Expectation Formation, Local Sampling and Belief Traps: A new Perspective on Education Choices*

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Abstract

Lack of diversity in higher education is partly driven by long-run belief distortions about admission chances at elite colleges. We depart from the rational expectation framework and propose a simple model of expectation formation in which students estimate their admission chances by sampling a pool of given size τ of peers who previously applied to elite colleges. Assuming students consider peers with ability as close as possible to their own, two types of inefficiencies arise in steady state: high-achieving disadvantaged students self-select out of elite colleges, and average students from advantaged families apply to elite colleges even though their true admission chances are null. We then explore the working of the model when students from several possibly dissimilar neighborhoods compete for the same positions, thereby highlighting externalities related to the comparative neighborhood compositions. Several policy instruments such as quotas or the mixing of neighborhoods are considered.

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1 Introduction

According to the rational expectations paradigm that is commonly used by economists, students when applying to elite colleges should form *correct* beliefs about their admission chances, in particular assessing correctly how the admission chances vary with observable characteristics such as the results obtained in ability tests. By contrast, sociologists argue that students are embedded in their social environment and obtain information by observing the decisions made by others, leading to mistakes and biases. We note that there is ample evidence that agents hold incorrect beliefs that are correlated across agents in the social network.[1]

In the rest of the paper, we propose a framework for understanding the two-way interactions between expectation formation and the social environment in the particular context of a career choice problem in which students have to decide whether or not to apply to elite colleges. Our approach uses the methodology of economics adopting an equilibrium formulation, but it takes inspiration from sociology when modeling the specific way students form their expectations based on their accessible environment.

Specifically, we assume that students differ in two dimensions: their ability (accessible through standardized test, say), and their cost of being rejected from elite colleges (that can alternatively be thought of as an opportunity cost induced by rejection). Students choose one out of two occupations: unqualified jobs on the labor market (or non-selective vocational training), and elite colleges. Elite colleges have limited seats and select only the best students up to their capacity. Importantly, we assume that students *do not form rational expectations regarding their admission chances*. We consider instead that they form their expectations by estimating the admission probability using a sample of past experiences from their peers in their neighborhood. This estimation procedure is constrained in three ways: First, the sample is endogenous and consists only of students in the neighborhood who applied in the past to elite colleges. This can be motivated on the ground that it is very hard in practice to have access to conterfactual information (here, the

[1] On the social network dimension, Altmejd et al. (2020) show that older sibling's enrollment in college increases a younger sibling's probability of enrolling in college at all.

admission outcomes of students who did not apply). Second, the sample must have a size no smaller than some threshold τ viewed as necessary to make the statistics derived from the sample sufficiently reliable. This can be viewed as a reduced-form approach formalizing the bias-variance trade-off faced by students in their estimation procedure. We note it is similar to the K-nearest neighbor algorithm that is very commonly used in machine learning and statistics. Third, students ask in priority peers with similar ability. This can be viewed as reflecting that students have the correct understanding that admission chances are related to ability.

We introduce the "local sampling equilibrium" representing steady states of a process in which students best respond to their subjective beliefs viewed as the aggregate empirical frequencies of admissions in the samples, and subjective beliefs are consistent with the above sequential sampling estimation procedure. It is parameterized by τ, the mass q of seats as well as the distributions of ability and costs in the various neighborhoods.

We first consider a one-neighborhood economy, highlighting the inefficiencies that arise as compared with the rational expectations equilibrium benchmark. We next explore a multi-neighborhood economy, highlighting the effect of asymmetries across neighborhoods in terms of welfare and the average ability of admitted students.

In our model with a continuum of students and no aggregate uncertainty, under rational expectations, students perfectly sort in each occupation based on their ability, and the equilibrium is efficient. Things are different in the sampling equilibrium. Two types of inefficiencies arise in the one-neighborhood case. First, some disadvantaged (i.e., with high rejection costs) students with ability above the admission threshold self-select out of elite colleges. Second, some advantaged students with ability below the required ability threshold apply to elite colleges but are rejected. This equilibrium mismatch is due to the fact that students with ability below the required threshold ability who apply and get rejected induce a *strategic externality* on high-achieving students by distorting their perceived admission chances downward, and conversely students with ability above the required ability threshold who apply and get accepted induce a strategic externality on low-achieving students by distorting their beliefs upward. These strategic externalities arise because

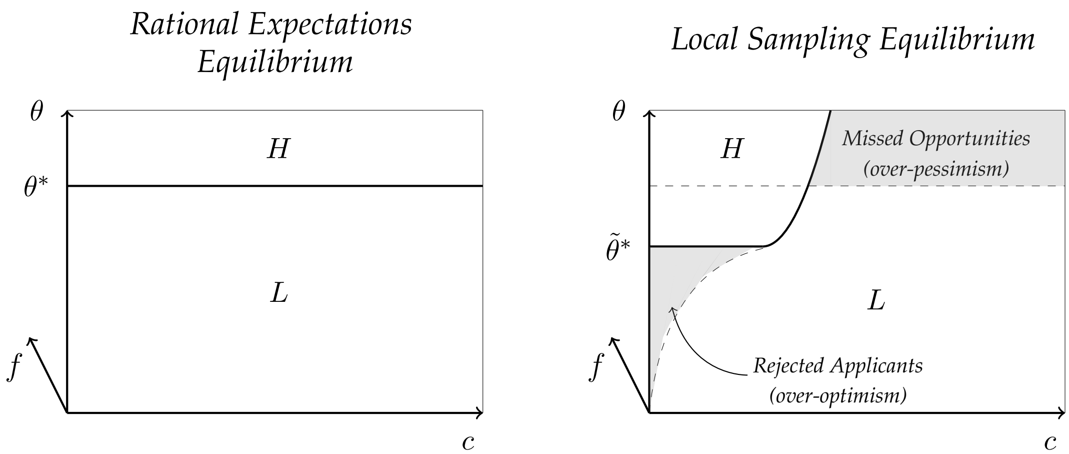


Figure 1: *The x-axis represents students' cost, the y-axis represents ability, and the z-axis is the population density. There are two occupations: H are elite colleges that have limited capacity, and L are jobs with no qualifications. (Left) Allocation of students to occupations in a rational expectations equilibrium. (Right) Allocation of students to occupations in a local sampling equilibrium. The shaded areas represent students who are mismatched: the top-right square corresponds to high-achieving disadvantaged students who self-select in non-selective colleges; the bottom-left triangle corresponds to average-achieving advantaged students who apply to elite colleges but are rejected.*

of the combination of rationing at elite colleges and the non-rational character of expectations, which leads both high and low ability students to rely on average-ability peers with different admission results to compute their admission chances. By contrast, there is no rationing on the labor market, hence there are no payoff-relevant distortions for students in the assessment of this alternative. See Figure 1 for a graphical representation of the rational expectations equilibrium and the local sampling equilibrium. We observe that the local sampling equilibrium moves gradually toward the rational expectations equilibrium, as one reduces the size τ of the samples. At the other extreme, when τ is large, all students have the same expectation about the admission chances irrespective of their type. The sampling procedure can be seen as inducing in students' minds a kind of regression to the mean when assessing the link between ability and the admission chance, where the mean admission rate is endogenously determined by the application strategy of students.

We provide a complete characterization of the welfare and the average ability of admitted students in a local sampling equilibrium when abilities and costs are independently distributed and the number of seats is small. We observe among other findings that a higher variance in the ability distribution induces a higher welfare loss.

Our second main investigation concerns the study of competition across several neighborhoods where students across all neighborhoods compete for the same seats, but, as already mentioned, sampling takes place locally, separately in each neighborhood. Our main question of interest concerns how asymmetries across neighborhoods in the relative size and/or in the ability/cost distributions affect the welfare in the various neighborhoods as well as the average quality of admitted students. Such an extension is important to understand how the aggregate characteristics of a neighborhood shape the education decisions made locally, which has been shown empirically to play an important role (Case and Katz, 1991; Kling et al., 2007; Chetty et al., 2016; Chetty and Hendren, 2018a,b).[2]

Specifically, we consider a two-neighborhood environment, and we provide a complete characterization of the sampling equilibrium when the number of seats is small and abilities and costs are independently distributed in each neighborhood.[3] When the two neighborhoods share the same ability distribution, we note that the neighborhood with larger cost (i.e., the neighborhood with the cost distribution having lower density around 0) gets fewer seats per head than the neighborhood with smaller costs. We also establish that if one neighborhood is much bigger than the other one, the bigger neighborhood gets almost all seats per head irrespective of the compositions of the neighborhoods, thereby formalizing how minorities may be hurt in a sampling equilibrium.

[2] Kling et al. (2007) found heterogeneous effects, with education outcomes improving for females, but degrading for males. Understanding these discrepancies across populations is an interesting avenue for further empirical and theoretical research.

[3] In the final section, we briefly investigate a case in which abilities and costs are not independenty distributed, and we observe that such a correlation may open the door to the possibility of multiple equilibria, even assuming symmetry across neighborhoods. Such a multiplicity may explain the emergence of inequality across neighborhoods, even when these have identical ex ante characteristics.

We next study the effect on redistribution, welfare as well as the average quality of admitted students of standard policy instruments such as quotas or the mixing of neighborhoods. We establish that when neighborhoods differ only in the distribution of costs, these policies have no effect on total welfare, despite allowing for redistribution between neighborhoods. We next characterize when quotas are welfare-enhancing when the distributions of abilities differ across neighborhoods. In particular, we establish that reserving a number of seats in proportion to the size of the neighborhood is welfare-enhancing if the neighborhood with larger cost is the one with smaller ability variance — assuming an equal mean ability across neighborhoods. We also establish that quotas have a negative impact on the average ability of admitted students (despite the presence of inefficiencies in the laissez-faire scenario). We also characterize the welfare impact of mixing, noting that in some cases mixing may be welfare-enhancing.

Related Literature At a methodological level, our model of belief formation can be viewed as offering a balance between strategic sophistication as usually considered in economics—which is at least partly empirically supported by some studies (Agarwal and Somaini, 2018)—and the embeddedness of students' beliefs as usually considered in sociology. This is to be contrasted with the "undersocialized" view of an atomic agent that forms correct beliefs independently from her environment as well as the "oversocialized" accounts of expectation formation in which students mechanically inherit the beliefs of their parents or have social capital fully account for educational choices (the sociology literature has departed from this Bourdieusian view in the last 20 years, see for instance Aschaffenburg and Maas (1997)).[4]

There is a growing empirical literature on expectation formation in education, broadly divided between beliefs on the returns to schooling and subjective admission chances.

Very few papers investigate subjective admission chances, which is the focus of our paper. Most notably, Hastings and Weinstein (2008) show that providing information about school quality and odds of admission to low-

[4]The distinction between undersocialized and oversocialized explanations is due to Granovetter (1985).

income families with high-achieving students increases application to good schools. It is unclear, however, if the effect is driven by growing awareness about these schools or changing expectations. Kapor et al. (2020) directly elicit admission probabilities of students facing a centralized school choice mechanism that rewards strategic behavior. They find that households play strategically, but do so with miscalibrated beliefs. Belief errors, however, do not seem to correlate with observable characteristics such as race or economic status. Finally, Altmejd et al. (2020) show that older sibling's enrollment in a better college increases a younger sibling's probability of enrolling in college at all, especially for families with low predicted probabilities of enrollment.

