Auction Design with Data-Driven Misspecifications: Inefficiency in Private Value Auctions with Correlation*

Philippe Jehiel and Konrad Mierendorff†

May 14, 2024

Abstract

We study the existence of efficient auctions in private value settings in which some bidders form their expectations about the distribution of their competitor's bids based on the accessible data from past similar auctions consisting of bids and ex post values. We consider steady-states in such environments with a mix of rational and data-driven bidders, and we allow for correlation across bidders in the signal distributions about the ex post values. After reviewing the working of the approach in second-price and first-price auctions, we show our main result that there is no efficient auction in such environments.

Keywords: Belief Formation, Auctions, Efficiency, Analogy-based Expectations
JEL Classification Numbers: D44, D82, D90

1 Introduction

Understanding which auction format if any ensures that the goods end up in the hands of the buyers who value them the most is not only of theoretical but also of practical interest.

*The first draft of this paper was completed before the tragic passing of the second co-author to whom this paper is dedicated. The content of this paper was presented by the first co-author in a preliminary shape as the Hurwicz lecture at the 2019 Conference on Economic Design in Budapest. Both co-authors thank the editor, referees, Dirk Bergemann, Alex Clyde, Andrew Ellis, Deniz Kattwinkel, Paul Klemperer, Laurent Lamy, Shengwu Li, Rani Spiegler, Giacomo Weber, Bob Wilson, Martin Weidner for helpful comments as well as the seminar audiences at SAET 2019, Oxford University, TSE, the 14th workshop on Economic Design and Institutions, UCLA, the Israeli theory seminar, the conference in honour of Konrad Mierendorff held at UCL, the 2nd KIT conference on market design, and the CREST. Jehiel thanks the European Research Council for funding (grant no 742816).

†Jehiel: Paris School of Economics and University College London (email: jehiel@enpc.fr); Mierendorff: Department of Economics, University College London (email: k.mierendorff@ucl.ac.uk)

As forcefully argued by Maskin (1992), a primary objective of privatizations is to ensure an efficient allocation of productive assets. More generally, the same efficiency concern applies to most auction settings that are organized by public authorities (for example, the U. S. Congress explicitly mandated the Federal Communications Commission to promote efficiency in its auctions of frequency bands for telecommunications sale of license auctions).

The academic view about efficient (one-object) auctions is as follows. In the private value setting, that is, when the private information held by any given buyer is a sufficient statistic for determining the value of the good to this buyer, the Vickrey or second-price auction is an efficient auction: Irrespective of whether there is correlation in the private information held by the various buyers and irrespective of potential asymmetries between buyers, the good ends up in the hand of the buyer who values it the most. This is not so with the first-price auction in which inefficiencies can arise in the private value setting, in particular in the presence of asymmetries or correlation. By contrast, in the interdependent value setting or when there are informational externalities between bidders, inefficiencies are unavoidable when private information is multi-dimensional no matter what auction format is used (see Maskin (1992) for an early illustration and Jehiel and Moldovanu (2001) for a general analysis of this).

In this paper, we revisit the possibility of efficient auctions in one-object private value settings assuming that some bidders, the less experienced ones, lack the ability to find out their best strategy, as usually considered in economic models. Specifically, such bidders referred to as novice observe some private signal (that is informative about their ex post valuation), and they also rely on the data accessible from past similar auctions played by other bidders, which are assumed to consist of bids and ex post values. In addition to the novice bidders, more experienced bidders can participate in the auction, and these are viewed as rational agents, thereby bidding optimally given the auction environment and the signals they receive.[1]

Our main result will be to establish that even in private value settings, there is no efficient auction when there is a mix of novice and experienced buyers, the private information is correlated among bidders, and the private information at the time of the auction is only a noisy signal about the ex post value of the buyer. That is, we suggest a novel

[1]From a more applied perspective, one can think of rational bidders as insiders having familiarity with the prevailing auction environment and of novice bidders as outsiders who would participate in such auctions for the first time and rely on AI or machine learning techniques applied to the available dataset to guide them on how to bid given their private information.

potential source of inefficiency in auctions that is related to the cognitive limitations of (some) bidders and not to the interdependence of the private information, as highlighted by the previous literature.

Specifically, we consider two-bidder one-object auctions in which at the time of the auction, a bidder's private information is a noisy signal about his (own) ex post value for the good, which is a simple way to capture the ex post uncertainty that prevails in many auction settings.[2] Importantly, we assume that after the auction is completed, what is publicly disclosed to new bidders is the profile of bids as well as the ex post values of the various bidders (the latter possibly with some lag), but *not* the signals observed by the bidders at the time of the auction. We believe that such disclosure assumptions are quite natural in a number of applications such as procurement auctions in which bids are often publicly disclosed afterwards, and upon completion of the contract, the details of the conditions of it become clear (making the assessment of the costs and resulting ex post valuations easier while, by contrast, accessing what the various contestants knew at the time of the auction remains very difficult even at that stage).[3]

It should be highlighted that we allow for correlation between signals, which will play a key role in the analysis. A practical way to think of correlation is that the distributions of private signals are influenced by unobserved conditions which are common to all bidders, thereby leading to correlation. But, despite the correlation and as already stressed, the setting is one of private values. Yet, novice players are assumed to be unaware of how signals and valuations are jointly distributed, and thus of the private value character of the auction. Instead, like econometricians or analysts would do, they construct a representation of the statistical links between the variables of interest based on the signal they receive as well as the dataset available to them. Specifically, as suggested above, observations from past auctions take the form (b_1, v_1, b_2, v_2) where b_j is the bid previously submitted by a subject in the role of bidder j and v_j is his ex post value. A novice bidder i constructs from the dataset the empirical distribution describing how b_j is distributed conditional on the various possible ex post values of bidder i. He also uses his own signal θ_i whose implication

[2]While the empirical literature on auctions has considered the possibility of noisy signals in private value settings, it has done so assuming bidders are risk averse (see Perrigne and Vuong (2022)). In our setting with non-rational bidders, ex post uncertainty plays a key role, even with risk neutral bidders.

