means that students ask a

[9] If a mass of students less than τ chooses H, then we divide by $\int_B \sigma^k(\tilde{\theta}, \tilde{c}) \, \mathrm{d}F(\tilde{\theta}, \tilde{c})$ instead of τ.

mass $\tau \in (0, 1]$ of students from the previous generation, where τ is interpreted as a confidence parameter. This parameter captures a bias-variance trade-off: if the sample it too small then subjective admission chances are unbiased because they are computed using students with similar ability, but the estimator is noisy.[10] Conversely, if the sample is too large then subjective admission chances are precisely estimated but they are more likely to be biased.

Second, students contact in priority peers with similar ability. This can be justified on the ground that if students know that the admission probability is somewhat correlated with their ability, then they might reduce bias by asking peers with similar ability.[11] From another perspective, one can view our sampling technology as the one inducing the smallest distortions away from rationality, so that any inefficiency identified within our setup is likely to persist with alternative sampling specifications. In Appendix B, we briefly discuss the case when bundling is made on similarity in c, and we illustrate how extra inefficiencies would arise in this case.

Note that students include in their sample only peers who *actually* applied to H in the previous period. Therefore, students make no inference using counterfactual outcomes—i.e., they are not asking their peers "What would have been your admission chances at x conditional on applying there?". Who is included in the sample is endogenous and typically differ for each player, even though sample size is identically equal to τ for each player. Concretely, the perimeter of the sample for H of low-ability disadvantaged students is very large because no close ties ever apply to H. Therefore, they will need to ask high-achieving peers who have very different characteristics which induce a large bias in the subjective admission chances. In general, a larger perimeter implies a larger bias because the sample includes students with very different characteristics, whereas a smaller perimeter implies a smaller bias.

[10]This is a reduced-form interpretation because there is no actual noise in the estimate as students sample from a continuum of peers.

[11]Alternatively, one can view this assumption as reflecting the hypthosesis that prior to applying to colleges, students have been grouped according to their ability, thereby leading to ties more naturally linked to ability.

Existence. We apply a fixed point argument on the mapping from subjective beliefs $p : \Theta \longrightarrow [0,1]$, to best responses σ as derived from Definition 1, compounded with the mapping from strategies σ to subjective beliefs as derived from Definition 5. The fixed point exists if each sub-map is continuous. It is easy to see that the best response σ has a threshold structure that varies continuously with p. Moreover, the sample bounds $b(\theta, \sigma)$ are continuous in the strategy profile σ, and so are subjective beliefs p. This shows the existence of a pure strategy local sampling equilibrium. (The formal proof appears in the Appendix).

Equilibrium Characterization. In the characterization below we make the simplifying assumption that cost and ability are independently distributed, i.e., $f(\theta, c) = h(\theta)g(c)$. Fixing ability and the subjective admission chances, students who apply to H have a cost $c < c^H(\theta, p(\theta))$ where

$$c^H(\theta, p(\theta)) = \frac{p(\theta)}{1 - p(\theta)}\theta.$$

The total mass of applicants to H is then:

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta, p(\theta))} f(\theta, c)\,\mathrm{d}c\,\mathrm{d}\theta.$$

In a local sampling equilibrium, the ability of the last student admitted to H, denoted θ^*, is such that the mass of applicants at H is equal to the capacity of elite colleges:

$$\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta, p(\theta))} f(\theta, c)\,\mathrm{d}c\,\mathrm{d}\theta = q.$$

Given our independence assumption, this equation can be simplified into:

$$\int_{\theta^*}^{\overline{\theta}} h(\theta) G\left(\frac{p(\theta)}{1 - p(\theta)}\theta\right) \mathrm{d}\theta = q \tag{1}$$

Let us now derive the equation that guarantees τ-consistency of subjective admission chances. The subjective admission chances of student (θ, c)

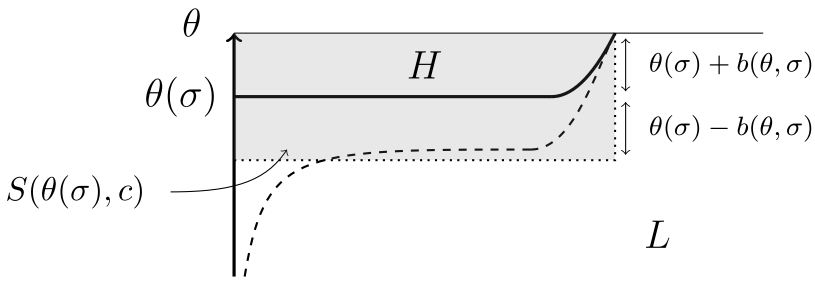

Figure 2: *Construction of the sample for the last student admitted at an elite college* $(\theta(\sigma), c)$ *in* (c, θ)*-space. The sample, represented in the shaded box, includes approximately a mass* τ *of students who applied to an elite college* H*. All students above the dashed line applied to* H *(i.e.* $\sigma(\theta, c) = 1$*) but only those above the solid line got admitted at an elite college. Rejected students exert a strategic externality on higher achieving students by distorting their estimated admission chances downard.*

are τ-consistent if they solve the following equation:

$$p(\theta) = \frac{1}{\tau} \int_{\max\{\underline{\theta}, \theta - b(\theta,\sigma)\}}^{\min\{\overline{\theta}, \theta + b(\theta,\sigma)\}} \int_{\underline{c}}^{c^H(\tilde{\theta}, p(\tilde{\theta}))} \mathbf{1}\left\{\tilde{\theta} > \theta^*\right\} \mathrm{d}F(\tilde{c}, \tilde{\theta}).$$

where $b(\theta, \sigma)$ is derived as explained above. With our independence assumption, this can be rewritten as:

$$p(\theta) \quad = \quad \frac{1}{\min(\tau, \tau(\sigma))} \int_{\max\{\underline{\theta}, \theta - b(\theta,\sigma)\}}^{\min\{\overline{\theta}, \theta + b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1 - p(\tilde{\theta})} \tilde{\theta}\right) \mathbf{1}\left\{\tilde{\theta} > \theta^*\right\} \mathrm{d}H(\tilde{\theta}) \quad (2)$$

where

$$\tau(\sigma) \quad = \quad \int_{\max\{\underline{\theta}, \theta - b(\theta,\sigma)\}}^{\min\{\overline{\theta}, \theta + b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1 - p(\tilde{\theta})} \tilde{\theta}\right) \mathrm{d}H(\tilde{\theta})$$

In equilibrium, θ^* must solve (1) given $p(\theta)$, and $p(\theta)$ must solve (2) for all students (θ, c) given θ^*.

We can now compare equation (1) with the equation that defines the last student admitted to H in a rational expectations equilibrium:

$$\int_{\theta^*}^{\overline{\theta}} h(\theta)\, \mathrm{d}\theta = q. \quad (3)$$

If there are students with sufficiently high costs—e.g. if g has full support on $\mathbb{R}_+$—any small belief distortion in equation (2) will induce self-selection among disadvantaged students: $c > c^H(\theta, p(\theta))$. Then, the term under the integral sign in (1) is smaller than in (3) because $G(c^H(\theta, p(\theta))) < 1$ as $c^H(\theta, p(\theta)) < c \leq \bar{c}$. Therefore, the ability of the last admitted student at H in a local sampling equilibrium θ^* must be smaller than in a rational expectations equilibrium to fill all the seats in equation (1).

We just proved that two types of inefficiencies arise in a local sampling equilibrium: high-achieving disadvantaged students self-select out of elite colleges even though their actual admission probability is one, and low-achieving advantaged students spend inefficient resources in applications at elite colleges even though their actual admission chances are zero. See Figure 1 in the introduction for a graphical representation of the two inefficiencies.

PROPOSITION 2 (Equilibrium Characterization)**.** *Suppose that g has full support on $\mathbb{R}_+$ and assume ability and cost are independent. For all $q < 1$ there exists $\theta^* \in (0,1)$ and $(\sigma(\theta), p(\theta))$ that solve (1) and (2) such that in this local sampling equilibrium students $N^H = \{(\theta, c) : \theta > \theta^*, c \leq c^H(\theta, p(\theta))\}$ obtain a seat at elite colleges and $N^L = N \setminus N^H$ go on the labor market. There are two types of inefficiencies:*

1. *Missed opportunities: all students (θ, c) with ability $\theta > \theta^*$ and cost $c > c^H(\theta, p(\theta))$ self-select out of elite colleges.*

2. *Inefficient applications: all students (θ, c) with ability $\theta < \theta^*$ and cost $c < c^H(\theta, p(\theta))$ apply to H but are rejected and suffer a cost $-c$.*

Observe that compared to the rational expectations equilibrium both the supply side and the demand side suffer from inefficiencies. On the supply side, belief distortion arises endogenously and leads to payoff-relevant mistakes for high-achieving students and low-achieving advantaged students. On the demand side, the quality of the pool of admitted students at elite colleges is lower than with rational expectations due to equilibrium mismatch.