The empirical literature on the perceived returns to schooling that is less directly related to our model has obtained mixed results. In Wisconsin, Dominitz and Manski (1994) find that the perceived returns from a Bachelor's degree compared to a high school diploma are positive. In Chile, Hastings et al. (2015) show that low-achieving disadvantaged students who apply to low-earning college degree programs overestimate earnings for past graduates by over 100%, while beliefs for high-achieving students are correctly centered. Conversely in the Dominican Republic, Jensen (2010) find that the perceived returns to secondary school are extremely low, despite high measured returns.

The first theoretical model of expectation formation on the returns to schooling is due to Manski (1993). He postulates an additive log-income equation, and he assumes that students infer the returns to schooling by taking the conditional expectation of log-income. If students omit to condition on ability—e.g., because they do not observe the ability of their peers—he shows that more low-ability and less high-ability students enroll to college.[5]

[5] Streufert (2000) considers a related model in which students infer the returns to schooling from a distribution of income that is truncated. More precisely, successful children who are more likely to leave their (disadvantaged) neighborhoods are under-sampled by new children of such neighborhoods, thereby leading to a potential downward bias in the estimated returns to schooling among disadvantaged children. That paper shares with our approach the local biased sampling idea, but the mechanism as well as the equilibrium approach (pursued here but not in Streufert (2000)) make the two very different. It may also be mentioned that in our approach low ability students typically overestimate their chance of success due to their consideration of students with high ability, a bias that has no counterpart in Streufert (2000).

Our model can be viewed as enriching that kind of approach to expectation formation about admission chances. Our analysis of the one-neighborhood scenario is similar to that of Manski when τ is large to the extent that as already mentioned our sampling procedure can then be interpreted as resulting from the lack of observability of other students' ability (and like us Manski (1993) assumes expectations are formed based on the selected sample of those choosing the activity of interest).[6] Allowing for smaller τ in our analysis of the one-neighborhood case allows us to move smoothly from the rational expectations paradigm to the case in which students do not condition their admission chances on ability as in Manski's model. Moreover, the study of competing neighborhoods has no counterpart in Manski (1993).

Our paper can be viewed as contributing to the recent literature on behavioral economics in education market design as recently surveyed in Rees-Jones and Shorrer (2023), even if that literature has mostly been concerned so far with explaining why students do not always report their true preferences in mechanisms (such as the Gale-Shapley student-prefered deferred acceptance mechanism) in which it is a (weakly) dominant strategy to report truthfully. By contrast, in our model there is no dominant strategy due to the presence of rejection costs and the non-rational aspect concerns the modelling of the expectations about the admission chances, as discussed above, a form of non-rational expectations not discussed in Rees-Jones and Shorrer (2023).

There is a vast literature on social learning illustrating that past cohorts' behavior influences the expectations of current cohorts (Banerjee, 1992; Bikhchandani et al., 1992; Ellison and Fudenberg, 1995). These papers, however, typically assume that agents have enough prior information to infer the outcome of counterfactual actions using Bayes' rule. Manski (2004) relaxes this assumption by considering a social learning environment in which students have no prior belief on the distribution of outcomes conditional on actions—as in our model. Hence students cannot infer anything on counterfactual actions. Only assuming the stationarity of the outcome distribution—as we do

[6] Other models of selection neglect in various environments include Esponda (2008); Jehiel (2018); Frick et al. (2022).

in this paper[7]—he shows that learning induces a process of sequential reduction in ambiguity, and insight different from the one developed here. Though similar in motivation, our papers differ with the social learning literature in two important ways: we account for strategic interactions among students (without strategic externalities all students would apply and be admitted to elite colleges), and we study how competition across neighborhoods can affect these biases.

Finally, several papers in behavioral game theory have introduced various departures from the rational expectation hypothesis. These include among others the cursed equilibrium Eyster and Rabin (2005), the analogy-based expectation equilibrium (Jehiel, 2005), the Berk-Nash equilibrium Esponda and Pouzo (2016) or the Bayesian Network Equilibrium (Spiegler, 2016). The spirit of our approach is maybe closest to Jehiel (2005) who introduces a model of coarse expectations in which players bundle actions into classes. In equilibrium, players best-respond to their analogy-based expectations, and expectations correctly represent the average behavior in every class. Our paper is based on a different learning rule where students average the outcome of an endogenously chosen group of players and do not bundle actions, whereas in Jehiel (2005) players average the outcome of an exogenously given bundle of actions using past observations from an exogenously given group of players (see however Jehiel and Mohlin (2022) for a model that endogenizes the analogy classes of Jehiel (2005) based on a similar bias-variance trade-off as the one used to motivate our sampling heuristic, see also Mohlin (2014) on the bias-variance trade-off).

2 SETUP

We introduce a stylized model of career choice with strategic students and rationing at elite colleges. In this Section, we consider a single neighborhood. Extension to more neighborhoods will be considered in the next Section. In this part, we normalize the mass of students to be 1 and we let q be the mass of seats at elite colleges. Students are indexed by their ability $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}_+$, and by their cost $c \in [\underline{c}, \overline{c}] \subseteq \mathbb{R}_+$. There is a probability distribution F

[7] Meaning that colleges never modify their admission criteria.

on $N \equiv [\underline{\theta}, \overline{\theta}] \times [\underline{c}, \overline{c}]$ with continuous density f that has full support. In the main text, we view the cost as an opportunity cost from being rejected from elite colleges.[8] That is, the cost c is incurred only when the student applies to elite colleges and gets rejected. In the Appendix, we discuss richer formulations for the opportunity cost and briefly develop the analysis for the application cost formulation (i.e., when the cost c is incurred when applying to elite colleges irrespective of whether the student is admitted or rejected). As we note later, our results obtained when q is small are the same whether c is a rejection cost or an application cost.

Students choose among two occupations: going directly on the labor market (or a non-selective vocational training) L, or applying to selective colleges H. The utility of attending an elite college is $U^H(\theta) = \theta$, whereas we assume that the utility of going directly on the labor market is $U^L(\theta) = 0$ for all θ.

Students can apply to only one occupation: the action space is then $A = \{L, H\}$. There is no rationing for going on the labor market. Elite colleges, however, have a limited number of seats and they select students with the highest ability (among the pool of applicants) up to their capacity $q \ll 1$.[9] The payoffs are as follows:

– If student (θ, c) goes on the labor market L her utility is 0.

– If student (θ, c) applies to H and obtains a seat, her utility is θ.

– If student (θ, c) applies to H but does not get a seat, she goes on the labor market and her utility is $-c$.

A natural interpretation of our model is that if a student applies to H and gets rejected, he loses some time before entering the job market resulting in a loss of c. A more elaborate interpretation would allow for three levels H, M and L of applications where L represents the labor marker and H and M represent high and medium ranked colleges respectively. The initial choice

[8]We have in mind that poorer students generally have a higher rejection cost since conditional on being rejected at an elite college, poorer students would have benefited more from going directly on the labor market.

[9]Our results are unchanged if colleges only receive a noisy signal about students' ability, as long as students know the signal used for selection purposes (for example, think of the signal as the student's performance in past exams).

is whether to apply to H or M and in case of rejection only L would be left. Our model corresponds to a stylized version of this, assuming there is limited capacity constraint on M.[10]

Getting back to our model, a strategy profile $\sigma : N \longrightarrow \Delta A$ is a (measurable) function from the population of students to mixed actions. This is a binary action game, hence we let $\sigma(\theta, c) \in [0, 1]$ simply denote the probability that student (θ, c) applies to H.

A key object that drives the choice of student (θ, c) is the subjective probability this student assigns to obtaining a seat at an elite college conditional on applying to H. In both the rational case and our approach, this subjective probability turns out to depend only on θ and we denote it by $p(\theta)$ accordingly. Based on $p(\theta)$, student (θ, c) applies to H whenever

$$p(\theta)\theta - (1 - p(\theta))c \geq 0$$

This leads to the following definition of an optimal strategy profile.[11]

DEFINITION 1. *σ is optimal given subjective beliefs $p(\cdot)$ if*

$$\sigma(\theta, c) = \begin{cases} 1 & \textit{when} \quad c \leq \frac{p(\theta)}{1-p(\theta)}\theta \\ 0 & \textit{when} \quad c > \frac{p(\theta)}{1-p(\theta)}\theta \end{cases}$$

For any strategy profile, let $\theta(\sigma)$ denote the cutoff at H such that any student with ability $\theta > \theta(\sigma)$ who applies to H is admitted. It is defined as follows: $\theta(\sigma) = \underline{\theta}$ when

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{c}}^{\overline{c}} \sigma(\theta, c) f(\theta, c)\, \mathrm{d}c\, \mathrm{d}\theta < q$$

[10]In our model, there is no signaling value to attending elite colleges given that $U^H(\theta) = \theta$ depends only on the characteristic θ of the student and not on who else is attending the elite colleges. This is to simplify matters and better isolate the sources of inefficiencies in our behavioral approach.

[11]For completeness, we assume that the student applies to H when indifferent, but how indifferences are resolved plays no role in the analysis.

Otherwise, $\theta(\sigma)$ is uniquely defined as the largest θ^* such that

$$\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{\overline{c}} \sigma(\theta, c) f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta = q$$

Subjective beliefs are rational when they are consistent with the admission cutoff, given the strategy profile.

DEFINITION 2. $p^R(\cdot)$ *is rationally consistent with* σ *if*

$$p^R(\theta) = \begin{cases} 1 & \textit{when} \quad \theta \geq \theta(\sigma) \\ 0 & \textit{when} \quad \theta < \theta(\sigma) \end{cases}$$

Therefore, the rational expectations equilibrium is defined as follows.

DEFINITION 3 (Rational Expectations Equilibrium)**.** σ^R *is a rational expectations equilibrium if there exist subjective beliefs* p^R *such that* σ^R *is optimal given* p^R *and* p^R *is rationally consistent with* σ^R*.*

Let us now characterize the unique rational expectations equilibrium—thus proving existence. Given the strategy profile σ and the consistency of beliefs, it is optimal to apply to H for all students with ability $\theta > \theta^*$ where $\theta^* = \theta(\sigma)$ as defined above. It follows that in a rational expectations equilibrium, the admission cutoff θ^* solves

$$\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{\overline{c}} f(c, \theta) \, \mathrm{d}c \, \mathrm{d}\theta = q \iff \theta^* = H^{-1}(1-q)$$

where H denoted the cdf of the marginal distribution of θ. In other words, the equilibrium allocation of students to occupations can be described by the cutoff strategy consisting in applying to H whenever θ is higher than $H^{-1}(1-q)$. This yields:

PROPOSITION 1 (Equilibrium Characterization)**.** *In the unique rational expectations equilibrium, students* $N^H = \{(\theta, c) : \theta > H^{-1}(1-q)\}$ *obtain a seat at elite colleges, and* $N^L = N \setminus N^H$ *go on the labor market.*

(All formal proofs appear in the Appendix). The rational expectations equilibrium induces perfect assortative matching as students sort across occupations based on their ability. Namely, high-achieving students go to elite colleges, and average- or low-ability students go on the labor market. No student applying to H gets rejected. See Figure 1 (Left) above for a graphical illustration of the equilibrium.