[3]In some applications, it may be argued that accessing losers' valuations is not as easy as accessing winners' valuations. In the later part of the paper, we note that our model and its analysis would remain unchanged if instead of assuming outside observers have access to the ex post valuations of losers we assumed such observers can generate unbiased (possibly poorly informative) estimates of those.

in terms of the distribution of his ex post valuation v_i is assumed to be known by him (independently of the auction data). The novice bidder then combines the two pieces of information assuming that θ_i and the opponent's current bid b_j provide independent information on v_i, and he accordingly forms a belief about how (v_i, b_j) is jointly distributed given the signal θ_i. He then best-responds to this belief given the rules of the auction.

We will be considering steady state environments in which there is a mixed (large) population of bidders (assigned to the role of i or j) composed of a share of novice bidders (whose expectations are formed as just informally explained and who should be thought of as being replaced in every generation) and a complementary share of experienced or rational bidders (who can be thought of as participating in every generation). In each generation, bidders are matched randomly to play the same auction game, and in steady state, the distributions of data generated are the same across generations. Steady states are referred to as Data-Driven Equilibria.

We are concerned with the efficiency properties of Data-Driven Equilibria, and more precisely, whether by a judicious choice of auction rule, one can implement an efficient allocation in such an equilibrium. Our insights are as follows. First, unless the distributions of signals of the two bidders are independent, data-driven bidders rely on a misspecified statistical model, and as a result choose suboptimal bidding strategies. In Section 3, we start illustrating this with Second-Price Auctions (SPA) in the (symmetric) binary case in which there are two possible ex post values. We show that unlike rational bidders, novice bidders do not bid their expected value when there are correlations. As in winner's curse models (see Milgrom and Weber (1982) for the classic analysis of such models), novice bidders make inferences about their ex post value from how the other bidder bids. In the case of positive correlation, this leads novice bidders to bid more than their expected value when they receive good signals (because in the neighborhood of large opponent's bids, their own ex post value is more likely to be high) and less than their expected value when they receive bad signals (for a symmetric reason). We provide an illustration of the equilibrium for parametric classes of distributions, and we describe how it is affected by the share of rational bidders and/or the correlation of signal distributions.

Clearly, the fact that novice and rational bidders do not bid in the same way leads to inefficiencies in the binary case, unless there is perfect correlation of the signals, or the bidders are all novice or all rational. For our parametric example, we observe that the normalized welfare loss in the data-driven equilibrium of the SPA is U-shaped in the share of novice bidders as well as in the degree of correlation. More generally, we show for the binary mixed population case that as soon as there are correlations, there is some welfare loss in the Second-Price Auction. We also consider First-Price Auctions (FPA), for which we also show that there must be inefficiencies whenever there is correlation.

Our main result concerns general auction-like mechanisms defined as mechanisms in which each bidder submits a real-valued bid, and an outcome is chosen as a function of the profile of bids with the restriction that if a bidder submits a higher bid, this bidder has more chance of winning the object. In Section 4, we provide a general inefficiency result. More precisely, we show that for generic joint distributions of signals, there is no auction-like mechanism that allows to obtain (or approximate) the first-best as a Data-Driven Equilibrium when ex post values can take at least three realizations. The intuition for this result is as follows. To obtain efficiency among rational bidders, only the Second-Price Auction or a strategically equivalent auction format can be used. This is so because with more than two ex post values there is generically a manifold of signal realizations corresponding to the same expected value for the object, but different beliefs about the signal realization of the other bidder, and if the payment in the auction were to depend on the own bid, then the belief about the opponent would affect the shape of the optimal bid, as in First-Price Auctions. Since in Second-Price auctions, novice bidders do not bid their expected value as also observed in the simplified binary case, we conclude that inefficiencies must occur.

Section 5 discusses the robustness of our analysis in the case in which losers' valuations cannot be observed with precision ex post, and when mechanisms other than auctions can be used. Section 6 concludes.

Related literature

Our paper relates to different strands of literature. After, we present our model, we discuss in more detail the mode of reasoning of novice bidders, and relate the data-driven equilibrium to the Berk-Nash equilibrium (Esponda and Pouzo, 2016), the Bayesian Network Equilibrium (Spiegler, 2016) and the analogy-based expectation equilibrium (Jehiel, 2005).

It is also worth mentioning the relation/difference of the Data-Driven Equilibrium with the cursed equilibrium (Eyster and Rabin, 2005). While Eyster and Rabin (2005) also consider the auction application, it should be mentioned that the cursed equilibrium gives predictions away from the Nash equilibrium, only in interdependent value settings (thus not in our private value setting). In some sense, cursed bidders behave as if they were in a private value setting when in interdependent value environments. By contrast, our data-driven bidders behave as if they were in an interdependent value setting when in private value settings with correlation.

One can also mention the strand of literature initiated by Li (2017) who introduced the idea of obviously dominant strategy. In Li's approach, bidders fail to identify their (weakly) dominant strategy in the second-price auction because they may entertain different expectations about their competitor's bidding behavior when considering different bids. By contrast, the expectation of novice bidders about the opponent's behavior is the same irrespective of the bid, but the underlying correlation and the possibility of multiple ex post values lead novice bidders to miss that they are in a private value setting, thereby leading them to fail to identify their optimal strategy in second-price auctions.

Regarding the robust mechanism design literature (Bergemann and Morris, 2005), we note a common motivation with our approach in the sense that in both cases it is emphasized that it may be hard to know what the beliefs of agents are. While the robust mechanism design literature uses this observation to motivate the desire to implement outcomes for a large range of (or even all) beliefs,[4] our paper explicitly suggests a method of expectation formation for bidders who do not have access to such information from past auctions.