We now describe comparative statics with respect to the confidence parameter τ. When $\tau \to 0$ students form their expectations using an infinitesimal sample of individuals. As it turns out, in our model this leads to rational

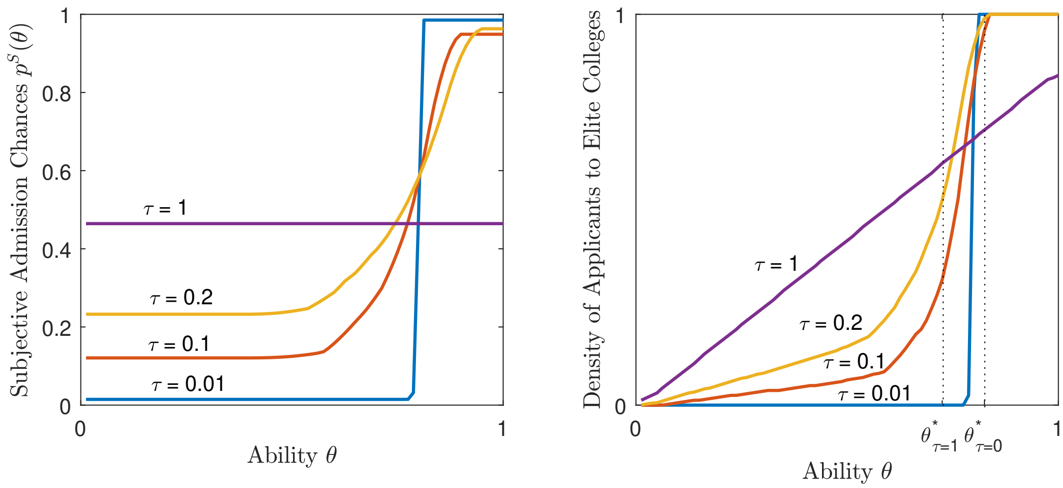

Figure 3: *(Left) Subjective admission chances as a function of student ability. Bias in subjective beliefs increases with the confidence parameter* τ. *(Right) Density of applicants to H as a function of student ability. As* τ *increases, the admission cutoff* $\tilde{\theta}^*$ *decreases, the number of self-selecting students (on the right of the cutoff) increases and the number of inefficient applicants (on the left of the cutoff) increases as well.*

expectations because students do not bias their estimate with dissimilar students. Indeed, taking the limit $\tau \to 0$ of the implicit equation (2) we can see that if $\theta < \theta^*$ then there is τ_* small enough such that $\theta + b(\theta, \sigma) < \theta^*$ and $\theta - b(\theta, \sigma) < \theta^*$. Therefore, the integral in (2) is zero, and we have $p(\theta, c) = 0$. Similarly, one can verify that for all $\theta > \theta^*$, $p(\theta, c) = 1$. Therefore, only the best students apply to elite colleges and the last student admitted in a local sampling equilibrium coincides with that of rational expectations.

Students, however, do not form expectations using one data point, formalized in our model by assuming that τ is strictly positive, away from 0. To reduce risk induced by imprecise estimates, they are more likely to include the outcome of multiple peers. In our model, belief distortions increase with the confidence level τ because students include peers with very different characteristics in their sample. Hence bias in the estimate stems from a selection bias that increases with τ. As $\tau \to 1$ (i.e., students include the entire population), the subjective beliefs of the entire population converge. In practice, we would expect intermediary values of τ so as to trade-off bias and precision of the estimate.

This comparative statics is illustrated in Figure 3 (Left). Figure 3 (Right) illustrates the two types of inefficiencies that arise in a local sampling equilibrium. We see that as the confidence parameter τ increases, the admission cutoff $\tilde{\theta}^*_\tau$ decreases. Subjective beliefs, however, move smoothly around this threshold hence the mass of student who apply to H with an ability that is below the cutoff $\tilde{\theta}^*_\tau$ is positive (inefficient applications), and the mass of students who apply to H with an ability that is above the cutoff is below one (missed opportunities).

PROPOSITION 3. *In any local sampling equilibrium, a higher confidence parameter τ leads to more self-selection from high-achieving disadvantaged students and to more inefficient applications from low-achieving advantaged students. As τ converges to 0, the local sampling equilibrium converges to the rational expectations equilibrium.*

Comment. The same conclusion does not hold when students sample in priority peers with similar cost c as opposed to peers with similar ability θ. As we show in Appendix, in this case, inefficiencies arise even as τ converges to 0 (essentially because it leads to subjective beliefs that depend on c when in reality they depend on θ).

3.1 Large sampling window $\tau = 1$

We conclude this section with the case in which τ is large enough so that students take the average success rate in the entire neighborhood as their subjective probability of acceptance. This case allows us to characterize more completely the sampling equilibrium, and the resulting analysis will be used in the next Section when several neighborhoods will compete for the same seats in elite colleges.

More precisely, when $\tau = 1$, the subjective admission belief $p(\theta)$ is constant independently of θ, which simplifies the analysis. To show this most simply, we continue to assume here that cost and ability are independently distributed. To fix ideas, we assume ability is uniformly distributed on $[0, 1]$ and we let as before G denote the cdf of the distribution of c. For a given

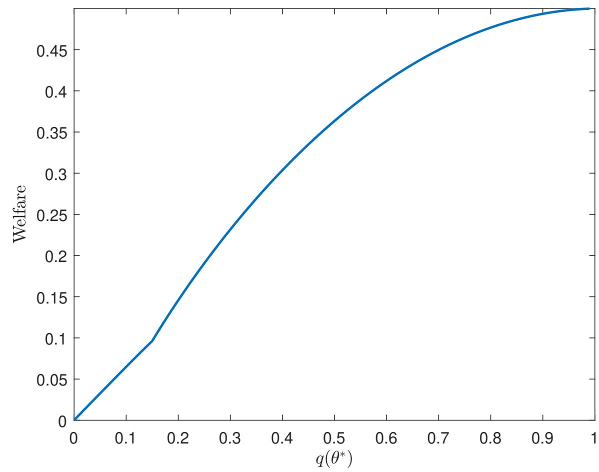

Figure 4: *Total welfare is monotonic in the number of allocated seats* $q(\theta^*)$.

capacity q, subjective admission chances are equal to the capacity at elite colleges divided by the mass of applicants:

$$p = q \Big/ \int_0^1 G\left(\frac{p}{1-p}\theta\right) \mathrm{d}\theta$$

Given that $p \mapsto p \int_0^1 G(\frac{p}{1-p}\theta) \mathrm{d}\theta$ is a strictly increasing function of p with value 0 at $p = 0$ and 1 at $p = 1$, we obtain that for each q there is a unique $\widetilde{p}(q)$ satisfying the above equation.

We can prove equilibrium uniqueness.

PROPOSITION 4. *When τ gets close to 1 and $f(\theta, c) = g(c)$, then the local sampling equilibrium is unique.*

As will be convenient when analyzing the multiple neighborhood case, it is useful to parameterize the equilibrium by the admission threshold θ^* defined for a given q by

$$\int_{\theta^*}^1 G\left(\frac{p}{1-p}\theta\right) \mathrm{d}\theta = q$$

where p is $\widetilde{p}(q)$ as previously defined. We also define a function which takes

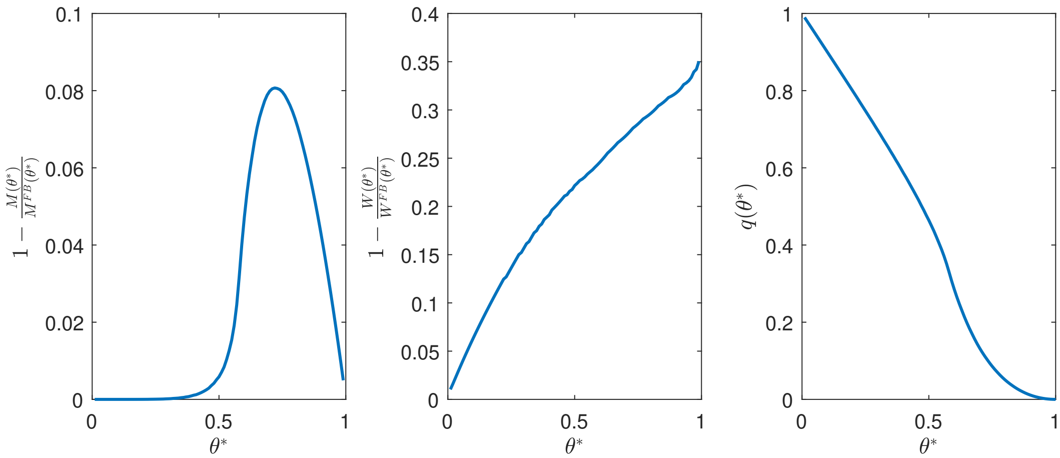

Figure 5: *Simulation for $G(\theta) = c/2$. (Left) Quality loss in a local sampling equilibrium as a function of the admission threshold θ^* (Middle) Welfare loss in a local sampling equilibrium as a function of the admission threshold θ^*. (Right) Number of seats allocated in equilibrium as a function of the admission threshold θ^*.*

the value 0 at an equilibrium belief p:

$$J(p;\theta^*) = \frac{\int_{\theta^*}^1 G(\frac{p}{1-p}\theta)\,\mathrm{d}\theta}{\int_0^1 G(\frac{p}{1-p}\theta)\,\mathrm{d}\theta} - p.$$

The first term should be understood as the ratio between the number of accepted students to the number of applicants. Hence, in equilibrium this should be equal to p when the sampling window is $\tau = 1$. To guarantee the existence of a root to the equation $J(p;\theta^*) = 0$, we make the following assumption.

Assumption 1. *The function $J(p;\theta^*)$ is decreasing in p.*

This assumption is satisfied when G is a uniform distribution, which we are going to assume in most of the analysis of the next section. Moreover, when G is a uniform distribution welfare is monotonically increasing in the number of allocated seats, as shown in Figure 4 (this holds for any support of g).