Define welfare as

$$W(\sigma) = \int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta,p(\theta))} \theta f(\theta, c)\, \mathrm{d}c\, \mathrm{d}\theta - \int_{\underline{\theta}}^{\theta^*} \int_{\underline{c}}^{c^H(\theta,p(\theta))} c f(\theta, c)\, \mathrm{d}c\, \mathrm{d}\theta$$

where $c^H(\theta, p(\theta))$ is the cost below which student (θ, c) applies to H conditional on admission chances $p(\theta)$. Moreover, define the average quality of the admitted students as

$$M(q) = \frac{\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta,p(\theta))} \theta f(\theta, c)\, \mathrm{d}c\, \mathrm{d}\theta}{\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta,p(\theta))} f(\theta, c)\, \mathrm{d}c\, \mathrm{d}\theta}$$

In the rational expectation equilibrium, $c^H(\theta, p(\theta)) = \overline{c}$ for all $\theta \geq H^{-1}(1-q)$ and $c^H(\theta, p(\theta)) = \underline{c}$ for all $\theta < H^{-1}(1-q)$. Rational expectations induce perfect sorting, and it results in a welfare optimal allocation of students to colleges as well as no wastes in rejection costs.

3 Expectation Formation and Belief Traps

In this section we introduce a simple model of expectation formation based on extrapolations and sampling, and we show, among other things, how it leads to persistent belief distortions and suboptimal application decisions.

Students have no prior over the distribution of admissions conditional on applications. Instead, students non-parametrically estimate their chance of being accepted by averaging the acceptance outcome of a sufficient mass τ of their peers who applied to elite colleges and who are closest to them in terms of ability. We think of the heuristic used by students as the continuous analog of the K-nearest beighbors alogorithm that is routinely used in statistics as a way to handle the bias-variance trade-off. We also believe it represents a plausible heuristic in line with considerations developed in Gigerenzer and Brighton (2009).

Formally, let $\mathcal{B}(N)$ denote the set of measurable subsets of N.

DEFINITION 4. *The sample for action H of student (θ, c) conditional on a strategy profile σ (from the previous generation) is*

$$S(\theta, c \mid \sigma) = \underset{B \in \mathcal{B}(N)}{\arg\inf} \left\{ \int_B |\theta - \tilde{\theta}| \, \mathrm{d}F(\tilde{\theta}, \tilde{c}) : \int_B \sigma(\tilde{\theta}, \tilde{c}) \, \mathrm{d}F(\tilde{\theta}, \tilde{c}) \geq \min(\tau, \tau(\sigma)) \right\}$$

where $\tau(\sigma) = \int_N \sigma(\tilde{\theta}, \tilde{c}) \, \mathrm{d}F(\tilde{\theta}, \tilde{c})$ is the total mass of students applying to H.

In words, S is the set with mass τ of students applying to H and having ability closest to θ. There is a convex penalty of including students with dissimilar ability, hence the sample $S(\theta, c \mid \sigma)$ is rectangular and it can be described by a simple index:

$$b(\theta, \sigma) = \inf \left\{ b > 0 : \int_{\max\{\underline{\theta}, \theta - b\}}^{\min\{\overline{\theta}, \theta + b\}} \int_{\underline{c}}^{\overline{c}} \sigma(\tilde{\theta}, \tilde{c}) \, \mathrm{d}F(\tilde{\theta}, \tilde{c}) \geq \tau \right\}.$$

This means that the sample for action H of student (θ, c) is obtained by taking all applicants with ability $\theta' \in [\theta - b(\theta, \sigma), \theta + b(\theta, \sigma)]$ regardless of their cost. See Figure 2 below for a graphical illustration.

We can now define subjective admission chances. As in the previous section, we denote by $\theta(\sigma)$ the admission cutoff at elite colleges given the strategy profile σ. The subjective admission chances at elite colleges H are obtained by averaging the experiences of the students in the sample.

DEFINITION 5. *Subjective admission chances at elite colleges p are τ-consistent with σ if*[12]

$$p(\theta) = \frac{1}{\min(\tau, \tau(\sigma))} \int_{S(\theta, c \mid \sigma)} \sigma(\tilde{\theta}, \tilde{c}) \mathbf{1}\{\tilde{\theta} > \theta(\sigma)\} \, \mathrm{d}F(\tilde{\theta}, \tilde{c}).$$

We now introduce our solution concept, the local sampling equilibrium, which requires optimality of actions and consistency of beliefs.

DEFINITION 6 (Local Sampling Equilibrium)**.** *σ is a local sampling equilibrium if there exists p such that σ is optimal given p and p is τ-consistent with σ.*

[12]If a mass of students less than τ chooses H, then we divide by $\int_B \sigma^k(\tilde{\theta}, \tilde{c}) \, \mathrm{d}F(\tilde{\theta}, \tilde{c})$ instead of τ.

We interpret this solution concept as the steady state of an intergenerational model of learning in which students of the current generation ask peers from the previous generation the outcome of their behavior. Therefore, this sample is completely endogenous as it depends on the strategy profile of the previous generation. Importantly, students know nothing ex-ante about the admission process: it could be either because schools do not disclose their admission criteria, or because students lack the ability to understand the admission process, or because they do not trust publicly disclosed information. Therefore, students entirely rely on the information provided by their social network. Of course, this is a stylized assumption and in practice we expect students to use a mix of information sources to form their expectations.

We made two assumptions on the learning process. First, students care about the precision of their estimate hence they must acquire a sufficient amount of data for each action. Formally, this means that students ask a mass $\tau \in (0, 1]$ of students from the previous generation, where τ is interpreted as a confidence parameter. This parameter captures a bias-variance trade-off: if the sample it too small then subjective admission chances are unbiased because they are computed using students with similar ability, but the estimator is noisy.[13] Conversely, if the sample is too large then subjective admission chances are precisely estimated but they are more likely to be biased.

Second, students contact in priority peers with similar ability. This can be justified on the ground that if students know that the admission probability is somewhat correlated with their ability, then they might reduce bias by asking peers with similar ability.[14] From another perspective, one can view our sampling technology as the one inducing the smallest distortions away from rationality, so that any inefficiency identified within our setup is likely to persist with alternative sampling specifications. In Appendix B, we briefly discuss the case when bundling is made on similarity in c, and we illustrate how extra inefficiencies would arise in this case.

[13] This is a reduced-form interpretation because there is no actual noise in the estimate as students sample from a continuum of peers.

[14] Alternatively, one can view this assumption as reflecting the hypthosesis that prior to applying to colleges, students have been grouped according to their ability, thereby leading to ties more naturally linked to ability.

Note that students include in their sample only peers who *actually* applied to H in the previous period. Therefore, students make no inference using counterfactual outcomes—i.e., they are not asking their peers "What would have been your admission chances at x conditional on applying there?". Who is included in the sample is endogenous and typically differ for each student, even though sample size is identically equal to τ for each student. Concretely, the perimeter of the sample for H of low-ability disadvantaged students is very large because no close ties ever apply to H. Therefore, they will need to ask high-achieving peers who have very different characteristics which induce a large bias in the subjective admission chances. In general, a larger perimeter implies a larger bias because the sample includes students with very different characteristics, whereas a smaller perimeter implies a smaller bias.

Existence. We apply a fixed point argument on the mapping from subjective beliefs $p : \Theta \longrightarrow [0,1]$, to best responses σ as derived from Definition 1, compounded with the mapping from strategies σ to subjective beliefs as derived from Definition 5. The fixed point exists if each sub-map is continuous. It is easy to see that the best response σ has a threshold structure that varies continuously with p. Moreover, the sample bounds $b(\theta, \sigma)$ are continuous in the strategy profile σ, and so are subjective beliefs p. This shows the existence of a pure strategy local sampling equilibrium. (The formal proof appears in the Appendix). In general, we are nit able to prove the uniqueness of a sampling equilibrium (even if this will be shown to be the case when q is small enough and θ and c are independently distributed).

Equilibrium Characterization. In the characterization below we make the simplifying assumption that cost and ability are independently distributed, i.e., $f(\theta, c) = h(\theta)g(c)$ and we denote by H and G the cumulative distributions of f and g, respectively. Fixing ability and the subjective admission chances, students who apply to H have a cost $c < c^H(\theta, p(\theta))$ where

$$c^H(\theta, p(\theta)) = \frac{p(\theta)}{1 - p(\theta)}\theta.$$

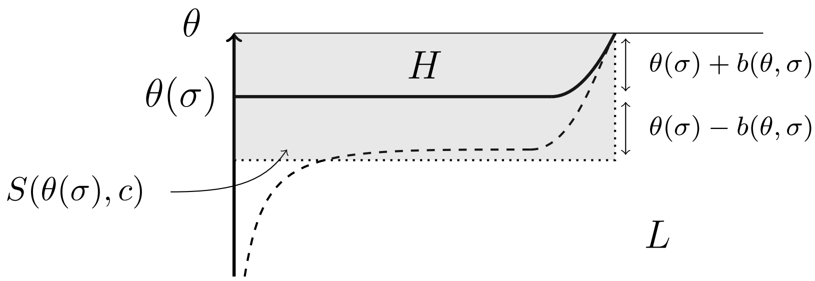


Figure 2: *Construction of the sample for the last student admitted at an elite college $(\theta(\sigma), c)$ in (c, θ)-space. The sample, represented in the shaded box, includes approximately a mass τ of students who applied to an elite college H. All students above the dashed line applied to H (i.e. $\sigma(\theta, c) = 1$) but only those above the solid line got admitted at an elite college. Rejected students exert a strategic externality on higher achieving students by distorting their estimated admission chances downard.*

The total mass of applicants to H is then:

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta, p(\theta))} f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta.$$

In a local sampling equilibrium, the ability of the last student admitted to H, denoted θ^*, is such that the mass of applicants at H is equal to the capacity of elite colleges:

$$\int_{\theta^*}^{\overline{\theta}} \int_{\underline{c}}^{c^H(\theta, p(\theta))} f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta = q.$$

Given our independence assumption, this equation can be simplified into:

$$\int_{\theta^*}^{\overline{\theta}} h(\theta) G\left(\frac{p(\theta)}{1 - p(\theta)} \theta\right) \mathrm{d}\theta = q \tag{1}$$

Let us now derive the equation that guarantees τ-consistency of subjective admission chances. The subjective admission chances of student (θ, c) are τ-consistent if they solve the following equation:

$$p(\theta) = \frac{1}{\min(\tau, \tau(\sigma))} \int_{\max\{\underline{\theta}, \theta - b(\theta, \sigma)\}}^{\min\{\overline{\theta}, \theta + b(\theta, \sigma)\}} \int_{\underline{c}}^{c^H(\tilde{\theta}, p(\tilde{\theta}))} \mathbf{1}\{\tilde{\theta} > \theta^*\} \, \mathrm{d}F(\tilde{c}, \tilde{\theta}).$$

where $b(\theta, \sigma)$ is derived as explained above. With our independence assumption, this can be rewritten as:

$$p(\theta) = \frac{1}{\min(\tau, \tau(\sigma))} \int_{\max\{\underline{\theta}, \theta - b(\theta,\sigma)\}}^{\min\{\overline{\theta}, \theta + b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1 - p(\tilde{\theta})}\tilde{\theta}\right) \mathbf{1}\{\tilde{\theta} > \theta^*\} \, \mathrm{d}H(\tilde{\theta}) \qquad (2)$$

where

$$\tau(\sigma) = \int_{\max\{\underline{\theta}, \theta - b(\theta,\sigma)\}}^{\min\{\overline{\theta}, \theta + b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1 - p(\tilde{\theta})}\tilde{\theta}\right) \mathrm{d}H(\tilde{\theta})$$

In equilibrium, θ^* must solve (1) given $p(\theta)$, and $p(\theta)$ must solve (2) for all students (θ, c) given θ^*.