It should also be mentioned that our disclosure assumptions are quite similar to those made in the empirical literature on auctions as initiated by Hendricks and Porter (1988). Several important differences between our approach and this literature should however be mentioned. First, we consider private value environments when Hendricks and Porter consider common value environments. Second, the empirical literature on auctions assumes that bidders behave optimally according to a Bayes Nash equilibrium, when in our approach, the behaviors of the less experienced bidders are assumed to be derived from the available data, leading to suboptimal behaviors (in a data-driven equilibrium).

Finally, from a technical point of view, our analysis makes use of some results developed in the literature on mechanism design with correlation. In particular, we borrow genericity arguments from Gizatulina and Hellwig (2017).

[4]Our result that with only rational bidders, an efficient auction must be strategically equivalent to a second-price auction belongs to the robust design literature, and as far as we know it is a new result not formally appearing in that literature.

2 Model

Mechanisms. We consider the allocation of a single object to two bidders $i = 1, 2$ via an auction or more general auction-like mechanism. To simplify notation, when we consider a generic bidder $i \in \{1, 2\}$, we denote the opponent by $j \neq i$. A Mechanism $M = [(B_i), q, p]$ consists of three elements: (i) feasible bids B_i for the two bidders. A profile of bids is denoted $b = (b_1, b_2) \in B := B_1 \times B_2$. (ii) an allocation rule $q : B \to [0, 1]^2$, $q(b) = (q_1(b), q_2(b))$, with $q_1(b) + q_2(b) \leq 1$, where $q_i(b)$ is the probability that bidder i gets the object if the bid profile b is submitted. (iii) A payment rule $p : B \to \mathbb{R}^2$, $p(b) = (p_1(b), p_2(b))$, where $p_i(b)$ denotes the payment bidder i has to make if the bid profile b is submitted.

Valuations. Ex-post, the value of the object for bidder i is denoted v_i. It can take values in $V = \left\{v^1, \ldots v^K\right\}$. Up to normalization, it is without loss to assume that $0 = v^1 < \ldots < v^K = 1$. When participating in a mechanism, each bidder receives a signal $\theta_i = (\theta_i^1, \ldots, \theta_i^K) \in \Theta := \Delta V$, where θ_i^k denotes the probability that $v_i = v^k$. A profile of types is denoted $\theta = (\theta_1, \theta_2)$. We assume that conditional on θ_i, v_i is independent of θ_j. As a consequence the expected valuation of a bidder only depends on her own interim type: $E[v_i|\theta] = E[v_i|\theta_i]$. In other words, we are considering a setting with *private values*. Signals are jointly distributed with cumulative distribution function $F(\theta)$ and density $f(\theta)$ defined over Θ^2, and our main interest is when θ_1 and θ_2 are not independently distributed. We assume throughout that the joint distribution is symmetric and has a continuous and positive density. When there is no confusion, we slightly abuse notation and denote marginal distributions $F_i(\theta_i)$ and $f_i(\theta_i)$ by $F(\theta_i)$ and $f(\theta_i)$; and conditional distributions $F_i(\theta_i|\theta_j)$ and $f_i(\theta_i|\theta_j)$, by $F(\theta_i|\theta_j)$ and $f(\theta_i|\theta_j)$.

Comment. To think more concretely of a setting with correlation, let bidder i 's ex post valuation satisfies $v_i = v(\varphi_i, \varepsilon_i)$ where bidder i would observe $\varphi_i = \eta_i + z$ at the time of the auction, η_i would be an efficiency parameter that applies to i only and z would represent an efficiency parameter associated to the common difficulty of the project while ε_i would be the realization of an idiosyncratic shock occurring ex post after the time of the auction. Clearly, such a setup can be cast into our θ_i, θ_j formulation, and assuming that the distribution of z is non-degenerate, the distributions of θ_i and θ_j would exhibit correlation, even in the (natural) case in which η_i and η_j are independently distributed.

Rational and Novice Bidders. Each bidder i is characterized by a *generalized type* $t_i = (\theta_i, s_i)$, where θ_i denotes the signal described before, and $s_i \in \{r, m\}$ specifies the *sophistication* of the bidder. We denote the set of general types by $T = \Theta \times \{r, m\}$. For simplicity we will call θ_i just the *type*. The probability that $s_i = r$ is denoted $\lambda \in (0, 1)$; we assume that it is independent of θ_i and across bidders. Bidder i is *rational* when $s_i = r$; and bidder i is novice or *misspecified* when $s_i = m$. Informally, the rational type correctly understands the environment, whereas the novice type holds beliefs that are endogenously determined by past observations of equilibrium outcomes of the mechanism he currently participates in. As we will see, this way of forming beliefs can lead to misspecifications, and accordingly we also refer to the novice type as the misspecified type.

We now make this precise. Fix a mechanism $M = [(B_i), q, p]$. A strategy of bidder i is a function $b_i : T \to B_i$, where as a shorthand we write $b_i(\theta_i, s_i) = b_i^{s_i}(\theta_i)$—that is, $b_i^r(\cdot)$ is the strategy of the rational type, and $b_i^m(\cdot)$ is the strategy of the misspecified type of bidder i.[5] A strategy profile is denoted by $b = (b_1, b_2) = (b_1^r, b_1^m, b_2^r, b_2^m)$ and we denote the space of all strategy profiles by $\mathcal{B}$.

For a rational type of bidder i, the expected utility of type θ_i when submitting bid $b_i \in B_i$, and assuming that bidder j bids according to $b_j(\cdot)$, is given by

$$U_i^r(b_i, \theta_i | b_j(\cdot)) = \mathbb{E}_f \left[v_i \, q_i(b_i, b_j(\theta_j, s_j)) - p_i(b_i, b_j(\theta_j, s_j)) | \theta_i \right],$$

where $\mathbb{E}_f$ is the expectation with respect to the correct distribution f and the probability λ.