It is useful to define measures of quality loss and welfare loss between a local sampling equilibrium and the first-best (achieved in a rational expectations equilibrium). Let the average ability (θ) of admitted students be

$$M(\theta^*) = \frac{\int_{\theta^*}^{1} G\left(\frac{p(\theta^*)}{1-p(\theta^*)}\theta\right)\theta \, \mathrm{d}\theta}{\int_{\theta^*}^{1} G\left(\frac{p(\theta^*)}{1-p(\theta^*)}\theta\right) \, \mathrm{d}\theta}$$

to be compared to $M^{FB}(\theta^*) = 1 - \frac{q(\theta^*)}{2}$, the corresponding first-best average quality when there are $q(\theta^*)$ seats. Similarly, $W(\theta^*)$ is defined as in Section 2 and $W^{FB} = q(\theta^*)(1 - \frac{q(\theta^*)}{2})$. In Figure 5, we plot the ratio of these quantities to assess the relative loss of quality and welfare induced by biased beliefs.

For a uniform distribution with support $(0, \overline{c})$, we can solve for equilibrium beliefs in closed form. Noting that $G(c) = \frac{c}{\overline{c}}$ on $[0, \overline{c}]$, subjective admission chances solve

$$p = q \Big/ \left[\int_0^1 \min\left\{\frac{\frac{p}{1-p}\theta}{\overline{c}}, 1\right\} \mathrm{d}\theta\right]. \tag{4}$$

We first consider the case in which $\min\left\{\frac{\frac{p}{1-p}\theta}{\overline{c}}, 1\right\} = \frac{\frac{p}{1-p}\theta}{\overline{c}}$ for all $\theta \in [0, 1]$. Then, the above condition amounts to a simple quadratic function and solving for p yields:

$$p = -q\overline{c} + \sqrt{q^2\overline{c}^2 + 2q\overline{c}} \tag{5}$$

We now characterize under what conditions the minimum does not bind. Substituting the expression for p in the following equation:

$$\frac{\frac{p}{1-p}}{\overline{c} - \underline{c}}\theta < 1$$

and solving for q yields

$$\hat{q}(\theta) = \frac{\overline{c}^2}{2\theta^2\left[(1 + \frac{\overline{c}}{\theta})^2\overline{c} - 2(1 + \frac{\overline{c}}{\theta})\frac{\overline{c}}{\theta}\overline{c}\right]}$$

Therefore, the subjective beliefs are given by (5) whenever the capacity at elite colleges verifies $q < \sup_\theta \hat{q}(\theta)$. If this condition is violated, however, we define $\hat{\theta}$ as the ability that solves $q = \hat{q}(\hat{\theta})$ and we decompose the integral in (4) into two integrals on the intervals $[0, \hat{\theta}]$ and $[\hat{\theta}, 1]$ and then solve for p accordingly.

Remark. For intermediate values of τ, it is not possible to obtain closed form solutions, therefore we run simulations (see Figure 3). As we already discussed, the bias increases with the confidence parameter τ because of selection of students with dissimilar characteristics in the sample.

4 COMPETING NEIGHBORHOODS

We consider now the case of multiple neighborhoods competing for the same positions. The neighborhood plays a role only in shaping the samples from which students form their subjective assessment, as we assume the sampling is made locally (only within the neighborhood to which the student belongs). The fact that students from the various neighborhoods compete for the same seats creates a linkage between the various neighborhoods as the threshold ability θ^* above which students get admitted has to be the same across neighborhoods. This linkage in turn induces externalities across neighborhoods the effects of which are the main subject of interest of this Section. To formalize the questions of interest, consider a two-neighborhood setup. Neighborhood $i = 1, 2$ consists of a unit mass of students with (θ_i, c_i) distributed according to distribution f_i and sampling window τ_i. Consider first neighborhood i in isolation, assume there is a mass q_i of seats available for students in this neighborhood and that students follow strategy σ_i. We let $\theta(\sigma_i, q_i)$ be the corresponding threshold admission ability in this neighborhood. It is computed as shown in Section 3. An equilibrium is formally defined as follows.

DEFINITION 7. *A local sampling equilibrium with competing neighborhoods $i = 1, 2$ (with characteristics f_i and τ_i) and total mass q of seats is a strategy profile (σ_1, σ_2) such that there exist q_1, q_2 satisfying*

1. *σ_i is a local sampling equilibrium in the neighborhood i with a mass q_i of seats;*

2. *$q_1 + q_2 = q$ and,*

3. $\theta(\sigma_1, q_1) = \theta(\sigma_2, q_2)$.

The definition of welfare W_i and average ability of admitted students M_i in neighborhood i are adapted accordingly. Denote $W = W_1 + W_2$ the aggregate welfare, and $M = \frac{q_1 M_1 + q_2 M_2}{q_1 + q_2}$ the average ability of admitted students.

In this section, we are interested in (i) the strategic interactions across neighborhoods, and (ii) how asymmetries across neighborhoods impact welfare and the average quality of admitted students. We consider asymmetries in sampling window $\tau_i \neq \tau_j$, and asymmetries in cost distributions—fixing the distribution of ability. When varying τ we will assume that τ is either 0 or 1 to make things simpler. When considering asymmetric distributions, we will consider that in both neighborhoods θ_i is uniformly distributed on $[0, 1]$ and c_i is distributed according to cdf G_i, independently of θ_i. G_i will be taken to be a uniform distribution on $[\underline{c}_i, \overline{c}_i]$ in most results and simulations. Toward the end of the Section, we briefly consider a case of symmetric neighborhoods in which the distributions of ability and cost are correlated. This serves to illustrate the possibility of multiple equilibria.

We also discuss the impact of several policies commonly considered to mitigate inequalities across neighborhoods: quotas in which each neighborhood receives a number of seats proportional to its size, place-based affirmative action according to which abilities in one neighborhood are boosted for the purpose of student admission, and the mixing of neighborhoods (i.e., directly changing the composition of neighborhoods).

4.1 Asymmetries in Sampling Window

We first investigate asymmetries in sampling windows, namely $\tau_i \neq \tau_j$. This arises naturally when neighborhoods are of different size, and students ask a fixed number of peers to construct their estimate. In this case, students in the smaller neighborhoods mechanically communicate with a larger fraction of their peers.

To keep things simple, we consider within our model an extreme situation where the sampling window in neighborhood i goes to zero (i.e., neighborhood i is very large) whereas in neighborhood j students contact all their peers (i.e., neighborhood j is very small).[12] We show that neighborhood j is disadvantaged and obtain less seats at elite colleges.

PROPOSITION 5. *Suppose that $G_i = G_j$ and consider a sequence (τ_i^n) such that $\tau_i^n \underset{n\to\infty}{\longrightarrow} 0$ and $\tau_j = 1$, then $\lim_{n\to\infty} q_i^n > q_j$. If G_i and G_j are uniform, this implies that $\lim_{n\to\infty} W_i^n > W_j$.*

To understand the result, observe that the set of admitted students is identical whether $\tau_i^n \longrightarrow 0$ or $G = \delta_0$ (point mass at zero cost). Indeed, as the sampling window becomes smaller, the sampling bias on p_i goes to zero and each student has an asymptotically unbiased estimator of his admission chances. Therefore, students apply to H if and only if $\theta_i \geq \theta^*$. Instead, when $G = \delta_0$ all students apply to H, and only students with $\theta_i \geq \theta^*$ are admitted. Therefore, the set of admitted students is identical in both cases. Now, when the cost distribution goes to zero, it is quite intuitive that students never self-select and take a larger number of seats at elite colleges. This is proven more precisely in Appendix.

4.2 Asymmetries in Cost Distribution

We now investigate asymmetries in cost distribution, i.e. $G_i \neq G_j$. This can arise due to differences in opportunity costs or in social norms for instance: the cost of not attending an elite college might be higher in some communities than others.

To illustrate the effect of such asymmetries most sharply, we start with an extreme situation where the cost is zero in neighborhood i (i.e. $G_i = \delta_0$) whereas the cost is arbitrary but non-zero in neighborhood j. We show that neighborhood j is disadvantaged and obtain less seats at elite colleges.

PROPOSITION 6. *Suppose that $\tau_i = \tau_j$, and $G_i = \delta_0$ but $G_j \neq \delta_0$. Then, $q_i > q_j$. If G_j is a uniform distribution, this implies that $W_i > W_j$.*

Moving to more general forms of cost asymmetries, comparative statics with respect to the cost distribution, however, are not always intuitive in our

[12] Even though in our model, neighborhoods i and j are of equal size, we could think of neighborhood j as being split into identically small neighborhoods as as to justify that $\tau_i \ggg \tau_j$, in line with the size asymmetry just suggested.

model. Using simulations, we show that a first order stochastic shift in G_j with respect to G_i does not necessarily imply that W_j/W_i decreases, as one might expect.

Specifically, in our simulations, we consider two cases. First, a situation in which costs are uniformly distributed on $[0, 0.1]$ in neighborhood i and uniformly distributed on $[0, 0.3]$ in neighborhood j. Second, a situation in which costs are uniformly distributed on $[0, 0.5]$ in neighborhood i and uniformly distributed on $[0.5, 1]$ in neighborhood j. (See Figure 6 below, case without quotas). In the second situation, the "disadvantaged" neighborhood has more seats than the "advantaged" neighborhood. This counter-intuitive effect is due to the fact that it is on average more "risky" for students in neighborhood j to apply to an elite college, hence only the best students apply to H. This increases the admission threshold, which induces more self-selection in neighborhood i. In equilibrium, students in j end up more optimistic about their admission chances than students in i, yielding $q'_j/q'_i > q_j/q_i$ and $W'_j/W'_i > W_j/W_i$.