We can now compare equation (1) with the equation that defines the last student admitted to H in a rational expectations equilibrium:

$$\int_{\theta^*}^{\overline{\theta}} h(\theta) \, \mathrm{d}\theta = q. \qquad (3)$$

If there are students with sufficiently high costs—e.g. if g has full support on $\mathbb{R}_+$—any small belief distortion in equation (2) will induce self-selection among disadvantaged students: $c > c^H(\theta, p(\theta))$. Then, the term under the integral sign in (1) is smaller than in (3) because $G(c^H(\theta, p(\theta))) < 1$ as $c^H(\theta, p(\theta)) < c \leq \bar{c}$. Therefore, the ability of the last admitted student at H in a local sampling equilibrium θ^* must be smaller than in a rational expectations equilibrium to fill all the seats in equation (1).

We just proved that two types of inefficiencies arise in a local sampling equilibrium: high-achieving disadvantaged students self-select out of elite colleges even though their actual admission probability is one, and low-achieving advantaged students spend inefficient resources in applications at elite colleges even though their actual admission chances are zero. See Figure 1 in the introduction for a graphical representation of the two inefficiencies.

PROPOSITION 2 (Equilibrium Characterization). *Suppose that g has full support on $\mathbb{R}_+$ and assume ability and cost are independent. For all $q < 1$ there exists $\theta^* \in (0, 1)$ and $(\sigma(\theta), p(\theta))$ that solve (1) and (2) such that in this local sampling* *equilibrium students $N^H = \{(\theta, c) : \theta > \theta^*, c \leq c^H(\theta, p(\theta))\}$ obtain a seat at elite colleges and $N^L = N \setminus N^H$ go on the labor market. In a sampling equilibrium characterized by the admission threshold θ^*, there are two types of inefficiencies:*

1. *Missed opportunities: all students (θ, c) with ability $\theta > \theta^*$ and cost $c > c^H(\theta, p(\theta))$ self-select out of elite colleges.*

2. *Inefficient applications: all students (θ, c) with ability $\theta < \theta^*$ and cost $c < c^H(\theta, p(\theta))$ apply to H but are rejected and suffer a cost $-c$.*

Observe that compared to the rational expectations equilibrium both the supply side and the demand side suffer from inefficiencies. On the supply side, belief distortion arises endogenously and leads to payoff-relevant mistakes for high-achieving students and low-achieving advantaged students. On the demand side, the quality of the pool of admitted students at elite colleges is lower than with rational expectations due to equilibrium mismatch.

We also note that in a sampling equilibrium, high ability students are overly pessimistic about their chances of success, and low ability students are overly optimistic. Further empirical studies should explore the validity of these predictions.

In our model, we assumed that the mass of students was 1. When the mass of students is m, the same analysis as above applies replacing the mass of seats q by q/m and the window threshold τ by τ/m. Keeping q/m and τ constant while varying m would thus lead in our model to consider the comparative statics as τ varies. Clearly, when m grows very large, it corresponds in our model (with normalized population size) to the case in which τ tends to 0 and then students form their expectations using an infinitesimal sample of individuals. As it turns out, in our model this leads to rational expectations because students do not bias their estimate with dissimilar ability students and admission rate depends only on ability. Indeed, taking the limit $\tau \to 0$ of the implicit equation (2) we can see that if $\theta < \theta^*$ then there is τ_* small enough such that $\theta + b(\theta, \sigma) < \theta^*$ and $\theta - b(\theta, \sigma) < \theta^*$. Therefore, the integral in (2) is zero, and we have $p(\theta, c) = 0$. Similarly, one can verify that for all $\theta > \theta^*$, $p(\theta, c) = 1$. Therefore, only the best students (with $\theta > \theta^*$) apply to elite colleges and the last student admitted in a local sampling equilibrium coincides with that of the rational expectations equilibrium.

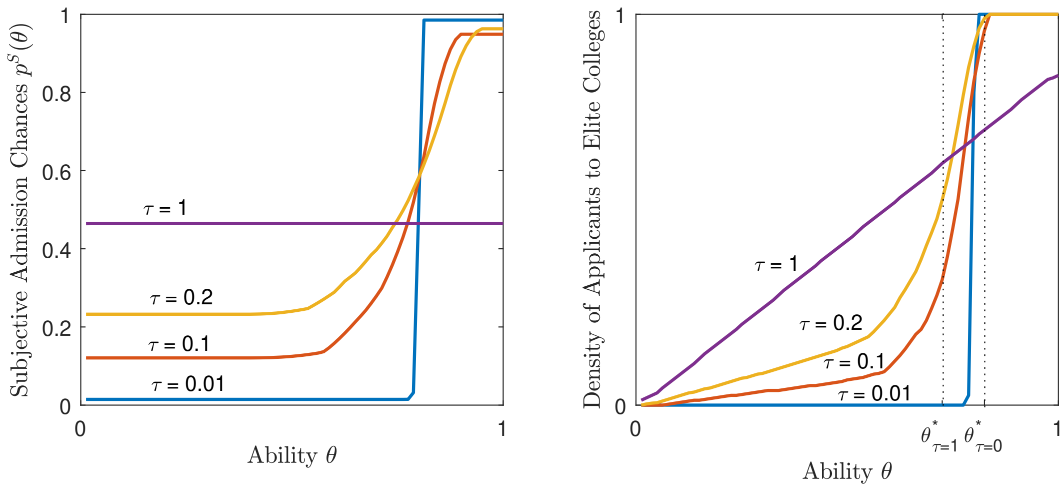


Figure 3: *(Left) Subjective admission chances as a function of student ability. Bias in subjective beliefs increases with the confidence parameter τ. (Right) Density of applicants to H as a function of student ability. As τ increases, the admission cutoff $\tilde{\theta}^*$ decreases, the number of self-selecting students (on the right of the cutoff) increases and the number of inefficient applicants (on the left of the cutoff) increases as well.*

Moving away from the limit $m = \infty$, students would not form expectations using one data point, formalized in our model by assuming that τ is strictly positive, away from 0. Obviously, belief distortions decrease with m (or increase with the confidence level τ in our normalized model) because students include peers with more different characteristics in their sample. Hence bias in the estimate stems from a selection bias that increases with τ. When τ becomes so large that students include the entire population in their sample, the subjective beliefs of the entire population converge to the same aggregate admission rate. This will arise when we consider the limit of a small q as then the number of students applying will be small leading students to consider the entire population of applicants (the mass of it will fall short of τ, irrespective of τ when q is small enough).

The comparative statics with respect to τ in our model with normalized population size is illustrated for the case in which θ and c are uniformly distributed on $[0, 1]$ in Figure 3 (Left). In this case, our simulations reveal that there is a unique local sampling equilibrium for all τ. Figure 3 (Right) illustrates the two types of inefficiencies that arise in a local sampling equilibrium. We see that as the confidence parameter τ increases, the admission cutoff $\tilde{\theta}^*_\tau$ decreases. Subjective beliefs, however, move smoothly around this threshold hence the mass of student who apply to H with an ability that is below the cutoff $\tilde{\theta}^*_\tau$ is positive (inefficient applications), and the mass of students who apply to H with an ability that is above the cutoff is below one (missed opportunities).

PROPOSITION 3. *As $\tau \longrightarrow 0$, the local sampling equilibrium converges to the rational expectations equilibrium. Assuming there is a unique local sampling equilibrium for all τ, a higher confidence parameter τ leads to more self-selection from high-achieving disadvantaged students and to more inefficient applications from low-achieving advantaged students.*

Comments.

1. When τ is so large (i.e. larger than 1) that students estimate their admission chances to be the aggregate admission rate in the entire neighborhood irrespective of their ability, an alternative interpretation of the sampling equilibrium is that students do not observe the ability of their peers leading them to consider the aggregate admission rate in the entire population to form their estimate. This is the scenario considered by Manski (1993) in a model focused on returns to schooling. Our heuristic procedure for more general τ — which is in the spirit of the K nearest neighbour algorithm — allows us to go smoothly from such a limit scenario to the rational expectations scenario by varying τ from 1 (or more) to 0.

2. The inefficiencies identified in a sampling equilibrium would not be the same if students had access to the counterfactual information of the admission outcome of those students who do not apply (still assuming that students consider a mass τ of students as close as possible to their own ability to form their expectations). Indeed, the fact that students only consider the pool of applicants implies that the bias is bigger when fewer students with nearby ability types apply. As a result, the bias is smaller for very large θ students than for very low θ students.

This implies that the missed opportunities (those $\theta > \theta^*$ who do not apply) concern more the students with ability not too far away from θ^* as compared with the inefficient applications (those $\theta < \theta^*$ who apply and get rejected) which concern also low θ students. This can be seen in Figure 3 where the slope of the subjective admission chance is steeper for θ above θ^* than it is for θ below θ^*.

3. The conclusion that as τ converges to 0, the rational expectations equilibrium obtains would not hold if students were to sample in priority peers with similar cost c as opposed to peers with similar ability θ (this can be inferred from the observation that in such a case, subjective beliefs would depend on c when in reality they depend on θ, see the Appendix for details).

3.1 Small Number of Seats

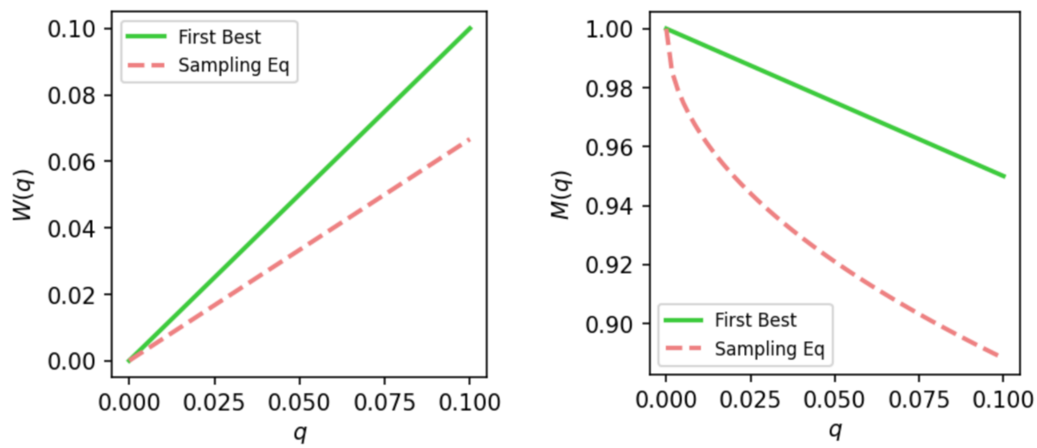
While it is somehow difficult to derive closed form descriptions of the sampling equilibrium, we now do so (through approximations) assuming that the number q of seats is small and that θ and c are independently distributed according to a smooth joint density $f(\theta, c) = h(\theta)g(c)$ with full support on $[0, 1] \times [0, 1]$. We note that assuming that q is small seems plausible for elite colleges that have limited capacity.