Next consider the novice type. When observing $\theta_i = (\theta_i^1, \dots, \theta_i^K)$, we assume this type correctly understands that without extra conditioning it means there is a probability θ_i^k that $v_i = v^k$. This understanding is assumed to be derived by access to data of the form (θ_i, v_i) that are unrelated to the auction environment, for example capturing situations in which the object is already owned by the agent (or the company for which the agent works). More precisely, such an agent is assumed to have access to infinitely many past instances of such cases in which the same signal θ_i was observed, and the novice bidder can then find out that the frequency with which $v_i = v^k$ in the corresponding pool coincides with θ_i^k, as implied by our definition of θ_i.

In order to choose how to bid in mechanism M, the novice type must also form a belief about the behavior of the opponent and how it relates to variables of interest. We assume

[5] We only consider pure strategies in our setting with continuous interim types.

that the novice type forms such a belief using past observations from the same mechanism played by similar bidders. We make the assumption that only bids and ex-post valuations are observable.

Assumption 1. *For each mechanism we consider, we assume that bidders have access to past observations of the form* (b_1, v_1, b_2, v_2) *from the same mechanism. The data about past mechanisms do not include the types* (θ_1, θ_2) *of past bidders.*

The idea behind this assumption is that bids are often disclosed after the auction, and as time goes by, outside observers get relatively precise signals about the ex-post valuations of the bidders.[6] On the other hand, (new) bidders typically do not have access to the beliefs (or signals) that past bidders in their respective role had at the time of bidding. In Section 5, we discuss situations in which, after the completion of the auction, bids and winners (but not losers)' ex post valuations are accessible, and we discuss the robustness of our analysis in this case.

Past data allow bidders to identify the joint distribution of observable variables. We abstract from issues of estimation, and assume that bidders can recover this distribution without estimation error. The novice bidder then forms a simple model that combines relevant information from the empirical distribution of (b_1, v_1, b_2, v_2), and his belief that his own v_i is distributed according to θ_i. To illustrate, consider an auction with possible bids $B_1 = B_2 = [0, \infty)$. To assess the payoff from different bids, a bidder need to know the joint distribution of her own valuation v_i and the opponent's bid b_j, conditional on his own type θ_i. The novice bidder combines the distribution of v_i given by his type θ_i with the joint distribution of v_i and the opponent's bid b_j learned from the data in a parsimonious way, taking the joint distribution to be

$$\mathbb{P}_m \left[v_i = v^k, b_j \leq b \middle| \theta_i \right] = \theta_i^k \times H_i(b|v^k) \tag{1}$$

where $H_i(b|v^k)$ is the c.d.f. of b_j conditional on $v_i = v^k$ that is obtained from the auction data. In other words, from the past available auction data, the novice bidder i is able to relate the distribution of opponent's bid to his own ex post value, and he views θ_i and b_j as providing independent sources of information about v_i.

[6] In the context of procurement auctions, this would amount to assuming that ex post one has a pretty good idea of the conditions of the contract, thereby making the assessment of the ex post valuations relatively easy.

Assuming that bidder $j \neq i$ bids according to $b_j(\cdot)$, the perceived expected utility of a novice bidder i with type θ_i when submitting bid $b_i \in B_i$ is given by:[7]

$$
\begin{aligned}
U_i^m(b_i, \theta_i | b_j(\cdot)) &= \mathbb{E}_m \left[v_i \, q_i(b_i, b_j(\theta_j, s_j)) - p_i(b_i, b_j(\theta_j, s_j)) | \theta_i \right], \\
&= \sum_{k=1}^{K} \theta_i^k \int_{B_j} [v^k \, q_i(b_i, b_j) - p_i(b_i, b_j)] dH_i(b_j | v^k),
\end{aligned}
$$

where $\mathbb{E}_m$ is the expectation formed according to the model described above. The analysis of the second-price auction in the next section will be helpful to illustrate more concretely how this works.

Equilibrium. To close the model, we assume that $H_i(\cdot | v^k)$ are equilibrium objects that are generated by the equilibrium strategy profile, and the misspecified type best-responds given her beliefs that are captured by $H_i(\cdot | v^k)$. In other words, we focus on steady state in which the data generated by new bidders follow the same distribution as those generated by previous bidders. Formally,

Definition 1. The strategy profile $b(\cdot)$ is a *"Data-Driven Equilibrium"* of the mechanism $M = [(B_i), q, t]$ if for all $i \neq j$, and for all $\theta_i \in \Theta$,

(a) $b_i^r(\theta_i) \in \arg\max_{b_i \in B_i} U_i^r(b_i, \theta_i | b_j(\cdot))$,

(b) $b_i^m(\theta_i) \in \arg\max_{b_i \in B_i} U_i^m(b_i, \theta_i | b_j(\cdot))$, where the distribution $H_i(b_j | v^k)$ used to compute U_i^m is derived from $b(\cdot)$, f, and $\text{Prob}[s_i = r] = \lambda$.

On the decision rules of novice and rational bidders.