4.3 Policy Instruments

We discuss the effect of three possible policy interventions. The first one consists in imposing quotas, pre-defining the number of seats each neighborhood should have in proportion to the size of the neighborhood. The second one considers an affirmative action policy consisting in boosting the ability index θ in one neighborhood in the admission process. Both these interventions amount to modifying the number of seats reserved to each neighborhood as compared with the laissez-faire outcome. Finally, the third policy consists in changing the compositions of the two neighborhoods by imposing some degree of mixing while leaving the equilibrium force determines the number of seats assigned to each neighborhood. When considering these interventions, we will discuss the effect in terms of welfare, in terms of expected quality of admitted students as well as a comparison of how the two neighborhoods benefit from the intervention.

Quotas. While we could consider the effect of more general quota specifications, we focus here on the case in which the two neighborhoods should have a number of seats proportional to their size, i.e. $q_i = q_j$. We investigate the impact on welfare (compared to the first-best allocation). We show that quotas are a redistribution tool across neighborhoods, but that they do not always lead to welfare gains. Indeed, in the uniform case with small capacities at elite colleges quotas are welfare neutral.

PROPOSITION 7. *Consider two neighborhoods with costs uniformly distributed on* $[0, \overline{c}_i]$ *and* $[0, \overline{c}_j]$ *with* $\overline{c}_i < \overline{c}_j$*. As the number* q *of seats gets small, quotas increase the welfare of neighborhood* i*, but have no effect on aggregate welfare at the first order.*

The redistributive advantage of quotas applies more generally as illustrated by the following Proposition.

PROPOSITION 8. *Consider two neighborhoods with costs uniformly distributed on* $[0, \overline{c}_i]$ *and* $[0, \overline{c}_j]$*. Suppose that* $q < \max\{\sup_\theta \hat{q}_i(\theta), \sup_\theta \hat{q}_j(\theta)\}$*. With quotas, subjective admission chances decrease in the advantaged neighborhood, and increase in the disadvantaged neighborhood compared to the case without quotas.*

The welfare neutrality result of Proposition 7, however, seems more specific to the conditions of that Proposition. As the next proposition shows, if inequality across neighborhoods is initially very large then quotas can increase the quality of admitted students which can increase aggregate welfare. This is illustrated under an extreme specification of our model with uniform distributions.

PROPOSITION 9. *Suppose that cost is zero for all students in neighborhood* i *and uniformly distributed on* $[0, \overline{c}_j]$ *in neighborhood* j*. As* $\overline{c}_j \longrightarrow 0$ *and* $q \longrightarrow 0$*, quotas increase aggregate welfare compared to the case without quotas.*

The intuition for the welfare-enhancing effect of quotas in Proposition 9 is as follows. Without quotas, many average-ability students from neighborhood i get admitted to an elite college because applications are costless for them, and many high-ability students from j self-select. With quotas, however, the best students from both groups get admitted, which raises welfare. Moreover, as cost in the poor neighborhood vanishes, inefficient applications in this neighborhood have no impact on welfare, thereby leading to a welfare advantage for the quota intervention.

In our simulations with uniform cost distribution with support $[0, 0.1]$ in neighborhood i and uniform cost distribution with support $[0, 0.3]$ in neighborhood j, we do not see much effect of quotas on welfare and ability. However, quotas have a significant redistributive effect. They are useful to transfer welfare from neighborhood i to neighborhood j, as shown in the next figure. Without quotas, the welfare in neighborhood j represents half of the welfare in neighborhood i for low capacity at elite colleges. Instead, with quotas, welfare in the two neighborhoods are roughly identical.

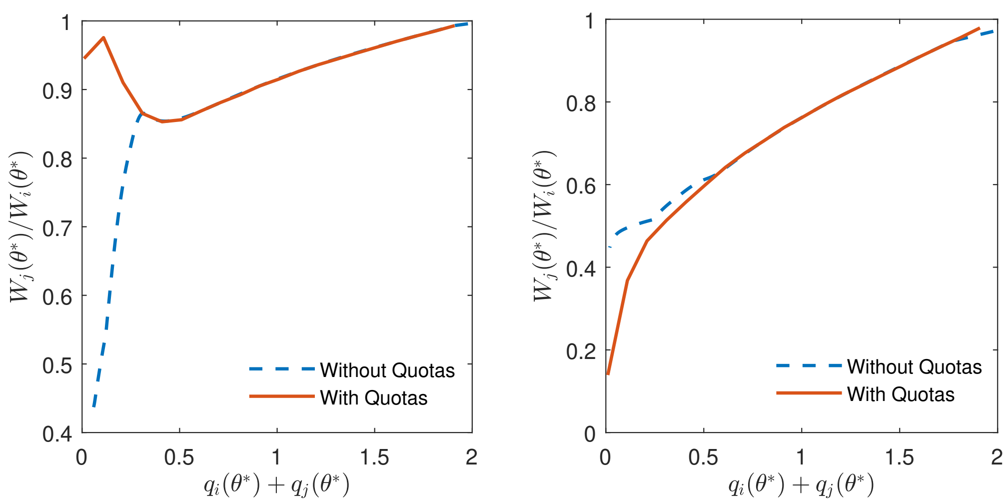

Figure 6: *Relative distribution of welfare across neighborhoods. (Left) Cost is uniformly distributed on* $[0, 0.1]$ *in* i *and* $[0, 0.3]$ *in* j*. (Right) Cost is uniformly distributed on* $[0, 0.5]$ *in* i *and* $[0.5, 1]$ *in* j*.*

As we mentioned in the previous section, when costs are high, some counter-intuitive statics may arise in terms of which neighborhood gets more seats. In such cases, quotas may reinforce inequalities, since then quotas would take away seats from the "disadvantaged neighborhood" as compared to the situation without quotas.

Place-Based Affirmative Action. Another common policy intervention is to provide a boost to the score of students from the disadvantaged neighborhood. We assume that there is a bonus $\kappa > 0$, such that a student in neighborhood i with ability θ is treated similarly as a student in neighborhood j with ability $\theta+\kappa$. As a result, the equilibrium condition 3 of Definition 7 should be replaced by $\theta(\sigma_i, q_i) + \kappa = \theta(\sigma_j, q_j)$ (with the other conditions being unchanged). Our main result is that in the uniform distribution case, when the number of seats is small, a small affirmative action intervention is welfare neutral and benefits the neighborhood enjoying the intervention.

PROPOSITION 10. *Consider two neighborhoods with costs uniformly distributed on $[0, \bar{c}_i]$ and $[0, \bar{c}_j]$ with $\bar{c}_i < \bar{c}_j$, and assume that $q < \max\{\sup_\theta \hat{q}_i(\theta), \sup_\theta \hat{q}_j(\theta)\}$. As the intervention κ gets small, affirmative action increases the welfare of neighborhood i, but has no effect on aggregate welfare at the first order.*

At some abstract level, the affirmative action policy κ has an effect similar to that of quotas to the extent that any κ can be associated with a quota policy in which the number of seats reserved to i corresponds to the number of seats obtained by i in equilibrium when the affirmative action policy κ prevails. The limiting case κ small would correspond to reserving slightly more seats to neighborhood i as compared with the laissez-faire. Proposition 10 establishes a neutrality result similar to that in Proposition 7 even if it does not follow from it (given that in Proposition 7 we assumed that the two neighborhoods should receive the same number of seats).

Mixed Neighborhoods. We investigate here whether moving students from the high cost neighborhood to the low cost neighborhood (and vice versa) increases welfare. Unlike quotas which do not change students' social network, this intervention exactly aims at reducing inequalities of social capital. We consider random reallocation, i.e. from two initial neighborhoods with cost distributions G_i and G_j we draw new neighborhoods from the following compound distributions:

$$\tilde{G}_i = \alpha G_i + (1-\alpha) G_j$$
$$\tilde{G}_j = \alpha G_j + (1-\alpha) G_i$$

The parameter α scales the equalization across neighborhoods: for $\alpha = 1$ there is no reallocation of students, and for $\alpha = \frac{1}{2}$ the new neighborhoods

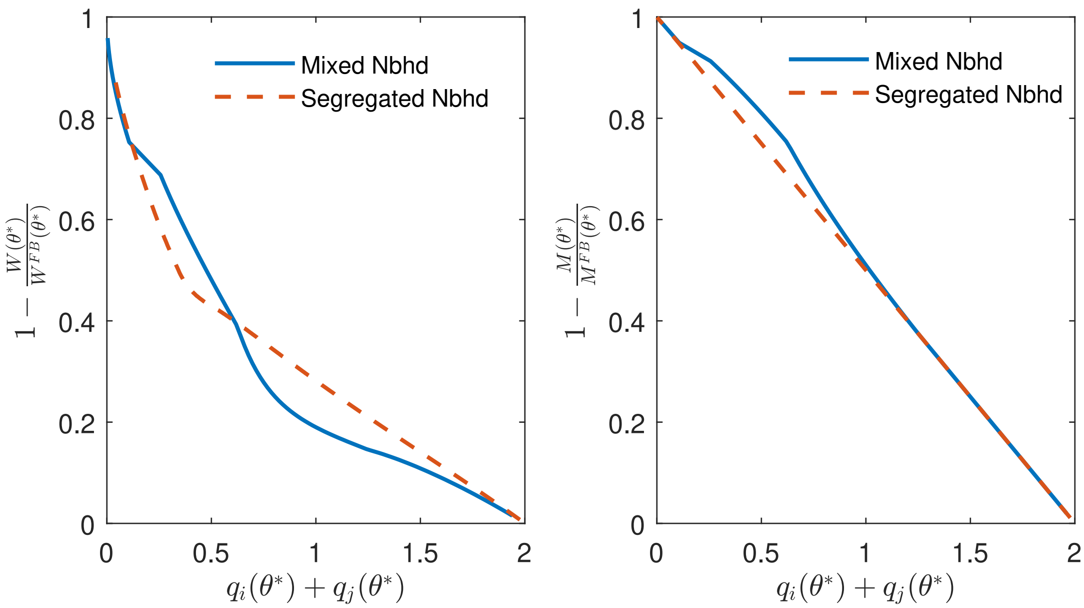

Figure 7: *Welfare loss and quality loss with respect to the first best allocation (i.e., rational expectations). Mixing is obtained with* $\alpha = 1/2$ *and cost is uniformly distributed on* $[0, 0.5]$ *in* i *and* $[0.5, 1]$ *in* j*.*

have equal cost distributions.