We first note that in the limit as q tends to 0, the mass μ of students applying to elite colleges must also converge to 0. This is true in the rational expectation case but also when τ is large so that it leads students to consider the aggregate acceptance rate, and it can be shown to apply to any intermediate value of τ.[15] Given this observation, it follows that for a fixed τ, when q is small enough, students make their choice whether or not to apply to H based on the aggregate acceptance rate p, and (θ, c) applies to H iff $c < \frac{p}{1-p}\theta$.[16] The

[15]This can be established as follows. Suppose by contradiction that the mass $\mu(q)$ of applicants is no smaller than $\mu^* > 0$ for all q. Then it is readily verified that for all θ, $p(\theta)$ should tend to 0 as q tends to 0 (since $p(\theta) < \frac{q}{\min(\mu^*, \tau)}$ and $\frac{q}{\min(\mu^*, \tau)} \to 0$ as $q \to 0$).

Since a student with ability θ would only apply if $c < \frac{p(\theta)}{1-p(\theta)}\theta$ and $G(\frac{p}{1-p}\theta)$ tends to 0 as p tends to 0, we would have that $\mu(q) = \int_0^1 G\left(\frac{p(\theta)}{1-p(\theta)}\theta\right) h(\theta)\, \mathrm{d}\theta$ tends to 0 as q tends to 0, leading to a contradiction.

[16]Given that p is small, the analysis presented here would be unchanged in the case in



fixed point determination of p in turn implies that

$$p = q \Big/ \int_0^1 h(\theta) G\left(\frac{p}{1-p}\theta\right) \mathrm{d}\theta.$$

In the next Proposition, we establish that the sampling equilibrium is unique when q is small enough and we provide approximations to $\theta^*(q)$, $p(q)$ as well as the welfare $W(q)$ and the average ability of admitted students $M(q)$ in terms of q for arbitrary densities h and g.

PROPOSITION 4. *When q is small, there is a unique sampling equilibrium. Moreover*

$$\begin{aligned}
p(q) &= \left(\frac{q}{g(0)E(\theta)}\right)^{1/2} + o(q^{1/2}) \\
\theta^*(q) &= 1 - \frac{1}{h(1)}\left(\frac{E(\theta)q}{g(0)}\right)^{1/2} + o(q^{1/2}) \\
W(q) &= \left(1 - \frac{E(\theta^2)}{2E(\theta)}\right) q + o(q) \\
M(q) &= 1 - \frac{1}{2h(1)}\left(\frac{E(\theta)q}{g(0)}\right)^{1/2} + o(q^{1/2})
\end{aligned}$$

which c would be an application cost instead of a rejection cost (this follows because p and $\frac{p}{1-p}$ are the same at the first order).

Note that in the first-best (or rational expectations) case, we have

$$\begin{aligned} p^{FB} &= 1 \\ \theta^{FB}(q) &= 1 - \frac{q}{h(1)} + o(q) \\ W^{FB}(q) &= q + o(q) \\ M^{FB}(q) &= 1 - \frac{q}{2h(1)} + o(q) \end{aligned}$$

Compared to the first best, we note that in the sampling equilibrium the admission cutoff is smaller, as well as welfare and average quality. The welfare loss is proportional to $\frac{E(\theta^2)}{2E(\theta)}$, meaning that it is larger when the distribution of ability is skewed towards high ability students. This is due to the fact that equilibrium mismatch is more costly (from a welfare perspective) when there are more high ability students. Interestingly, welfare is always increasing in the number of seats. Conversely, the average ability is always decreasing in the number of seats, as expected. In a sampling equilibrium, the average quality decreases faster than in a rational expectation equilibrium, since we deviate from 1 by $\sqrt{q}$ which is greater than q for small values of q. The drop in average quality is also proportional to $\sqrt{E(\theta)}$ in a sampling equilibrium, whereas it is independent from the type distribution in a rational expectation equilibrium. For comparison, we plot welfare and average quality as a function of seats q in a sampling equilibrium and a rational expectation equilibrium for a uniform distribution.

4 Competing Neighborhoods

We consider now the case of multiple neighborhoods competing for the same positions. The neighborhood plays a role only in shaping the samples from which students form their subjective assessment, as we assume the sampling is made locally (only within the neighborhood to which the student belongs). The fact that students from the various neighborhoods compete for the same seats creates a linkage between the various neighborhoods as the threshold ability θ^* above which students get admitted has to be the same across neighborhoods. This linkage in turn induces externalities across neighborhoods the effects of which are the main subject of interest of this Section.

To formalize the questions of interest, consider a two-neighborhood setup. Neighborhood $i = 1, 2$ consists of a mass m_i of students with (θ_i, c_i). We let $f_i(\theta_i, c_i)$ denote the mass-normalized distribution of (θ_i, c_i). That is, (θ_i, c_i) is distributed according to $m_i f_i(\theta_i, c_i)$. And we let τ_i denote the sampling window in neighborhood i.

Consider first neighborhood i in isolation, assume there is a mass q_i of seats available for students in this neighborhood and that students follow strategy σ_i (q_i will be endogenized in equilibrium). We let $\theta(\sigma_i, q_i)$ be the corresponding threshold admission ability in this neighborhood. It is computed as shown in Section 3 using there the mass-normalized mass of seats q_i/m_i and the mass-normalized sampling window τ_i/m_i. An equilibrium is formally defined as follows.

DEFINITION 7. *A local sampling equilibrium with competing neighborhoods $i = 1, 2$ (with characteristics f_i and τ_i) and total mass q of seats is a strategy profile (σ_1, σ_2) such that there exist q_1, q_2 satisfying*

1. *σ_i is a local sampling equilibrium in the neighborhood i with a mass q_i of seats;*
2. *$q_1 + q_2 = q$ and,*
3. *$\theta(\sigma_1, q_1) = \theta(\sigma_2, q_2)$.*

We consider the case in which θ and c are independently distributed in each neighborhood, i.e. $f_i(\theta, c) = h_i(\theta) g_i(c)$ for $i = 1, 2$. In the next proposition, we let q be small for a fixed ratio m_1/m_2. In this case, students in neighborhood i consider the aggregate admission rate in neighborood i to decide whether to apply to elite colleges. Formally, the aggregate admission rate in neighborhood i is given by $p_i = \frac{q_i}{\mu_i}$ where μ_i is the mass of applicants in neighborhood i and q_i is the mass of seats obtained in neighborhood i. As in Section 2, a student in neighborhood i with characteristics (θ_i, c_i) applies to elite colleges whenever $c_i < \frac{p_i}{1-p_i} \theta_i$.

PROPOSITION 5. *As q gets small, the sampling equilibrium with competing neighborhoods is unique and characterized by*

$$\begin{aligned} q_i &= \frac{m_i h_i(1)^2 g_i(0)/E(\theta_i)}{m_1 h_1(1)^2 g_1(0)/E(\theta_1) + m_2 h_2(1)^2 g_2(0)/E(\theta_2)} q + o(q) \\ \frac{p_1}{p_2} &= \frac{E(\theta_2)}{E(\theta_1)} \frac{h_1(1)}{h_2(1)} + o(1) \end{aligned}$$

Proposition 5 allows us to see how the seats are distributed across neighborhoods as a function of the primitives m_i, g_i and h_i. We can establish the following comparative statics:

- When $h_1 = h_2$ (same distribution of θ_i), then $p_1 = p_2$ and $\frac{q_1}{m_1 g_1(0)} = \frac{q_2}{m_2 g_2(0)}$. Thus, if $g_1(0) > g_2(0)$, neighborhood 1 gets in relative share more seats than neighborhood 2. The neighborhood with a smaller opportunity cost (defined here as $\arg\max g_i(0) = 1$) applies more to elite colleges and obtains relatively more seats.

- When $h_1(1) = h_2(1)$ and $g_1(0) = g_2(0)$, if $E(\theta_1) > E(\theta_2)$, then $p_1 < p_2$ and $q_1/m_1 < q_2/m_2$. Namely, the neighborhood with a higher average quality is more pessimistic about admission chances and obtains fewer seats at elite colleges. This counter-intuitive result is explained by the fact that only the highest quality students from neighborhood 2 are admitted to elite colleges, raising the subjective admission chances in this neighborhood. The equilibrium effect is strong enough to have more students admitted from the neighborhood with lower average quality.

- When $E(\theta_1) = E(\theta_2)$ and $g_1(0) = g_2(0)$, if $h_1(1) < h_2(1)$, then $p_1 < p_2$ and $q_1/m_1 < q_2/m_2$. The interpretation is similar to the previous case.

In the next proposition, we consider a different exercise in which neighborhood 1 is assumed to be very big in size m_1.

PROPOSITION 6. *We consider the limit $q \to \infty$, $\frac{q}{m_1+m_2} \to 0$ and $m_1 \to \infty$ keeping fixed $\tau_1 = \tau_2 = \tau$ as well as m_2 and $h_i(\theta_i)$, $g_i(c_i)$. In this limit, neighborhood 1 gets almost all seats per capita as compared with neighborhood 2. Moreover since welfare is increasing in q (locally around 0) as established in the one neighborhood case, neighborhood 1 is favored in terms of welfare per capita.*

The intuition for this result is as follows. Neighborhood 1 is assumed here to be very big so that the sampling heuristic leads students in neighborhood 1 to form rational expectations in the limit. By contrast, neighborhood 2 is of more limited size leading students in this neighborhood to rely on the aggregate admission rate in that neighborhood to make their decisions whether to apply to elite colleges. The analysis in the previous section and the requirement that the admission threshold should be the same in both neighborhoods leads then to the result that the big neighborhood obtains most seats in relative share. This result can receive an interesting interpretation given that the neighborhood in our analysis is only used to determine the pool from which students make their sample. It shows that everything else being equal minority groups may be significantly hurt relative to majority groups simply due to the more biased estimate of minority students that results from minority students only sampling the minority group and thereby not seeing enough applicants with nearby characteristics in their observed pool.

4.1 Policy Instruments

We discuss the effect of two possible policy interventions. The first one consists in imposing quotas, pre-defining the number of seats each neighborhood should have in proportion to the size of the neighborhood. The second one consists in changing the compositions of the two neighborhoods by imposing some degree of mixing while leaving the equilibrium force determines the number of seats assigned to each neighborhood. When considering these interventions, we will discuss the effect in terms of welfare, in terms of expected quality of admitted students as well as a comparison of how the two neighborhoods benefit from the intervention.

Quotas. We investigate the effect on welfare of two types of quotas (assuming the mass q of seats is small and the two neighborhoods are of comparable size, i.e. Proposition 5 applies in the laissez-faire case).[17] First, we consider giving to each neighborhood an equal number of seats proportionally to their

[17] Total welfare is defined as the sum of welfare in the two neighbrohoods.

size, meaning $\frac{q_1}{m_1} = \frac{q_2}{m_2}$. Second, we consider reallocating seats to the neighborhood with the highest opportunity cost.