In our proposed approach, the novice bidder i views his signal θ_i and the opponent's bid b_j as providing independent sources of information about v_i. More formally, bidder i upon observing θ_i is concerned with the joint distribution of (v_i, b_j) conditional on θ_i so as to determine his best-response in the auction. This joint distribution can be factorized as $P(v_i, b_j \mid \theta_i) = P(v_i \mid \theta_i) P(b_j \mid v_i, \theta_i)$ where $P(v_i \mid \theta_i)$ is assumed to be known to bidder i (using the above notation θ_i^k indicates the probability that $v_i = v^k$ given θ_i). In the correct representation, v_i and θ_j are independently distributed conditional on θ_i (since we are in a private value environment). Since b_j only depends on θ_j, it follows

[7] Clearly, we do not have in mind that $b_j(\cdot)$ is known by i. As explained above, the decision rule of the novice bidder i is based on H_i, which she recovers from the data that are accessible from past mechanisms.

that v_i and b_j are independently distributed conditional on θ_i. Thus, the correct joint distribution satisfies $P(b_j \mid v_i, \theta_i) = P(b_j \mid \theta_i)$. By contrast, the novice bidder i assumes that $P(b_j \mid v_i, \theta_i) = P(b_j \mid v_i)$ as if θ_i and b_j (or θ_j) were independently distributed conditional on v_i.[8]

The incorrect representation of novice bidders is motivated as follows. First, since novice bidders do not see the type θ_i of past bidders i in the auction data, they cannot empirically assess $P(b_j \mid v_i, \theta_i)$. Second, seeing (b_1, v_1, b_2, v_2) in the auction data makes it easy to non-parametrically estimate $P(b_j \mid v_i)$, and this is the probability considered by novice bidders i in a data-driven equilibrium.

One way to think of the incorrect representation of novice bidders is in terms of a misspecified model in which bidder i would wrongly believe that he lives in a world with interdependent values in which bidders i and j's information would be independent conditional on v_i.[9] To state this more concretely, remember that bidder i 's information θ_i is a noisy signal about v_i, and let the novice bidder i (mistakenly) believe that bidder j observes a noisy signal θ_j about $h(v_i, \eta_j)$ where η_j is an auxiliary variable whose distribution is viewed by the novice bidder i as being independent of v_i and $h(v_i, \eta_j)$ can be thought of representing j's valuation (neither the function h nor the distribution of η_j need to be known by the novice bidder i). With our Assumption that the novice bidder i only sees (b_1, v_1, b_2, v_2) from past auctions, in a Berk-Nash equilibrium of the corresponding setting (see Esponda and Pouzo (2016) for a formal definition), the novice bidder would behave as in our data-driven equilibrium (given that he would think that θ_i and $b(\theta_j)$ are drawn from independent distributions conditional on v_i).

Another way to think of the incorrect representation is to view the novice bidders as using data similarly as non-structural statisticians would do, thereby not attempting to construct a structural model involving unobserved variables but instead directly working

[8] Using the language of Bayesian network (Spiegler, 2016), one can think of the incorrect representation of the novice bidder i through the lens of a Direct Acyclic Graph in which the novice bidder i would wrongly believe that v_i is a cause of θ_i and b_j with no direct causal links between θ_i and b_j (whereas the true DAG would be one where there is a common cause to v_i and v_j but v_i (resp v_j) is the only cause of θ_i (resp. θ_j). This can also be formulated using the language of the analogy-based expectation equilibrium, in particular considering the payoff-relevant analogy partition (see Jehiel and Koessler (2008) and also Jehiel (2022) for a discussion of the link between the Bayesian Network approach and the analogy-based expectation equilibrium).

[9] It should be mentioned that the empirical literature on auctions often struggles to decide if a certain environment falls better into the Private Value or Common value camp (see, for example the discussion in Perrigne and Vuong (2022)). It is then plausible that novice bidders could be misspecified along the lines of thinking they are in a common value environment when the true environment has private values.

with the correlation of observed variables.

Observe that in our environment, the novice bidder i is only assumed to know how the signal θ_i he currently observes maps into the odds that the ex post value is $v_i = v^k$ (an understanding he derives from data that are unrelated to the auction, see above). It is then hard/impossible for the novice bidder i to make use of b_i or v_j in the auction data (b_1, v_1, b_2, v_2) he observes, since he has no idea how information may have been distributed among agents assigned to the role of bidder i in these past auctions (the novice bidder i need not know how θ_i is distributed in the auction). Moreover, in his use of the data (b_j, v_i) and signal θ_i, the novice bidder makes the additional assumption that θ_i and b_j are independently distributed conditional on v_i, which can, from the non-structural econometrics perspective, be motivated on the ground that it is the simplest statistical model consistent with the observations (also inducing maximum entropy, as sometimes considered in statistical physics, see Jayne, 1957).[10]

Regarding rational types, one way to think of them is that they have a full understanding of the auction environment including the distribution of types and of the decision rules. However, our preferred way to think of rational types is to view them as experienced bidders who would have learned the best bidding strategy given the environment, and thus behave as if they had such a complete understanding (with no need to assume that each aspect of the environment is understood or known in isolation).

3 Standard Auctions

Before considering general auction-like mechanisms and presenting the main result of the paper, we apply the model to standard auctions. This helps us illustrate the working of data-driven equilibria.

To start with a simple case, we assume here that $|V| = 2$, so that the type of each bidder is one-dimensional. More specifically, we assume that $V = \{0, 1\}$, so that the type can be written as one number $\theta_i \in [0, 1]$, that specifies the probability that bidder i's ex-post valuation is $v_i = 1$. Note that this implies that θ_i is also the interim expected

[10] From a different but related perspective, such an independence assumption can also possibly be motivated on the ground that the novice bidder would see no reason to impose some extra correlation as such a correlation would not be motivated by any empirical measure and it would typically be considered ad hoc (this preference for independence unless proven wrong by the data can be viewed as a formalization of the principle of insufficient reason or Occam's razor, see also Jehiel (2022) for further discussion of a similar assumption in the context of the analogy-based expectation equilibrium).

value of bidder i. In the following, we explain the equilibrium logic of our model for two standard auctions formats, the Second-Price Auction and the First-Price Auction. To compute concrete bidding equilibria, we will consider two specific families of distributions that allow us to vary the correlation between θ_1 and θ_2.

Example 1. θ_1 and θ_2 are uniformly distributed on $[0, 1]$. They are perfectly correlated (i.e., $\theta_1 = \theta_2$) with probability α, or else they are independently distributed with probability $1 - \alpha$ where $\alpha \in [0, 1]$.