Our main analytical result is that mixing is welfare neutral in the uniform case when the number of seats is small.

PROPOSITION 11. *Consider two neighborhoods with costs uniformly distributed on* $[0, \bar{c}_i]$ *and* $[0, \bar{c}_j]$*. Suppose that* $q < \max\{\sup_\theta \hat{q}_i(\theta), \sup_\theta \hat{q}_j(\theta)\}$*. Subjective admission chances* p_i, p_j *are independent of the degree of mixing* α*. Therefore, mixing is neutral on welfare and average quality.*

More general analysis of the effect of mixing has proven difficult. Figure 7 reports simulations where mixing can be welfare-enhancing.

4.4 Equilibrium Multiplicity

We conclude by investigating the role of equilibrium multiplicity on belief traps. In most of the analysis so far, we have assumed that cost and ability are independent. It turns out that this independence rules out equilibrium multiplicity.

PROPOSITION 12. *Suppose that for each neighbourhood $i = 1, 2$, $J_i(p, \theta^*)$ is monotonically decreasing in both arguments, then the local sampling equilibrium is unique.*

In some practical situations, however, it could be that cost and ability are not independently distributed. When multiple neighborhoods compete for the same seats at elite colleges, this can lead to equilibria with belief traps in which one neighborhood takes many more seats than the other—even if both neighborhoods are ex-ante identical. Clearly, given the symmetry of the problem, any neighborhood can take the role of being favored in equilibrium, thereby illustrating the possibility of multiple equilibria with possibly very strong asymmetries. The following Proposition illustrates this in an extreme form.

PROPOSITION 13. *Suppose that in both neighborhoods there is a mass α of students with $(\theta, c) = (0, 0)$ and a mass $1 - \alpha$ of students with $(\theta, c) > (0, 0)$ (with arbitrary distribution) and assume $1 > q > 0$. Then for α small enough, there is an equilibrium in which all seats at H are taken by students from neighborhood i.*

In the equilibrium of Proposition 13, only very low ability students in neighborhood j apply to elite college, and they all get rejected. They apply to elite colleges because their costs of rejection are negligible (even null in the formal statement). But, by applying and being rejected such students create a strong negative externality on high ability students in neighborhood j, as the latter get convinced they are better off not applying (even for moderate rejection costs). On the other hand, when $1 > q > 0$ and α is small enough, one can guarantee that at the same time there is a mass no less than q of students from neighborhood i with $\theta > 0$ who apply to H, thereby ensuring that all seats are taken by students in i.[13] While extreme, we believe that Proposition 13 is suggestive that multiple equilibria can easily arise in our setting (in particular when low rejection cost students tend to have low ability), which in turn may suggest that some asymmetries in outcome may sometimes be the result of historical factors rather than fundamental asymmetries.

[13]The limit $\alpha \to 0$ corresponds to the model studied above and when $q < 1$ and there is only one neighborhood, the equilibrium is such that the demand for the elite college in that neighborhood exceeds the capacity, thereby providing the required property.

5 Conclusion

We have introduced a model of expectation formation in a career choice problem. Unlike the rational expectations framework, students were assumed to have no prior information and no prior belief as to how elite colleges admit students. We have assumed instead that students non-parametrically estimate the distribution of outcomes conditional on actions by averaging past experiences from their peers with similar characteristics. Formally, we have introduced a new solution concept—the local sampling equilibrium—in which players best respond to their subjective expectations, and expectations are consistent with the average outcomes of their peers. We believe this provides a coherent framework for thinking the strategic interactions between expectation formation and the social environment.

We have derived three main results. First, expectation formation leads to belief traps whereby high-achieving disadvantaged students self-select out of elite colleges, and average-ability advantaged students take their seats at elite colleges. This is due to the fact that average students create a strategic externality on high-achieving students by distorting their perceived admission chances toward the mean. This leads to multiple inefficiencies: on the supply side, high-achieving disadvantaged students go on the labor market instead of attending elite colleges, whereas low-achieving advantaged students spend resources applying to elite colleges even though their actual admission chances are zero. On the demand side, the pool of admitted students is of lower quality compared to the rational expectations benchmark.

Second, in our setting with multiple neighborhoods, we have suggested that a decrease in the average cost in one neighborhood may have a negative impact on self-selection in *other* neighborhoods. This type of cross-neighborhood externality arises because rationing at elite colleges acts as a propagation mechanism of local demand shocks. Indeed, a reduction of cost in one neighborhood induces a higher admission cutoff, leading to a lower admission rates in other neighborhoods hence more self-selection. This may suggest that growth inequality across locations disproportionately benefits advantaged neighborhoods at the expense of poor neighborhoods.

Finally, we have suggested a potential benefit of quotas and affirmative action in mitigating the effects of neighborhood inequalities. While more work would be needed to quantify these effects and study their robustness, we believe that our model of expectation formation can serve as a building block in empirical studies on education choices.[14]

[14]For example, our insights and framework can be used to shed light on why there is little empirical support for the "mismatch hypothesis" which asserts that affirmative action policies results in minority students being admitted to colleges for which they are otherwise unqualified—leading to lower graduation rates and eventually harming minority students.

Appendix A Application Cost

An alternative to the opportunity cost is to consider an application cost: all else equal, it is harder for disadvantaged students to apply to elite colleges because they don't have access to peers or professional who can help them in the process.

The payoffs are as follows:

- If student (θ, c) goes on the labor market L her utility is 0.
- If student (θ, c) applies to H and obtain a seat, her utility is $\theta - c$.
- If student (θ, c) applies to H but does not get a seat, she goes on the labor market and her utility is $-c$.

Student (θ, c) applies to H whenever $p(\theta)\theta - c \geq 0$, that is, whenever $c^H(\theta, p) \leq p(\theta)\theta$. Define welfare as

$$W(\sigma) = \int_{\theta^*}^{\overline{\theta}} \int_0^{c^H} \theta f(\theta, c)\, \mathrm{d}c\, \mathrm{d}\theta - \int_0^1 \int_0^{c^H} c f(\theta, c)\, \mathrm{d}c\, \mathrm{d}\theta$$

It is readily verified that with one neighborhood: (i) the rational expectation equilibrium with application cost is identical than with opportunity cost, and (ii) the local-sampling equilibrium with application cost is identical than with opportunity cost (up to the thresholds c^H). Therefore, with one neighborhood the analysis and the qualitative predictions are very similar.

With multiple neighborhoods, the welfare effect of policy instruments is similar with application cost and opportunity cost. For instance, quotas are welfare neutral with uniform cost.

Proposition 14. *Consider two neighborhoods with costs uniformly distributed on $[0, \overline{c}_i]$ and $[0, \overline{c}_j]$. As $q \longrightarrow 0$, quotas have no effect on aggregate welfare at the first order.*

Proof of Proposition 14. First, we derive subjective admission chances in the case with quotas. We consider the following neighborhood specific quotas: $q_i = q_j = \frac{q}{2}$. The subjective admission chances for each neighborhoods write:

$$p_i = \sqrt{\overline{c}_i q} \qquad p_j = \sqrt{\overline{c}_j q}$$

The neighborhood specific admission cutoff $\tilde{\theta}_i^*$ solves

$$\int_{\tilde{\theta}_i^*}^{1} \frac{p_i}{\overline{c}_i} \theta \, \mathrm{d}\theta = \frac{q}{2} \iff \tilde{\theta}_i^* = \sqrt{1 - q\frac{\overline{c}_i}{p_i}}. \tag{6}$$

The admission cutoff in neighborhood j is similar, replacing p_i with p_j.

Second, we approximate $W(\theta^*)$ at the first order and show that it is independent of $\overline{c}_i$ and $\overline{c}_j$. As $q \longrightarrow 0$, we have

$$W_i(\theta^*) = p_i \frac{1 - (\theta^*)^3}{3\overline{c}_i} - \frac{p_i^2}{6\overline{c}_i}.$$

For $q \longrightarrow 0$, we make the following approximation: $(\theta^*)^3 \approx 1 - 3\sqrt{q\overline{c}_i}$. Therefore we obtain $W_i \approx \frac{5}{6}q$ at the first order. Hence, $W(\theta^*) \approx \frac{5}{3}q$ is independent of $\overline{c}_i, \overline{c}_j$. □

APPENDIX B BUNDLING ON COST

In the main text, we assume that students bundle peers based on ability (and indirectly based on cost when there are multiple neighborhoods). Here we explore the converse where students bundle peers based on cost directly. Overall, this increases the various inefficiencies (as students bundle on a dimension that is irrelevant for admissions), up to the point that as $\tau \longrightarrow 0$ we do not converge to the rational expectation equilibrium, unlike the case with bundling on ability.