PROPOSITION 7. *We assume that q is small enough and we consider the following policies:*

1. *Giving an equal number of seats to each neighborhood proportionally to their size is welfare improving compared to laissez faire if*

$$\arg\min_i \frac{h_i(1)^2 g_i(0)}{E(\theta_i)} = \arg\min_i \frac{E(\theta_i^2)}{E(\theta_i)}.$$

2. *Giving more seats to the neighborhood with higher opportunity cost (interpreted as neighborhood $\arg\min_i g_i(0)$) is welfare improving compared to laissez faire if*

$$\arg\min_i g_i(0) = \arg\min_i \frac{E(\theta_i^2)}{E(\theta_i)}.$$

Whether quotas are welfare improving or not depends only on the distribution of ability, not the distribution of costs. Moreover, if two neighborhoods have the same mean ability but one neighborhood has lower variance, reserving seats for this neighborhood is welfare improving.

In the next Proposition, we assume that the two neighborhoods have the same size and we establish that when the mass of seats q is small enough, assigning to each neighborhood the same mass of seats $q/2$ always deteriorates the average quality of admitted students as compared with the laissez faire.[18]

PROPOSITION 8. *We assume that q is small enough and that the two neighborhoods are of equal size $m_1 = m_2 = 1$. Reserving the same mass of seats $q/2$ to each neighborhood reduces the average quality of admitted students.*

The conventional wisdom is that quotas may deteriorate the quality of admitted students because without quotas, seats are allocated efficiently. This intuition holds true in the rational expectations paradigm where seats are

[18]Formally, the overall average quality of admitted students is defined as $M = \frac{q_1 M_1 + q_2 M_2}{q_1 + q_2}$ where M_i is the average quality of admitted students in neighborhood i and as before q_i is the mass of seats in neighborhood i.

allocated efficiently. Surprisingly, the same conclusion that quotas have a negative impact on the average quality of admitted students holds true also in our sampling equilibrium environment, despite the fact that seats are not allocated efficiently.

Of course, another effect of quotas is that the neighborhood receiving less seats in proportion to its size in laissez faire benefits in terms of relative welfare from a policy that assigns seats in proportion to the size of the neighborhood. Altogether, this observation together with Propositions 7 and 8 can be used to assess the pros and cons of quotas in a sampling equilibrium environment.

Mixed Neighborhoods. We investigate whether moving students from the high cost neighborhood to the low cost neighborhood (and vice versa) increases welfare. Unlike quotas which do not change students' social network, this intervention exactly aims at reducing inequalities of social capital. We consider random reallocation, i.e. from two initial neighborhoods with distributions F_i and F_j we draw new neighborhoods from the following compound distributions:

$$\tilde{F}_i = \alpha F_i + (1-\alpha)F_j$$
$$\tilde{F}_j = \alpha F_j + (1-\alpha)F_i$$

The parameter α scales the equalization across neighborhoods: for $\alpha = 1$ there is no reallocation of students, and for $\alpha = \frac{1}{2}$ the new neighborhoods have equal cost distributions. In order to preserve the independence while mixing, it should be that the heterogeneity is either only on g_i or only on h_i.

PROPOSITION 9. *1. If $h_1 = h_2$, then mixing has no effect on aggregate welfare for small enough q.*

2. If $m_1 = m_2$, $g_1 = g_2$, a complete mixing (i.e., $\alpha = \frac{1}{2}$) is welfare enhancing for *small enough q whenever $1 - \frac{E(\theta_1^2)+E(\theta_2^2)}{2(E(\theta_1)+E(\theta_2))}$ is no smaller than*

$$\frac{h_1(1)^2/E(\theta_1)}{h_1(1)^2/E(\theta_1)+h_2(1)^2/E(\theta_2)}\left(1-\frac{E(\theta_1^2)}{2E(\theta_1)}\right) + \frac{h_2(1)^2/E(\theta_2)}{h_1(1)^2/E(\theta_1)+h_2(1)^2/E(\theta_2)}\left(1-\frac{E(\theta_2^2)}{2E(\theta_2)}\right)$$

3. *If $m_1 = m_2$, $g_1 = g_2$, $E[\theta_1] = E[\theta_2]$, and $h_1(1) = h_2(1)$ then mixing has no effect on aggregate welfare.*

4. *If $m_1 = m_2$, $g_1 = g_2$, $E[\theta_1] \neq E[\theta_2]$, $E[\theta_1^2] = E[\theta_2^2]$, and $h_1(1) = h_2(1)$ then aggregate welfare is monotonically increasing in mixing α.*

Note that we maintain fixed either h or g across neighborhoods to preserve the independence between ability and cost in the compound distribution. We observe that mixing may sometimes be good for total welfare when the two neighborhoods have different distributions of ability. The result that when neighbors differ in average ability, then mixing has a positive effect on welfare may be viewed in the perspective of the Moving to Opportunity experiment showing that moving disadvantaged students to more advantaged neighborhoods improve college attendance and efficiency (Chetty et al., 2016; Chetty and Hendren, 2018a,b).[19]

4.2 When Abilities and Costs Correlated: A Potential Extra Source of Belief Traps

In the previous analysis, we assumed that ability and opportunity costs are independenty distributed. In some contexts, it could be that cost and ability are not independently distributed. When multiple neighborhoods compete for the same seats at elite colleges, this can lead to equilibria with belief traps in which one neighborhood takes many more seats than the other—even if both neighborhoods are ex-ante identical. Clearly, given the symmetry of the problem, any neighborhood can take the role of being favored in equilibrium, thereby illustrating the possibility of multiple equilibria with possibly very

[19]The effect of mixing on the average quality of admitted students is somehow cumbersome and can go either way in general.

strong asymmetries. The following Proposition illustrates this in an extreme form.[20]

PROPOSITION 10. *Suppose that in both neighborhoods there is a mass α of students with $(\theta, c) = (0, 0)$ and a mass $1 - \alpha$ of students with $(\theta, c) > (0, 0)$ (with arbitrary distribution) and assume $1 > q > 0$.[21] For α small enough, there is an equilibrium in which all seats at H are taken by students from neighborhood i.*

In the equilibrium of Proposition 10, only very low ability students in neighborhood j apply to elite college, and they all get rejected. They apply to elite colleges because their costs of rejection are negligible (even null in the formal statement). But, by applying and being rejected such students create a strong negative externality on high ability students in neighborhood j, as the latter get convinced they are better off not applying (even for moderate rejection costs). On the other hand, when $1 > q > 0$ and α is small enough, one can guarantee that at the same time there is a mass no less than q of students from neighborhood i with $\theta > 0$ who apply to H, thereby ensuring that all seats are taken by students in i.[22] While extreme, we believe that Proposition 10 is suggestive that multiple equilibria can easily arise in our setting (in particular when low rejection cost students tend to have low ability), which in turn may suggest that some asymmetries in outcome may sometimes be the result of historical factors rather than fundamental asymmetries.[23] Note that there is no explicit externality of low ability students on high ability students, since they do not impact the admission cutoff, but only an equilibrium-induced externality on expectation formation. This con-

[20]The theme of exploring the possibility of multiple equilibria in otherwise symmetric contexts is recurrent in the literature (see, in particular as a canonical illustration, Bénabou (1993) in the context of competing jurisdictions with composition externalities).

[21]This does not assume a vanishing number of seats, but it is possible to construct a sequence of economies parameterized by (q, α) such that q tends to 0 and one neighborhood gets all the q seats in a local sampling equilibrium, thereby establishing the multiplicity even as q tends to 0.

[22]The limit $\alpha \to 0$ corresponds to the model studied above and when $q < 1$ and there is only one neighborhood, the equilibrium is such that the demand for the elite college in that neighborhood exceeds the capacity, thereby providing the required property.

[23]To make this insight fits with our previous results in which the mass of seats was assumed to be small, one could consider a sequence of such economies indexed by the mass q_n of students, let q_n tends to 0 as n tends to infinity, and for each n, let α_n be small enough so that Proposition 10 applies.

trasts with models of segregation that consider explicit externalities, such as Benabou (1993).

5 Conclusion

We have introduced a model of expectation formation in a career choice problem. Unlike the rational expectations framework, students were assumed to have no prior information and no prior belief as to how elite colleges admit students. We have assumed instead that students non-parametrically estimate the distribution of outcomes conditional on actions by averaging past experiences from a sufficient mass τ of applicants with nearby ability characteristics. Formally, we have introduced a new solution concept—the local sampling equilibrium—in which sudents best respond to their subjective expectations, and expectations are consistent with the average observation made in the sample viewed as sufficiently reliable and representative. We believe this provides a coherent framework for thinking of the strategic interactions between expectation formation and the social environment.

We have derived three main results. First, expectation formation leads to belief traps whereby relatively high ability disadvantaged students self-select out of elite colleges, and less able advantaged students take their seats at elite colleges. This is due to the fact that average students create a strategic externality on high-achieving students by distorting their perceived admission chances toward the mean. This leads to multiple inefficiencies: on the supply side, high-achieving disadvantaged students go on the labor market instead of attending elite colleges, whereas low-achieving advantaged students spend resources applying to elite colleges even though their actual admission chances are zero. On the demand side, the pool of admitted students is of lower quality compared to the rational expectations benchmark.

Second, we have extended our study of local sampling equilibrium to the case of competing neighborhoods, and we have shown among other things that minority groups may end up getting a disproportionately small number of seats in elite collleges and how neighborhoods with lower rejection costs may end up getting a disproportionately high number of seats in elite colleges. This type of cross-neighborhood externality arises because rationing at elite colleges acts as a propagation mechanism of local demand shocks. Indeed, a reduction of cost in one neighborhood induces a higher admission cutoff, leading to a lower admission rates in other neighborhoods hence more self-selection. This may suggest that growth inequality across locations disproportionately benefits advantaged/majority neighborhoods at the expense of poor/minority neighborhoods.

Finally, we have studied the pros and cons of using quotas and/or of mixing neighborhoods in terms of total welfare and the quality of admitted students. In particular, we have shown that quotas (imposing that the number of seats in a neighborhood be proportional to its size) may sometimes be welfare-enhancing in contrast to the predictions obtained under rational expectations while the quality of admitted students was shown to be reduced with quotas similarly as in the rational expectations case. While more work would be needed to quantify these effects and study their robustness, we believe that our model of expectation formation can serve as a building block in empirical studies on education choices.

Appendix A Application Cost

An alternative to the opportunity cost is to consider an application cost: all else equal, it is harder for disadvantaged students to apply to elite colleges because they don't have access to peers or professional who can help them in the process.

The payoffs are as follows:

– If student (θ, c) goes on the labor market L her utility is 0.

– If student (θ, c) applies to H and obtain a seat, her utility is $\theta - c$.

– If student (θ, c) applies to H but does not get a seat, she goes on the labor market and her utility is $-c$.

Student (θ, c) applies to H whenever $p(\theta)\theta - c \geq 0$, that is, whenever $c^H(\theta, p) \leq p(\theta)\theta$. Define welfare as

$$W(\sigma) = \int_{\theta^*}^{\overline{\theta}} \int_0^{c^H} \theta f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta - \int_0^1 \int_0^{c^H} c f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta$$

It is readily verified that with one neighborhood: (i) the rational expectation equilibrium with application cost is identical than with opportunity cost, and (ii) the local-sampling equilibrium with application cost is identical than with opportunity cost (up to the thresholds c^H). Therefore, with one neighborhood the analysis and the qualitative predictions are very similar.

With multiple neighborhoods, the welfare effect of policy instruments is similar with application cost and opportunity cost. For instance, quotas are welfare neutral with uniform cost.