Example 2. The joint density is given by

$$f(\theta_1, \theta_2) = \frac{2+\alpha}{2}\left(1 - |\theta_1 - \theta_2|\right)^{\alpha}$$

where $\alpha \in [0, \infty)$.

In each of these specifications, the parameter α determines the correlation between the two types where $\alpha = 0$ corresponds to the independent case and the supremum of α ($\alpha = 1$ in Example 1 and $\alpha = \infty$ in Example 2) corresponds to perfect correlation. Example 1 involves distributions that are admittedly less smooth than in Example 2, but it will allow us to derive some analytic solutions in some limiting cases whereas for Example 2 only simulations will be provided.

3.1 Second-price Auction

In a second-price auction, the rational type has a weakly dominant strategy since values are private. Hence she bids her interim expected value. We have

$$b^r(\theta_i) = \theta_i,$$

where b^r refers to the rational type's strategy. We denote the inverse by $\theta^r(b_i)$, which is of course equal to b_i in this case.

Now consider the misspecified type and consider a symmetric equilibrium, that is $b_i^m(\cdot) = b_j^m(\cdot) = b^m(\cdot)$. Suppose the equilibrium strategy $b^m(\cdot)$ is strictly increasing with inverse $\theta^m(b_i)$. In equilibrium, the distribution of b_j conditional on $v_i = 1$ is

$$H^{\mathrm{SPA}}(b \mid v_i = 1) = \frac{\mathbb{P}_f\left[b_j \leq b, v_i = 1\right]}{\mathbb{P}_f\left[v_i = 1\right]}, \tag{2}$$

where $H^{\mathrm{SPA}}(\cdot)$ refers to this distribution for the SPA. Note that the misspecified type learns the correct joint distribution of v_i and b_j from the data. Hence we have used the correct probabilities $\mathbb{P}_f$ on the right-hand side. In the denominator, we have the unconditional probability of $v_i = 1$ which is given by the (ex-ante) expectation of the random variable $\tilde{\theta}_i$. In the numerator, the probability $\mathbb{P}_f[b_j \leq b, v_i = 1]$ is obtained by averaging $\mathbb{P}_f[b_j \leq b, v_i = 1|\tilde{\theta}_i]$ over the (ex-ante) random variable $\tilde{\theta}_i$. Since b_j is a function of θ_j and s_j, and the *generalized type* (θ_j, s_j) and v_i are independent *conditional on* $\tilde{\theta}_i$, we have:

$$
\begin{aligned}
H^{\mathrm{SPA}}(b \mid v_i = 1) &= \frac{\mathbb{E}_{\tilde{\theta}_i}\left[\mathbb{P}_f[b_j \leq b|\tilde{\theta}_i] \times \mathbb{P}_f[v_i = 1|\tilde{\theta}_i]\right]}{\mathbb{E}[\tilde{\theta}_i]} \\
&= \frac{\mathbb{E}_{\tilde{\theta}_i}\left[\left(\lambda \mathbb{P}_f[b^r(\theta_j) \leq b|\tilde{\theta}_i] + (1-\lambda)\mathbb{P}_f[b^m(\theta_j) \leq b|\tilde{\theta}_i]\right) \times \mathbb{P}_f[v_i = 1|\tilde{\theta}_i]\right]}{\mathbb{E}[\tilde{\theta}_i]} \\
&= \frac{1}{\mathbb{E}[\tilde{\theta}_i]} \int_0^1 \left[\lambda F(b \mid \tilde{\theta}_i) + (1-\lambda) F(\theta^m(b) \mid \tilde{\theta}_i)\right] \tilde{\theta}_i \, f(\tilde{\theta}_i) d\tilde{\theta}_i.
\end{aligned}
$$

In the second line we decomposed the probability $\mathbb{P}_f[b_j \leq b|\tilde{\theta}_i]$ into the probability that a rational and a misspecified type bid below b, conditional on $\tilde{\theta}_i$. If the opponent is rational, the probability of $b_j \leq b$ is given by $\mathbb{P}_f[b^r(\theta_j) \leq b|\tilde{\theta}_i] = F(\theta^r(b) \mid \tilde{\theta}_i) = F(b \mid \tilde{\theta}_i)$, and if the opponent is misspecified it is given by $\mathbb{P}_f[b^m(\theta_j) \leq b|\tilde{\theta}_i] = F(\theta^m(b) \mid \tilde{\theta}_i)$. The term $\tilde{\theta}_i$ in the third line is just $\mathbb{P}_f[v_i = 1|\tilde{\theta}_i]$. We obtain a similar expression for the distribution of b_j conditional on $v_i = 0$:

$$
H^{\mathrm{SPA}}(b \mid v_i = 0) = \frac{1}{\mathbb{E}[1-\tilde{\theta}_i]} \int_0^1 \left[\lambda F(b \mid \tilde{\theta}_i) + (1-\lambda) F(\theta^m(b) \mid \tilde{\theta}_i)\right] (1-\tilde{\theta}_i) \, f(\tilde{\theta}_i) d\tilde{\theta}_i
$$

where the expectation in the integral differs from that in $H^{\mathrm{SPA}}(b \mid v_i = 1)$ since $P_f[v_i = 0|\tilde{\theta}_i] = (1-\tilde{\theta}_i)$, and outside the integral $\mathbb{E}[1-\tilde{\theta}_i]$ is the unconditional probability $P_f[v_i = 0]$.[11]

In a symmetric equilibrium of the second-price auction, the misspecified type's bid for θ_i solves

$$
\max_b \left\{ \theta_i H^{\mathrm{SPA}}(b \mid v_i = 1) - \theta_i \int_0^b x dH^{\mathrm{SPA}}(x|v_i = 1) - (1-\theta_i) \int_0^b x dH^{\mathrm{SPA}}(x|v_i = 0) \right\}
$$

[11]Note that $H^{\mathrm{SPA}}(b \mid v_i = 0) = \mathbb{P}_f\left[b_j \leq b, v_i = 0\right] / \mathbb{P}_f\left[v_i = 0\right]$