PROPOSITION 15. *Suppose that students bundle peers only based on cost c. Then, as $\tau \longrightarrow 0$, the local sampling equilibrium does not converge to the rational expectation equilibrium.*

Proof. Let $A(\tau) \subset [\underline{c}, \overline{c}] \times [\underline{\theta}, \overline{\theta}]$ be the set of admitted students in any local sampling equilibrium. For this to converge to a REE, we must have $\lim_{\tau \to 0} A(\tau) = [\underline{c}, \overline{c}] \times [\theta^*, \overline{\theta}]$ and $p(c) = 0$ or 1 for any c. But notice that $\lim_{\tau \to 0} A(\tau) = [\underline{c}, \overline{c}] \times [\theta^*, \overline{\theta}]$ implies $\lim_{\tau \to 0} p(c) = 1 - \theta^*$ which is not equal to 0 or 1, a contradiction. □

APPENDIX C PROOFS

Existence of Local Sampling Equilibria. Consider the following scheme:

$$p \longmapsto \sigma^{BR}(p,\cdot) \longmapsto b(\sigma^{BR},\cdot) \longmapsto p(b)$$

By Tychonoff's theorem, the scheme is compact-valued $p(b) \in [0,1]^{\Theta}$. Hence to obtain a fixed point, we just need to prove that the scheme is continuous. Fix a subjective belief map $p : \Theta \longrightarrow [0,1]$. The action space is binary and the subjective admission chances p enter payoffs linearly, hence σ^{BR} is the following measurable threshold strategy:

$$\sigma^{BR}(p,\cdot) = \begin{cases} 1 & \text{if} \quad p(\cdot) \geq \gamma(\cdot) \\ 0 & \text{if} \quad p(\cdot) < \gamma(\cdot) \end{cases}$$

where $\gamma(\theta, c) = \frac{c}{\theta+c}$. Take any converging sequence $p_n \longrightarrow p$. We need to show that $p \longmapsto \sigma^{BR}(p,\cdot)$ is continuous in the L^1-weak topology, namely

$$\int \sigma^{BR}(p_n,(\theta,c))\,\mathrm{d}F \longrightarrow \int \sigma^{BR}(p,(\theta,c))\,\mathrm{d}F.$$

We have

$$\int \sigma^{BR}(p_n,(\theta,c))\,\mathrm{d}F = \int \mathbf{1}\left\{p_n(\theta) \geq \gamma(\theta,c)\right\}\,\mathrm{d}F.$$

Therefore, continuity follows from Lebesgue's dominated convergence theorem. We now show the continuity of $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$. By Berge's maximum theorem, $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$ is upper-hemicontinuous. The loss function $|\theta - \tilde{\theta}|$ is strictly quasi-convex, hence $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$ is continuous. Finally, the continuity of $b \longmapsto p(b)$ follows directly from the integrability of p together with the continuity of the functions $\max\{\cdot,\cdot\}$ and $\min\{\cdot,\cdot\}$. Therefore, by the Schauder fixed point theorem the set of local sampling equilibria is nonempty. □

Proof of Proposition 1. Fix the admission cutoff at $\theta^* = H^{-1}(1-q)$. If

$\sigma^R(\theta, c) = 1$ for all $\theta > H^{-1}(1-q)$ and 0 otherwise, then the beliefs

$$p^R(\theta) = \begin{cases} 1 & \text{when} \quad \theta > H^{-1}(1-q) \\ 0 & \text{when} \quad \theta < H^{-1}(1-q) \end{cases}$$

are rationally consistent with σ^R by definition of θ^*. Given these subjective beliefs, $\sigma^R(\theta, c) = 1$ for all $\theta > H^{-1}(1-q)$ and 0 otherwise is optimal. Therefore, (σ^R, p^R) is a rational expectations equilibrium.

We now prove uniqueness. Suppose that $\sigma(\theta, c) < 1$ for some (positive mass of) $\theta > \theta^*$ and $\sigma(\theta, c) > 0$ for some (positive mass of) $\theta < \theta^*$. By belief consistency, students with ability $\theta > \theta^*$ know that $p^R(\theta) = 1$ (i.e. they can obtain a seat at H for sure) hence they have a profitable deviation. □

Proof of Proposition 2. First we show that all students (θ, c) with ability $\theta > \theta^* = H^{-1}(1-q)$ and cost $c > c^H(\theta^*, p(\theta^*))$ self-select out of elite colleges. Student (θ, c) applies to H only if $p(\theta^*) \geq \frac{c}{\theta^*+c}$. As long as $q < 1$ and $\tau > 0$, we must have $p(\theta^*) < 1$ because the last admitted student (θ^*, c) includes rejected students in her sample. Therefore, as $\lim_{c \to \infty} \frac{c}{\theta^*+c} = 1$ there must exist a positive g-measure of costs such that $p(\theta^*) < \frac{c}{\theta^*+c}$ because g has full support on $\mathbb{R}_+$. This proves that self-selection arises in equilibrium.

Second, we show that students with ability $\theta < \tilde{\theta}^*$ and cost $c < c^H(\tilde{\theta}^*, p(\tilde{\theta}^*))$ apply to H but are rejected. Student (θ, c) with $\theta = \tilde{\theta}^* - \varepsilon$ for $\varepsilon > 0$ arbitrarily small applies to H only if $p(\theta^*) \geq \frac{c}{\tilde{\theta}^*+c}$. As long as $q < 1$ and $\tau > 0$, $p(\theta^*) > 0$ because this student includes in her sample admitted peers for ε small enough. Therefore, as $\lim_{c \to 0} \frac{c}{\tilde{\theta}^*+c} = 0$ there must exist a positive g-measure of costs such that $p(\tilde{\theta}^*) > \frac{c}{\tilde{\theta}^*+c}$ because g has full support on $\mathbb{R}_+$. This proves that inefficient applications arise in equilibrium. □

Proof of Proposition 3. We can rewrite the implicit equation for subjective beliefs as follows:

$$p - \frac{1}{\tau} \int_{\underline{\theta}}^{\bar{\theta}} \mathbf{1}_{\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \cap \{\tilde{\theta} > \tilde{\theta}^*\}} G\left(\frac{p(\tilde{\theta})}{1 - p(\tilde{\theta})}\right) \mathrm{d}H(\tilde{\theta}) = 0 \tag{7}$$

We first consider the case in which $\tau \to 1$. By definition of $b(\theta, \sigma)$ we have

$\lim_{\tau\to 1}\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \supseteq \{\tilde{\theta} > \tilde{\theta}^*\}$. Therefore,

$$\lim_{\tau\to 1}\left[p(\theta) - \frac{1}{\tau}\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{1}_{\{\theta-b(\theta,\sigma)<\tilde{\theta}<\theta+b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1-p(\tilde{\theta})}\right) \mathrm{d}H(\tilde{\theta})\right] = 0$$
$$\iff p = \int_{\underline{\theta}}^{\overline{\theta}} G\left(\frac{p}{1-p}\right) \mathrm{d}H(\tilde{\theta})$$

where the second line uses the fact that, as $\tau \to 1$, the subjective probability becomes independent of θ.

We now consider the case $\tau \to 0$. There are two cases to consider.

Case 1: There exists τ_* small enough such that $\theta + b(\theta,\sigma) < \tilde{\theta}^*$. Then we have $\lim_{\tau\to 1}\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \cap \{\tilde{\theta} > \tilde{\theta}^*\} = \emptyset$. Hence taking the integral in equation (7) is zero, and we directly have that $p(\theta) = 0$.

Case 2: There exists τ^* small enough such that $\theta - b(\theta,\sigma) > \tilde{\theta}^*$. Then we have $\lim_{\tau\to 1}\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \subseteq \{\tilde{\theta} > \tilde{\theta}^*\}$. Therefore,

$$\lim_{\tau\to 0}\left[p(\theta) - \frac{1}{\tau}\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{1}_{\{\theta-b(\theta,\sigma)<\tilde{\theta}<\theta+b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1-p(\tilde{\theta})}\right) \mathrm{d}H(\tilde{\theta})\right] = 0$$

Take $p(\theta) = 1$ and using the fact that $\lim_{x\to\infty} G(x) = 1$ we can rewrite the above equation as follows:

$$\lim_{\tau\to 0}\left[1 - \frac{1}{\tau}\int_{\theta-b(\theta,\sigma)}^{\theta+b(\theta,\sigma)} h(\tilde{\theta})\,\mathrm{d}\tilde{\theta}\right] = 0$$

By L'Hospital's rule and Leibniz integral rule,

$$\lim_{\tau\to 0}\frac{\int_{\theta-b(\theta,\sigma)}^{\theta+b(\theta,\sigma)} h(\tilde{\theta})\,\mathrm{d}\tilde{\theta}}{\tau} = \lim_{\tau\to 0}\Big[h(\theta+b(\theta,\sigma)) + h(\theta-b(\theta,\sigma))\Big]\frac{\partial b(\theta,\sigma)}{\partial\tau} \tag{8}$$

By definition, $b(\theta,\sigma)$ is the smallest $b > 0$ that solves:

$$\int_{\theta-b}^{\theta+b} h(\theta)\,\mathrm{d}\theta > \tau \iff \underbrace{H(\theta+b) - H(\theta-b) - \tau}_{=\Phi(b,\tau)} > 0$$

We apply the implicit function theorem to obtain the derivative of $b(\theta, \tau)$:

$$\frac{\partial \Phi}{\partial b}\frac{\partial b}{\partial \tau} + \frac{\partial \Phi}{\partial \tau} = 0 \iff \frac{\partial b}{\partial \tau} = \frac{1}{h(\theta + b) + h(\theta - b)}$$