Proposition 11. *Consider two neighborhoods with costs uniformly distributed on $[0, \overline{c}_i]$ and $[0, \overline{c}_j]$. As $q \longrightarrow 0$, quotas have no effect on aggregate welfare at the first order.*

Proof of Proposition 11. First, we derive subjective admission chances in the case with quotas. We consider the following neighborhood specific quotas: $q_i = q_j = \frac{q}{2}$. The subjective admission chances for each neighborhoods write:

$$p_i = \sqrt{\overline{c}_i q} \qquad p_j = \sqrt{\overline{c}_j q}$$

The neighborhood specific admission cutoff $\tilde{\theta}_i^*$ solves

$$\int_{\tilde{\theta}_i^*}^{1} \frac{p_i}{\overline{c}_i} \theta \, \mathrm{d}\theta = \frac{q}{2} \iff \tilde{\theta}_i^* = \sqrt{1 - q\frac{\overline{c}_i}{p_i}}. \tag{4}$$

The admission cutoff in neighborhood j is similar, replacing p_i with p_j.

Second, we approximate $W(\theta^*)$ at the first order and show that it is independent of $\overline{c}_i$ and $\overline{c}_j$. As $q \longrightarrow 0$, we have

$$W_i(\theta^*) = p_i \frac{1 - (\theta^*)^3}{3\overline{c}_i} - \frac{p_i^2}{6\overline{c}_i}.$$

For $q \longrightarrow 0$, we make the following approximation: $(\theta^*)^3 \approx 1 - 3\sqrt{q\overline{c}_i}$. Therefore we obtain $W_i \approx \frac{5}{6}q$ at the first order. Hence, $W(\theta^*) \approx \frac{5}{3}q$ is independent of $\overline{c}_i, \overline{c}_j$. □

APPENDIX B BUNDLING ON COST

In the main text, we assume that students bundle peers based on ability (and indirectly based on cost when there are multiple neighborhoods). Here we explore the converse where students bundle peers based on cost directly. Overall, this increases the various inefficiencies (as students bundle on a dimension that is irrelevant for admissions), up to the point that as $\tau \longrightarrow 0$ we do not converge to the rational expectation equilibrium, unlike the case with bundling on ability.

PROPOSITION 12. *Suppose that students bundle peers only based on cost c. Then, as $\tau \longrightarrow 0$, the local sampling equilibrium does not converge to the rational expectation equilibrium.*

Proof. Let $A(\tau) \subset [\underline{c}, \overline{c}] \times [\underline{\theta}, \overline{\theta}]$ be the set of admitted students in any local sampling equilibrium. For this to converge to a REE, we must have $\lim_{\tau \to 0} A(\tau) = [\underline{c}, \overline{c}] \times [\theta^*, \overline{\theta}]$ and $p(c) = 0$ or 1 for any c. But notice that $\lim_{\tau \to 0} A(\tau) = [\underline{c}, \overline{c}] \times [\theta^*, \overline{\theta}]$ implies $\lim_{\tau \to 0} p(c) = 1 - \theta^*$ which is not equal to 0 or 1, a contradiction. □

APPENDIX C PROOFS

Existence of Local Sampling Equilibria. Consider the following scheme:

$$p \longmapsto \sigma^{BR}(p,\cdot) \longmapsto b(\sigma^{BR},\cdot) \longmapsto p(b)$$

By Tychonoff's theorem, the scheme is compact-valued $p(b) \in [0,1]^{\Theta}$. Hence to obtain a fixed point, we just need to prove that the scheme is continuous. Fix a subjective belief map $p : \Theta \longrightarrow [0,1]$. The action space is binary and the subjective admission chances p enter payoffs linearly, hence σ^{BR} is the following measurable threshold strategy:

$$\sigma^{BR}(p,\cdot) = \begin{cases} 1 & \text{if} \quad p(\cdot) \geq \gamma(\cdot) \\ 0 & \text{if} \quad p(\cdot) < \gamma(\cdot) \end{cases}$$

where $\gamma(\theta, c) = \frac{c}{\theta + c}$. Take any converging sequence $p_n \longrightarrow p$. We need to show that $p \longmapsto \sigma^{BR}(p,\cdot)$ is continuous in the L^1-weak topology, namely

$$\int \sigma^{BR}(p_n,(\theta,c))\,\mathrm{d}F \longrightarrow \int \sigma^{BR}(p,(\theta,c))\,\mathrm{d}F.$$

We have

$$\int \sigma^{BR}(p_n,(\theta,c))\,\mathrm{d}F = \int \mathbf{1}\left\{p_n(\theta) \geq \gamma(\theta,c)\right\}\,\mathrm{d}F.$$

Therefore, continuity follows from Lebesgue's dominated convergence theorem. We now show the continuity of $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$. By Berge's maximum theorem, $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$ is upper-hemicontinuous. The loss function $|\theta - \tilde{\theta}|$ is strictly quasi-convex, hence $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$ is continuous. Finally, the continuity of $b \longmapsto p(b)$ follows directly from the integrability of p together with the continuity of the functions $\max\{\cdot,\cdot\}$ and $\min\{\cdot,\cdot\}$. Therefore, by the Schauder fixed point theorem the set of local sampling equilibria is nonempty. $\square$

Proof of Proposition 1. Fix the admission cutoff at $\theta^* = H^{-1}(1-q)$. If

$\sigma^R(\theta, c) = 1$ for all $\theta > H^{-1}(1-q)$ and 0 otherwise, then the beliefs

$$p^R(\theta) = \begin{cases} 1 & \text{when} \quad \theta > H^{-1}(1-q) \\ 0 & \text{when} \quad \theta < H^{-1}(1-q) \end{cases}$$

are rationally consistent with σ^R by definition of θ^*. Given these subjective beliefs, $\sigma^R(\theta, c) = 1$ for all $\theta > H^{-1}(1-q)$ and 0 otherwise is optimal. Therefore, (σ^R, p^R) is a rational expectations equilibrium.

We now prove uniqueness. Suppose that $\sigma(\theta, c) < 1$ for some (positive mass of) $\theta > \theta^*$ and $\sigma(\theta, c) > 0$ for some (positive mass of) $\theta < \theta^*$. By belief consistency, students with ability $\theta > \theta^*$ know that $p^R(\theta) = 1$ (i.e. they can obtain a seat at H for sure) hence they have a profitable deviation. □

Proof of Proposition 2. First we show that all students (θ, c) with ability $\theta > \theta^* = H^{-1}(1-q)$ and cost $c > c^H(\theta^*, p(\theta^*))$ self-select out of elite colleges. Student (θ, c) applies to H only if $p(\theta^*) \geq \frac{c}{\theta^*+c}$. As long as $q < 1$ and $\tau > 0$, we must have $p(\theta^*) < 1$ because the last admitted student (θ^*, c) includes rejected students in her sample. Therefore, as $\lim_{c\to\infty} \frac{c}{\theta^*+c} = 1$ there must exist a positive g-measure of costs such that $p(\theta^*) < \frac{c}{\theta^*+c}$ because g has full support on $\mathbb{R}_+$. This proves that self-selection arises in equilibrium.

Second, we show that students with ability $\theta < \tilde{\theta}^*$ and cost $c < c^H(\tilde{\theta}^*, p(\tilde{\theta}^*))$ apply to H but are rejected. Student (θ, c) with $\theta = \tilde{\theta}^* - \varepsilon$ for $\varepsilon > 0$ arbitrarily small applies to H only if $p(\theta^*) \geq \frac{c}{\tilde{\theta}^*+c}$. As long as $q < 1$ and $\tau > 0$, $p(\theta^*) > 0$ because this student includes in her sample admitted peers for ε small enough. Therefore, as $\lim_{c\to 0} \frac{c}{\tilde{\theta}^*+c} = 0$ there must exist a positive g-measure of costs such that $p(\tilde{\theta}^*) > \frac{c}{\tilde{\theta}^*+c}$ because g has full support on $\mathbb{R}_+$. This proves that inefficient applications arise in equilibrium. □

Proof of Proposition 3. We can rewrite the implicit equation for subjective beliefs as follows:

$$p - \frac{1}{\tau} \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{1}_{\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \cap \{\tilde{\theta} > \tilde{\theta}^*\}} G\left(\frac{p(\tilde{\theta})}{1 - p(\tilde{\theta})}\right) \mathrm{d}H(\tilde{\theta}) = 0 \tag{5}$$

We first consider the case in which $\tau \to 1$. By definition of $b(\theta, \sigma)$ we have

$\lim_{\tau\to 1}\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \supseteq \{\tilde{\theta} > \tilde{\theta}^*\}$. Therefore,

$$\lim_{\tau\to 1}\left[p(\theta) - \frac{1}{\tau}\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{1}_{\{\theta-b(\theta,\sigma)<\tilde{\theta}<\theta+b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1-p(\tilde{\theta})}\right)\,\mathrm{d}H(\tilde{\theta})\right] = 0$$
$$\iff p = \int_{\underline{\theta}}^{\overline{\theta}} G\left(\frac{p}{1-p}\right)\,\mathrm{d}H(\tilde{\theta})$$

where the second line uses the fact that, as $\tau \to 1$, the subjective probability becomes independent of θ.

We now consider the case $\tau \to 0$. There are two cases to consider.

Case 1: There exists τ_* small enough such that $\theta + b(\theta,\sigma) < \tilde{\theta}^*$. Then we have $\lim_{\tau\to 1}\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \cap \{\tilde{\theta} > \tilde{\theta}^*\} = \emptyset$. Hence taking the integral in equation (5) is zero, and we directly have that $p(\theta) = 0$.

Case 2: There exists τ^* small enough such that $\theta - b(\theta,\sigma) > \tilde{\theta}^*$. Then we have $\lim_{\tau\to 1}\{\theta - b(\theta,\sigma) < \tilde{\theta} < \theta + b(\theta,\sigma)\} \subseteq \{\tilde{\theta} > \tilde{\theta}^*\}$. Therefore,

$$\lim_{\tau\to 0}\left[p(\theta) - \frac{1}{\tau}\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{1}_{\{\theta-b(\theta,\sigma)<\tilde{\theta}<\theta+b(\theta,\sigma)\}} G\left(\frac{p(\tilde{\theta})}{1-p(\tilde{\theta})}\right)\,\mathrm{d}H(\tilde{\theta})\right] = 0$$

Take $p(\theta) = 1$ and using the fact that $\lim_{x\to\infty} G(x) = 1$ we can rewrite the above equation as follows:

$$\lim_{\tau\to 0}\left[1 - \frac{1}{\tau}\int_{\theta-b(\theta,\sigma)}^{\theta+b(\theta,\sigma)} h(\tilde{\theta})\,\mathrm{d}\tilde{\theta}\right] = 0$$

By L'Hospital's rule and Leibniz integral rule,

$$\lim_{\tau\to 0}\frac{\int_{\theta-b(\theta,\sigma)}^{\theta+b(\theta,\sigma)} h(\tilde{\theta})\,\mathrm{d}\tilde{\theta}}{\tau} = \lim_{\tau\to 0}\Big[h(\theta+b(\theta,\sigma)) + h(\theta-b(\theta,\sigma))\Big]\frac{\partial b(\theta,\sigma)}{\partial \tau} \tag{6}$$

By definition, $b(\theta,\sigma)$ is the smallest $b > 0$ that solves:

$$\int_{\theta-b}^{\theta+b} h(\theta)\,\mathrm{d}\theta > \tau \iff \underbrace{H(\theta+b) - H(\theta-b) - \tau}_{=\Phi(b,\tau)} > 0$$

We apply the implicit function theorem to obtain the derivative of $b(\theta, \tau)$:

$$\frac{\partial \Phi}{\partial b}\frac{\partial b}{\partial \tau} + \frac{\partial \Phi}{\partial \tau} = 0 \iff \frac{\partial b}{\partial \tau} = \frac{1}{h(\theta + b) + h(\theta - b)}$$

Substituting this expression in equation (6) concludes the proof. $\square$

Proof of Proposition 4

Step 1. ($p \to 0$ as $q \to 0$): By contradiction, suppose that there exists b with $p > b > 0$ for all q, then all (θ, c) such that $c < \frac{b}{1-b}\theta$ apply to H. The mass of applicants $m(q)$ is no smaller than $m^* = \Pr(c < \frac{b}{1-b}\theta)$ (and $m^* > 0$ where use is made of the full support assumption). But then $p(q) = \frac{q}{m(q)} \to 0$ and we get a contradiction.