To obtain an equilibrium we have to determine a bidding strategy b^m and the implied H^{SPA} such that b^m is optimal for the misspecified type give belief H^{SPA}. Taking the first-order condition for b we obtain that at $b = b^m(\theta_i)$:

$$\theta_i \frac{\partial H^{\text{SPA}}}{\partial b}(b \mid v_i = 1) - \theta_i b \frac{\partial H^{\text{SPA}}}{\partial b}(b \mid v_i = 1) - (1 - \theta_i) \frac{\partial H^{\text{SPA}}}{\partial b}(b \mid v_i = 0) = 0$$

or

$$b^m(\theta_i) = \frac{\theta_i h^{\text{SPA}}(b^m(\theta_i) \mid v_i = 1)}{\theta_i h^{\text{SPA}}(b^m(\theta_i) \mid v_i = 1) + (1 - \theta_i) h^{\text{SPA}}(b^m(\theta_i) \mid v_i = 0)}$$

where $h^{\text{SPA}}(\cdot \mid v_i)$ denotes the pdf associated with $H^{\text{SPA}}(\cdot \mid v_i)$.

When $H^{\text{SPA}}(\cdot \mid v_i = 1) = H^{\text{SPA}}(\cdot \mid v_i = 0)$ (which arises in the independent case), the above expression simplifies into the familiar expression $b^m(\theta_i) = \theta_i$, which corresponds to the bid of the rational bidder. When $H^{\text{SPA}}(\cdot \mid v_i = 1)$ and $H^{\text{SPA}}(\cdot \mid v_i = 0)$ are different (which arises when there is correlation), $b^m(\theta_i)$ is typically different from θ_i. The novice bidder i uses the bid of j to refine his perceived assessment of the odds of v_i. When $h^{\text{SPA}}(b^m(\theta_i) \mid v_i = 1) > h^{\text{SPA}}(b^m(\theta_i) \mid v_i = 0)$, he bids more than θ_i because bidder i perceives that when j bids $b_j = b^m(\theta_i)$ it conveys an extra information that $v_i = 1$. And he bids less than θ_i when $h^{\text{SPA}}(b^m(\theta_i) \mid v_i = 1) < h^{\text{SPA}}(b^m(\theta_i) \mid v_i = 0)$. This is a reasoning similar to the one we are familiar with in auction settings with interdependent valuations. It arises here in a private value setting due to the erroneous representation of the novice bidder i.

To illustrate how it works, consider first Example 1, assuming either that $\lambda = 1$ (all bidders are rational) or $\lambda = 0$ (all bidders are novice).

In Example 1 with $\lambda = 1$, everyone bids according to $b^r(\theta_i) = \theta_i$ and thus

$$\begin{aligned} H^{\text{SPA}}(b \mid v_i = 1) &= \frac{(1-\alpha)\int_0^1 b\theta d\theta + \alpha \int_0^b \theta d\theta}{\int_0^1 \theta d\theta} \\ &= (1-\alpha)b + \alpha b^2 \end{aligned}$$

Similarly,

$$\begin{aligned} H^{\text{SPA}}(b \mid v_i = 0) &= \frac{(1-\alpha)\int_0^1 b(1-\theta) d\theta + \alpha \int_0^b (1-\theta) d\theta}{\int_0^1 (1-\theta) d\theta} \\ &= (1-\alpha)b + \alpha b(2-b) \end{aligned}$$

This in turn implies that $b^m(\theta)$ should be a solution to

$$b = \frac{\theta(1-\alpha+2\alpha b)}{\theta(1-\alpha+2\alpha b)+(1-\theta)(1-\alpha+2\alpha(1-b))}$$

yielding

$$b^m(\theta) = \frac{-(1-\alpha)-2\alpha(1-2\theta)+\left[(1-\alpha+2\alpha(1-2\theta))^2+8\theta(1-\alpha)\alpha(2\theta-1)\right]^{1/2}}{4\alpha(2\theta-1)}.$$

It is readily verified that $b^m(0)=0$, $b^m(1)=1$, $b^m(1/2)=1/2$ and $b^m(\theta)<\theta$ (resp. $b^m(\theta)>\theta$) whenever $\theta \in (0,\frac{1}{2})$ (resp. $\theta \in (\frac{1}{2},1)$). Moreover, as the correlation parameter increase to $\alpha=1$, $b^m(\theta)$ approaches the step function in which $b^m(\theta)=0$ for $\theta \in (0,\frac{1}{2})$ and $b^m(\theta)=1$ for $\theta \in (\frac{1}{2},1)$.

Consider now the same example with $\lambda=0$. We have that $b^m(\theta)$ should satisfy

$$b^m(\theta) = \frac{\theta\left(\alpha\theta+(1-\alpha)/2\right)}{\theta\left(\alpha\theta+(1-\alpha)/2\right)+(1-\theta)\left(\alpha(1-\theta)+(1-\alpha)/2\right)}$$

since bidder j bids $b^m(\theta)$ when $\theta_j=\theta$, which implies that bidder i has $\theta_i=\theta$ with probability α and otherwise θ_i has mean $1/2$, which translates in a probability $\alpha\theta+(1-\alpha)/2$ (resp. $(1-\theta)\left(\alpha(1-\theta)+(1-\alpha)/2\right)$) that $v_i=1$ (resp. $v_i=0$).

It is readily verified in this case too that $b^m(0)=0$, $b^m(1)=1$, $b^m(1/2)=1/2$ and $b^m(\theta)<\theta$ (resp. $b^m(\theta)>\theta$) whenever $\theta \in (0,\frac{1}{2})$ (resp. $\theta \in (\frac{1}{2},1)$). However, the difference between $b^m(\theta)$ and $b^r(\theta)$ can be seen to be smaller than when $\lambda=0$. To see an illustration of this, consider the case in which the correlation parameter increases to $\alpha=1$. While we had a step function moving from 0 to 1 around $\theta=1/2$ when $\lambda=1$, we have that $b^m(\theta)$ now corresponds to $\frac{\theta^2}{\theta^2+(1-\theta)^2}$ when $\lambda=0$.