Substituting this expression in equation (8) concludes the proof. □

Proof of Proposition 4. By contradiction, suppose that there exist two equilibria A and B. Without loss of generality suppose that we have $p^A > p^B$. By independence, the set of students who apply to H in equilibrium A is a superset of the set of students who apply to H in equilibrium B because subjective admission chances are higher in A. Note however that p is the ratio of seats to the number of applicants, i.e. $p = q \Big/ \left[\int_0^1 \min\left\{\frac{\frac{p}{1-p}\theta}{\bar{c}}, 1\right\} \mathrm{d}\theta\right]$. Hence, we must have $p^A < p^B$, a contradiction. □

Proof of Proposition 5. When $\lim_n \tau_i^n = 0$, we already showed that $p_i = \mathbf{1}\{\theta_i \geq \theta^*\}$, hence $\{i : \sigma_i = 1\} = \{i : \theta_i \geq \theta^*\}$ and $q_i = |\{i : \sigma_i = 1 \text{ and } \theta_i \geq \theta^*\}| = |\{i : \theta_i \geq \theta^*\}|$. By contradiction, suppose that $q_i < q_j$. Then $|\{j : \sigma_j = 1 \text{ and } \theta_j \geq \theta^*\}| > |\{i : \theta_i \geq \theta^*\}|$. Note that $\{j : \sigma_j = 1 \text{ and } \theta_j \geq \theta^*\} \subseteq \{j : \theta_j \geq \theta^*\}$, hence $|\{j : \theta_j \geq \theta^*\}| \geq |\{j : \sigma_j = 1 \text{ and } \theta_j \geq \theta^*\}|$. But then, $|\{j : \theta_j \geq \theta^*\}| > |\{i : \theta_i \geq \theta^*\}|$, which contradicts the fact that $f_i = f_j$. □

Proof of Proposition 6. When $G_i = \delta_0$ we have that $\sigma_i = 1$ for all i. Hence, $q_i = |\{i : \sigma_i = 1 \text{ and } \theta_i \geq \theta^*\}| = |\{i : \theta_i \geq \theta^*\}|$. We conclude using the same reasoning as in the proof of Proposition 5. □

Proof of Proposition 7. First, we derive subjective admission chances in the case with quotas. We consider the following neighborhood specific quotas: $q_i = q_j = \frac{q}{2}$. The subjective admission chances (computed in Section 3.1) for each neighborhoods write:

$$p_i = -\frac{\bar{c}_i q}{2} + \sqrt{\frac{\bar{c}_i^2 q^2}{4} + \bar{c}_i q} \qquad p_j = -\frac{\bar{c}_j q}{2} + \sqrt{\frac{\bar{c}_j^2 q^2}{4} + \bar{c}_j q}$$

The neighborhood specific admission cutoff $\tilde{\theta}_i^*$ solves

$$\int_{\tilde{\theta}_i^*}^1 \frac{p_i}{\overline{c}_i(1-p_i)}\theta \, \mathrm{d}\theta = \frac{q}{2} \iff \tilde{\theta}_i^* = \sqrt{1 - q\overline{c}_i\frac{1-p_i}{p_i}}. \tag{9}$$

The admission cutoff in neighborhood j is similar, replacing p_i with p_j.

Second, we approximate $W(\theta^*)$ at the first order and show that it is independent of $\overline{c}_i$ and $\overline{c}_j$. As $q \longrightarrow 0$, we have

$$W_i(\theta^*) = \frac{p_i}{1-p_i}\frac{1-(\theta^*)^3}{3\overline{c}_i} - \left(\frac{p}{1-p}\right)^2 \frac{(\theta^*)^3}{6\overline{c}_i}.$$

Again for $q \longrightarrow 0$, we make the following approximations: $p_i \approx \sqrt{2q\overline{c}_i}$, $\frac{p_i}{1-p_i} \approx p_i$ and $(\theta^*)^3 \approx 1 - \frac{3}{2}\sqrt{2q\overline{c}_i}$. Therefore we obtain $W_i \approx \frac{2}{3}q$ at the first order. Hence, $W(\theta^*) \approx \frac{4}{3}q$ is independent of $\overline{c}_i, \overline{c}_j$. □

Proof of Proposition 8. We already derived subjective admission chances in the case of quotas in Proposition 7. Therefore, we derive them in the case without quotas.

When there are no quotas, both neighborhoods compete for the same q seats. Subjective admission chances in neighborhoods i are obtained by dividing the number of seats by the mass of applicants in this neighborhood:

$$p_i = \frac{q_i}{\int_0^1 \min\left\{\frac{p_i}{\overline{c}_i(1-p_i)}\theta, 1\right\} \mathrm{d}\theta} \tag{10}$$

where $q_i + q_j = q$ are the seats taken by students from neighborhoods i and j in equilibrium. We consider first the case in which the minimum does not bind in both neighborhoods. The above equation rewrite:

$$p_i = 2q_i\overline{c}_i\frac{1-p_i}{p_i} \tag{11}$$

The market clearing condition in neighborhood i writes:

$$\int_{\tilde{\theta}_i^*}^1 \min\left\{\frac{p_i}{\overline{c}_i(1-p_i)}\theta, 1\right\} \mathrm{d}\theta = q_i \iff 1 - \tilde{\theta}_i^{*2} = 2q_i\overline{c}_i\frac{1-p_i}{p_i}$$

Together with the fact that admission cutoffs must be equal across neighborhoods $\tilde{\theta}_i^* = \tilde{\theta}_j^*$, this shows that subjective beliefs are identical $p_i^{\textit{no quotas}} = p_j^{\textit{no quotas}} = p^{\textit{no quotas}}$.

Using the market clearing condition together with the identity $q = q_i + q_j$ we obtain the number of seats taken by each neighorhoods in equilibrium:

$$q_i = q\frac{\overline{c}_j}{\overline{c}_i + \overline{c}_j}$$

Solving the quadratic form (11) and substituting the expression for q_j yields a closed form solution for subjective beliefs:

$$p^{\textit{no quotas}} = -q\frac{\overline{c}_i\overline{c}_j}{\overline{c}_i + \overline{c}_j} + \sqrt{\left(q\frac{\overline{c}_i\overline{c}_j}{\overline{c}_i + \overline{c}_j}\right)^2 + 2q\frac{\overline{c}_i\overline{c}_j}{\overline{c}_i + \overline{c}_j}}.$$

Now consider the situation with quota $q_i = q + j = q/2$, and assume that $\overline{c}_i > \overline{c}_j$. It is readily verified that i receives more seats with quotas than without quotas and given the monotonicity of $q_i \to p_i(q_i)$, we conclude that $p_i^{\textit{quotas}} > p^{\textit{no quotas}} > p_j^{\textit{quotas}}$. □

Proof of Proposition 9 All students in neighborhood i are indifferent hence apply to H. Then without quotas all students $\{(\theta, c) : \theta \geq \theta^*\}$ are admitted to H in neighborhood i. In neighborhood j, all students $\left\{(\theta, c) : \theta \geq \theta^* \text{ and } c \leq \frac{p_j}{\overline{c}_j(1-p_j)}\theta\right\}$ are admitted to H. Overall welfare is

$$W(\theta^*) = \underbrace{\int_{\theta^*}^1 \theta \,\mathrm{d}\theta}_{i} + \underbrace{\int_{\theta^*}^1 \int_0^{c^H} \theta \,\mathrm{d}c\,\mathrm{d}\theta - \int_0^{\theta^*} \int_0^{c^H} c g_j(c) \,\mathrm{d}c\,\mathrm{d}\theta}_{j} \tag{12}$$

With quotas, each neighborhood has $q/2$ reserved seats. In neighborhood j, quotas must increase subjective admission chances p_j. Indeed, as $\overline{c}_j \longrightarrow 0$ the best students in both neighborhoods apply to H hence the admission cutoff solves $2(1 - \theta^*_{quotas}) = q$. Instead, without quotas the admission cutoff solves $1 - \theta^* = q$, which is strictly smaller than with quotas. This raises the quality of admitted students in both neighborhoods, which increases the first two terms in the welfare equation (12). Now the third term vanishes as $\overline{c}_j \longrightarrow 0$, which yields the result. □

Proof of Proposition 10. Consider one neighborhood with c uniformly distributed on $[0, \overline{c}]$ and q seats. As explained in subsection 3.1, for q small enough, we have

$$p = -q\overline{c} + \sqrt{q^2\overline{c}^2 + 2q\overline{c}}$$

which defines $p(q)$.

Moreover, the condition relating the admission threshold to p and q writes

$$\frac{p}{1-p}\frac{1-(\theta^*)^2}{2\overline{c}} = q$$

which implicitly defines a function $\theta^*(q)$ (using $p = p(q)$ as just defined to eliminate the dependence in p).

Finally, total welfare in this neighborhood writes

$$\begin{aligned} \widetilde{W}(p,\theta^*) &= \frac{1}{\overline{c}}\int_{\theta^*}^{1}\frac{p}{1-p}\theta^2 d\theta - \frac{1}{2\overline{c}}\int_0^{\theta^*}(\frac{p}{1-p}\theta)^2 d\theta \\ \widetilde{W}(p,\theta^*) &= \frac{1}{\overline{c}}\frac{p}{1-p}\frac{1-(\theta^*)^3}{3} - \left(\frac{p}{1-p}\right)^2\frac{1}{2\overline{c}}\frac{(\theta^*)^3}{3} \end{aligned}$$

Totally differentiating w.r.t q yields

$$\frac{dW}{dq}(q) = \frac{\partial\widetilde{W}}{\partial p}\frac{dp}{dq} + \frac{\partial\widetilde{W}}{\partial\theta^*}\frac{d\theta^*}{dq}.$$

Consider now a two neighborhood case with cost distributions uniformly distributed on $[0, \overline{c}_i]$ and $[0, \overline{c}_j]$ in i and j, respectively.