Step 2. (Approximation of p in terms of θ^*): We have

$$\begin{aligned} p &= \frac{\int_{\theta^*}^1 h(\theta) G(\frac{p}{1-p}\theta) d\theta}{\int_0^1 h(\theta) G(\frac{p}{1-p}\theta) d\theta} \\ &\approx \frac{\int_{\theta^*}^1 h(\theta) \frac{p}{1-p}\theta g(0) d\theta}{\int_0^1 h(\theta) \frac{p}{1-p}\theta g(0) d\theta} \\ &= \frac{\int_{\theta^*}^1 h(\theta)\theta d\theta}{\int_0^1 h(\theta)\theta d\theta} \\ &= \frac{\int_{\theta^*}^1 h(\theta)\theta d\theta}{E(\theta)} \end{aligned}$$

using a 1st order Taylor approximation of G around 0. This approximation also implies that $\theta^* \to 1$ as $q \to 0$ to ensure that $p \to 0$ (from Step 1). This in turn implies that (using $h(\theta) \approx h(1)$ when θ is in $(\theta^*, 1)$, with $h(1) > 0$ because of full support):

$$p \approx \frac{h(1)}{E(\theta)} \frac{1 - (\theta^*)^2}{2}.$$

Step 3. (Approximation of θ^* in terms of q):

$$
\begin{aligned}
q &= \int_{\theta^*}^{1} h(\theta) G\left(\frac{p}{1-p}\theta\right) d\theta \\
&\approx \int_{\theta^*}^{1} h(\theta) p g(0) \theta d\theta \\
&\approx p g(0) h(1) \frac{1-(\theta^*)^2}{2}
\end{aligned}
$$

where the first approximation uses $\frac{p}{1-p} \approx p$ and $G(p\theta) \approx p\theta g(0)$, and the second approximation uses $h(\theta) \approx h(1)$ for $\theta \in (\theta^*, 1)$.

Writing θ^* in terms of q, we get

$$
\theta^* = 1 - \frac{1}{h(1)} \left(\frac{E(\theta)}{g(0)}\right)^{1/2} q^{1/2} + o(q^{1/2}).
$$

Step 4. (Approximation of p in q): From steps 3 and 4 we get

$$
p = \left(\frac{E(\theta)}{g(0)}\right)^{1/2} q^{1/2} + o(q^{1/2}).
$$

Step 5. (Approximation of $W(q)$): We have

$$
\begin{aligned}
W(q) &= \int_{\theta^*}^{1} \theta h(\theta) G\left(\frac{p}{1-p}\theta\right) d\theta - \int_0^{\theta^*} h(\theta) \left[\int_0^{\frac{p\theta}{1-p}} c g(c) dc\right] d\theta \\
&\approx g(0) p h(1)(1-\theta^*) - \int_0^{\theta^*} g(0) \frac{p^2\theta^2}{2} h(\theta) d\theta \\
&\approx g(0) p h(1)(1-\theta^*) - \frac{g(0)E(\theta^2)}{2} p^2 \\
&\approx q - \frac{E(\theta^2)}{2E(\theta)} q \\
&= \left(1 - \frac{E(\theta^2)}{2E(\theta)}\right) q + o(q)
\end{aligned}
$$

using the above approximations.

Step 6. (Approximation of $M(q)$): We have

$$\begin{aligned}
M(q) &= \frac{\int_{\theta^*}^{1} \theta h(\theta) G(\frac{p}{1-p}\theta) d\theta}{\int_{\theta^*}^{1} h(\theta) G(\frac{p}{1-p}\theta) d\theta} \\
&\approx \frac{\left(1-\theta^{*3}\right)/3}{\left(1-\theta^{*2}\right)/2} \\
&= 1 - \frac{1}{2h(1)} \left(\frac{E(\theta)}{g(0)} \right)^{1/2} q^{1/2} + o(q^{1/2})
\end{aligned}$$

using the above approximations. □

Proof of Proposition 5. At equilibrium, the admission cutoff must be equal between neighborhood 1 and 2. Using the formulas derived in Proposition 4, we have

$$\begin{aligned}
& 1 - \frac{1}{h_1(1)} \left(\frac{E[\theta_1] q_1}{g_1(0)} \right)^{\frac{1}{2}} = 1 - \frac{1}{h_2(1)} \left(\frac{E[\theta_2] q_2}{g_2(0)} \right)^{\frac{1}{2}} \\
\iff & q_1 = \frac{g_1(0)}{g_2(0)} \left(\frac{h_1(1)}{h_2(1)} \right)^2 \frac{1/E[\theta_1]}{1/E[\theta_2]}
\end{aligned}$$

Using the fact that $q_2 = q - q_1$, and rearranging the above formula yields the desired result. We obtain p_1/p_2 using the closed form from Proposition 4. □

Proof of Proposition 6. From Proposition 4 we know that q_1 is the product of two terms:

$$\lim_{m_1 \to \infty} \frac{m_1 h_1(1)^2 g_1(0)/E(\theta_1)}{m_1 h_1(1)^2 g_1(0)/E(\theta_1) + m_2 h_2(1)^2 g_2(0)/E(\theta_2)} = 1 \qquad \lim_{q \to \infty} q = \infty$$

Therefore, the limit of the product yields $q_1 \longrightarrow \infty$. Conversely, we have $q_2 \longrightarrow 0$ using the fact that $\frac{1}{m_1+m_2} q \to 0$ implies $\frac{am_2}{bm_1+cm_2} q \to 0$ for any positive constant a, b, c and m_2. □

Proof of Proposition 7. From the one neighborhood case, we know that aggregate welfare is (at the first order):

$$W = \left(1 - \frac{E[\theta_1^2]}{2E[\theta_1]}\right) q_1 + \left(1 - \frac{E[\theta_2^2]}{2E[\theta_2]}\right) q_2$$

Therefore, aggregate welfare can be increased only by giving more seats to neighborhood $\arg\min_i \frac{E[\theta_1^i]}{E[\theta_i]}$. Now if we give seats proportionally to the size of the neighborhood $q_i = \frac{q_i}{m_i}$, this will benefit the neighborhood that currently has the smallest number of seats, i.e. $\arg\min_i \frac{h_i(1)^2 g_i(0)}{E[\theta_i]}$. Alternatively, giving more seats to the most disadvantaged neighborhood will benefit neighborhood $\arg\min_i g_i(0)$. □

Proof of Proposition 8. Letting $A_i = h_i(0)^2 g_i(0)/E(\theta_i)$, the value of M after the intervention is

$$M^{AA} = 1 - \frac{1}{2}\left[\left(\frac{q}{2A_1}\right)^{1/2} + \left(\frac{q}{2A_2}\right)^{1/2}\right] + o(q^{1/2})$$

The laissez-faire value of M is

$$M^{LF} = 1 - \left(\frac{q}{A_1 + A_2}\right)^{1/2} + o(q^{1/2})$$

That $M^{LF} > M^{AA}$ follows from Jensen's inequality noting that $x \to x^{-1/2}$ is convex. □

Proof of Proposition 9.

1. Aggregate welfare is the same as in the one neighborhood case and only depend on h_1, h_2.

2. Aggregate welfare when $\alpha = \frac{1}{2}$ writes

$$\begin{aligned}&\frac{1}{2}\left(1 - \frac{\frac{1}{2}E[\theta_1^2] + \frac{1}{2}E[\theta_2^2]}{E[\theta_1] + E[\theta_2]}\right) q + \frac{1}{2}\left(1 - \frac{\frac{1}{2}E[\theta_1^2] + \frac{1}{2}E[\theta_2^2]}{E[\theta_1] + E[\theta_2]}\right) q \\ &= 1 - \frac{E[\theta_1^2] + E[\theta_2^2]}{2(E[\theta_1] + E[\theta_2])}\end{aligned}$$

   which should be no less than aggregate welfare when $\alpha = 0$.

3. In this case we have $q_1 = q_2 = q/2$. Hence aggregate welfare writes

$$\left(1 - \frac{\alpha E[\theta_1^2] + (1-\alpha)E[\theta_2^2]}{2E[\theta]}\right)\frac{q}{2} + \left(1 - \frac{\alpha E[\theta_2^2] + (1-\alpha)E[\theta_1^2]}{2E[\theta]}\right)\frac{q}{2}$$
$$= \left(2 - \frac{E[\theta_1^2] + E[\theta_2^2]}{2E[\theta]}\right)\frac{q}{2}$$

which is independent of α.

4. In this case, aggregate welfare writes

$$q\Bigg[\left(1 - \frac{E[\theta^2]}{2(\alpha E[\theta_1] + (1-\alpha)E[\theta_2])}\right)\frac{\frac{1}{\alpha E[\theta_1]+(1-\alpha)E[\theta_2]}}{\frac{1}{\alpha E[\theta_1]+(1-\alpha)E[\theta_2]} + \frac{1}{\alpha E[\theta_2]+(1-\alpha)E[\theta_1]}}$$
$$+ \left(1 - \frac{E[\theta^2]}{2(\alpha E[\theta_2] + (1-\alpha)E[\theta_1])}\right)\frac{\frac{1}{\alpha E[\theta_2]+(1-\alpha)E[\theta_1]}}{\frac{1}{\alpha E[\theta_1]+(1-\alpha)E[\theta_2]} + \frac{1}{\alpha E[\theta_2]+(1-\alpha)E[\theta_1]}}\Bigg]$$

which can be verified is an increasing function of α.

$\square$

Proof of Proposition 10. Suppose that all seats are taken by students from neighborhood i ($q_i = q$). First note that some seats must be allocated to high ability students (otherwise they have a profitable deviation as $p = 1$ if only students with $(\theta, c) = (0, 0)$ apply). Therefore, the admission cutoff satisfies $\theta^* > 0$. Moreover, for α small the admission probability converges to $\frac{q}{\zeta}$ where ζ is the fraction of high ability students who apply to H, with $\theta\frac{q}{\zeta} > c(1 - \frac{q}{\zeta})$. Hence all seats are occupied by students from neighborhood i for this ζ. Suppose that all low-ability students from neighborhood j apply to H but are being rejected because $\theta_j = 0 < \theta^*$. Then we have $p_j = 0$, and no high-ability student in neighborhood j applies to H. There are no profitable deviations and beliefs are consistent, hence this is a local sampling equilibrium. $\square$

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