As already established, in the laissez-faire case, we have

$$\begin{aligned} p_i &= p_j \\ \overline{c}_i q_i &= \overline{c}_j q_j \end{aligned}$$

It is then readily verified using the expression of $\widetilde{W}(p,\theta^*)$ that around the laissez-faire, we have that

$$\frac{dW_i(q_i)}{dq_i} = \frac{dW_j(q_j)}{dq_j}.$$

Letting $W(\kappa)$ denote the total welfare induced by the policy intervention, we have that

$$\frac{dW}{d\kappa} = \left(\frac{dW_i(q_i)}{dq_i} - \frac{dW_j(q_j)}{dq_j}\right)\frac{dq_i}{d\kappa}$$

which is null given our observation that $\frac{dW_i(q_i)}{dq_i} = \frac{dW_j(q_j)}{dq_j}$ at the laissez-faire. □

Proof of Proposition 11. Subjective admission chances in neighborhoods i are obtained by dividing the number of seats by the mass of applicants in this neighborhood:

$$p_i = \frac{q_i}{\int_0^1 \alpha \min\left\{\frac{p_i}{\bar{c}_i(1-p_i)}\theta, 1\right\} + (1-\alpha)\min\left\{\frac{p_i}{\bar{c}_j(1-p_i)}\theta, 1\right\} \,\mathrm{d}\theta} \tag{13}$$

where $q_i + q_j = q$ are the seats taken by students from neighborhoods i and j in equilibrium. We consider first the case in which the minimum does not bind in both neighborhoods. The above equation rewrite:

$$p_i = 2q_i\frac{1-p_i}{p_i}\left[\frac{\alpha}{\bar{c}_i} + \frac{1-\alpha}{\bar{c}_j}\right]^{-1} \tag{14}$$

and in neighborhood j:

$$p_j = 2q_j\frac{1-p_j}{p_j}\left[\frac{\alpha}{\bar{c}_j} + \frac{1-\alpha}{\bar{c}_i}\right]^{-1} \tag{15}$$

The market clearing condition in neighborhood i writes:

$$\int_{\tilde{\theta}_i^*}^1 \alpha \min\left\{\frac{p_i}{\bar{c}_i(1-p_i)}\theta, 1\right\} + (1-\alpha)\min\left\{\frac{p_i}{\bar{c}_j(1-p_i)}\theta, 1\right\} \,\mathrm{d}\theta = q_i$$
$$\iff 1 - \tilde{\theta}_i^{*2} = 2q_i\frac{1-p_i}{p_i}\left[\frac{\alpha}{\bar{c}_i} + \frac{1-\alpha}{\bar{c}_j}\right]^{-1}$$

Together with the fact that admission cutoffs must be equal across neighborhoods $\tilde{\theta}_i^* = \tilde{\theta}_j^*$, this shows that subjective beliefs are identical $p_i = p_j = p$.

Solving for q_i yields

$$q_i = q\left(\frac{\alpha}{\bar{c}_j}+\frac{1-\alpha}{\bar{c}_i}\right)^{-1}\left[\left(\frac{\alpha}{\bar{c}_j}+\frac{1-\alpha}{\bar{c}_i}\right)^{-1}+\left(\frac{\alpha}{\bar{c}_i}+\frac{1-\alpha}{\bar{c}_j}\right)^{-1}\right]^{-1}$$

Solving the quadratic form (15) yields a closed form solution for subjective beliefs:

$$p = \frac{-q_i+\sqrt{q_i^2+2q_i\left(\frac{\alpha}{\bar{c}_i}+\frac{1-\alpha}{\bar{c}_j}\right)}}{\left(\frac{\alpha}{\bar{c}_i}+\frac{1-\alpha}{\bar{c}_j}\right)}.$$

which is constant in α. Therefore, for small q welfare and average quality is independent of α. □

Proof of Proposition 12. Consider two equilibria, denoted A and B, such that

$$\begin{cases} q_1^A > q_1^B \\ q_2^A < q_2^B \end{cases} \iff \begin{cases} p_1(q_1^A) > p_1(q_1^B) \\ p_2(q_2^A) < p_2(q_2^B) \end{cases}$$

where we used the monotonicity of subjective admission chances. By definition of H, the following system must hold in equilibrium for the cutoff θ^{*A} that is common in both neighborhoods:

$$\begin{cases} J_1(p_1(q_1^A), \theta^{*A}) = 0 \\ J_2(p_2(q_2^A), \theta^{*A}) = 0 \end{cases}$$

But now, by monotonicity of H, we must have

$$\begin{cases} J_1(p_1(q_1^B), \theta^{*A}) > 0 \\ J_2(p_2(q_2^B), \theta^{*A}) < 0 \end{cases}$$

which contradicts the fact that B is an equilibrium. □

Proof of Proposition 13. Suppose that all seats are taken by students from neighborhood i ($q_i = q$). First note that some seats must be allocated to high ability students (otherwise they have a profitable deviation as $p = 1$ if only students with $(\theta, c) = (0, 0)$ apply). Therefore, the admission cutoff satisfies $\theta^* > 0$. Moreover, for α small the admission probability converges to $\frac{q}{\zeta}$ where ζ is the fraction of high ability students who apply to H, with $\theta \frac{q}{\zeta} > c(1 - \frac{q}{\zeta})$. Hence all seats are occupied by students from neighborhood i for this ζ. Suppose that all low-ability students from neighborhood j apply to H but are being rejected because $\theta_j = 0 < \theta^*$. Then we have $p_j = 0$, and no high-ability student in neighborhood j applies to H. There are no profitable deviations and beliefs are consistent, hence this is a local sampling equilibrium. □

References

Agarwal, N. and Somaini, P. (2018). Demand Analysis using Strategic Reports: An application to a School Choice Mechanism. *Econometrica*, 86(2):391–444.

Alon, S. and Tienda, M. (2005). Assessing the "Mismatch" Hypothesis: Differences in College Graduation Rates by Institutional Selectivity. *Sociology of education*, 78(4):294–315.

Altmejd, A., Barrios Fernández, A., Drlje, M., Hurwitz, M., Kovac, D., Mulhern, C., Neilson, C., Smith, J., and Goodman, J. (2020). O Brother, Where Start Thou? Sibling Spillovers on College and Major Choice in Four Countries.

Aschaffenburg, K. and Maas, I. (1997). Cultural and educational careers: The Dynamics of Social Reproduction. *American sociological review*, pages 573–587.

Banerjee, A. V. (1992). A Simple Model of Herd Behavior. *The quarterly journal of economics*, 107(3):797–817.

Bikhchandani, S., Hirshleifer, D., and Welch, I. (1992). A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades. *Journal of political Economy*, 100(5):992–1026.

Dominitz, J. and Manski, C. F. (1994). Eliciting Student Expectations of the Returns to Schooling. Technical report, National Bureau of Economic Research.

Ellison, G. and Fudenberg, D. (1995). Word-of-Mouth Communication and Social Learning. *The Quarterly Journal of Economics*, 110(1):93–125.

Esponda, I. (2008). Behavioral Equilibrium in Economies with Adverse Selection. *American Economic Review*, 98(4):1269–91.

Esponda, I. and Pouzo, D. (2016). Berk–nash equilibrium: A framework for modeling agents with misspecified models. *Econometrica*, 84(3):1093–1130.

Eyster, E. and Rabin, M. (2005). Cursed equilibrium. *Econometrica*, 73(5):1623–1672.

Granovetter, M. (1985). Economic Action and Social Structure: The Problem of Embeddedness. *American journal of sociology*, 91(3):481–510.

Hastings, J., Neilson, C. A., and Zimmerman, S. D. (2015). The Effects of Earnings Disclosure on College Enrollment Decisions. Technical report, National Bureau of Economic Research.

Hastings, J. S. and Weinstein, J. M. (2008). Information, School Choice, and Academic Achievement: Evidence from two Experiments. *The Quarterly journal of economics*, 123(4):1373–1414.

Jehiel, P. (2005). Analogy-Based Expectation Equilibrium. *Journal of Economic theory*, 123(2):81–104.

Jehiel, P. (2018). Investment Strategy and Selection Bias: An Equilibrium Perspective on Overoptimism. *American Economic Review*, 108(6):1582–97.

Jehiel, P. and Mohlin, E. (2022). Cycling and Categorical Learning in Decentralized Adverse Selection Economies. *Working paper*.

Jensen, R. (2010). The (Perceived) Returns to Education and the Demand for Schooling. *The Quarterly Journal of Economics*, 125(2):515–548.

Kapor, A. J., Neilson, C. A., and Zimmerman, S. D. (2020). Heterogeneous Beliefs and School Choice Mechanisms. *American Economic Review*, 110(5):1274–1315.

Manski, C. F. (1993). Adolescent Econometricians: How do Youth Infer the Returns to Schooling? In *Studies of Supply and Demand in Higher Education*, pages 43–60. University of Chicago Press.

Manski, C. F. (2004). Social Learning from Private Experiences: The Dynamics of the Selection Problem. *The Review of Economic Studies*, 71(2):443–458.

Mohlin, E. (2014). Optimal Categorization. *Journal of Economic Theory*, 152:356–381.