Moving away from Example 1, we obtain in general a differential equation for b^m that depends both on λ and the distribution $f(\theta_1,\theta_2)$. Solving the differential equation numerically for the joint distribution from Example 2, we get the bid-functions illustrated in Figure 3.1.

We obtain insights in line with those obtained in Example 1. We see that increasing the correlation leads to stronger deviations from the rational bid. Moreover, the sensitivity of b^m with respect to λ becomes stronger if the correlation is stronger. Generally, for fixed correlation, increasing the share of misspecified types $(1-\lambda)$ leads to smaller deviations

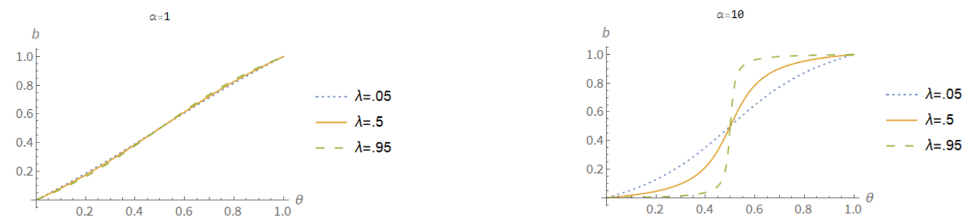

Figure 1: SPA bid-function $b^m(\theta_i)$, $\alpha \in \{1, 10\}$ (left to right)

from rationality. Bidding against mainly rational types, a misspecified type's behavior exhibits strong deviations from rationality,[12] and we observe that the presence of other misspecified types has a dampening effect. It may also be mentioned that the rational bidder is always better off conditional on being matched with a novice bidder than with a rational bidder. This can perhaps best be seen when we approach perfect correlation in which case the payoff is 0 when facing a rational bidder but is positive when facing a novice bidder and $\theta < 1/2$. But, the insight that rational bidders are better off when matched with novice bidders applies more generally, noting that the bid distribution of the novice bidder is a mean-preserving spread of the bid distribution of the rational bidder.

Intuition. The reasoning leading to the derivation of b^m follows a logic similar to that in classic analysis of winner's curse models (see Milgrom and Weber, 1982). We observe from Figure 3.1 that the misspecified type overbids for $\theta_i > 1/2$ and underbids for $\theta_i < 1/2$. What explains this behavior? To understand this, it is useful to shut down the (dampening) equilibrium effect of misspecified types and assume that $\lambda \approx 1$. The crucial observation is that the m-type believes that conditional on $v_i = 1$, the opponent's bid distribution is strong. This is because in the data, v_i and b_j are positively correlated: Observations with $v_i = 1$ are more likely to be generated when $\tilde{\theta}_i$ is high. Due to the positive correlation between θ_i and θ_j, this implies that b_j is also likely to be high. Conversely, the m-type believes that conditional on $v_i = 0$, the opponent's bid distribution is weak.

For an m-type with low θ_i, consider the incentives to decrease the bid below $b = \theta_i$. In this range reducing the bid has a large effect on the winning probability conditional on $v_i = 0$ (the m-type believes that conditional on $v_i = 0$, the opponents bid's are concentrated on a low range) and little effect on the winning probability conditional on $v_i = 1$ (where

[12] Numerical computations indicate that even if $\lambda \to 1$, the slope of b^m remains bounded, where the bound depends on α. In other words, b^m does *not* converge to a step function according to the numerical results.

the m-type believes the opponents bid's are concentrated on a high range). Therefore, the m-type believes that by shading the bid, he can cut the losses from winning with $v_i = 0$, without a strong reduction of the gains from winning when $v_i = 1$.

For a high θ_i, this logic is reversed. Consider the incentives to increase the bid above $b = \theta_i$ when θ_i is high. The bid is now in a range where the m-type believes that increasing the bid mainly affects the winning probability conditional on $v_i = 1$ and has less effect on the winning probability conditional on $v_i = 0$. Hence, he thinks overbidding increases the profits from winning with $v_i = 1$, while only modestly increasing the losses from winning with $v_i = 0$. This leads to bids above θ_i for high types of the misspecified bidder.[13]

Inefficiency of the Second-Price Auction. While the distortions observed in the example are specific to the parametric classes of distributions, we can show generally that the SPA is not efficient whenever both rational and misspecified types arise with positive probability, and the types of the two bidders are correlated.[14,15]

Proposition 1. *Consider any joint distribution* $f(\theta_1, \theta_2)$ *such that* $Corr[\theta_1, \theta_2] \neq 0$*. If* $\lambda \in (0, 1)$ *then any equilibrium of the second-price auction in which the rational types of both bidder play their dominant strategies is inefficient.*

Proof. All omitted proofs can be found in Appendix A. □

Revenue and Efficiency. Continuing our illustration for the parametric class in Example 2, we show how revenue and (relative) efficiency of the allocation varies with (a) the share of rational types λ and (b) the correlation between θ_1 and θ_2—that is, the parameter α.

[13] The dampening effect of lower values of λ can be understood as follows. Take a value of θ_i larger (resp. smaller) than 0.5. Rational bidders bid less (resp. more) than misspecified bidders. Thus, bidder j ties with the equilibrium bid of a misspecified agent, for a larger (resp smaller) value of θ_j when bidder j is rational than when he is misspecified. Given the correlation between θ_i and θ_j, this in turn gives rise to a bigger winner's curse-like correction when λ is bigger, thereby explaining the dampening effect of decreasing the share of rational types.

[14] Correlation is a sufficient condition for an inefficiency. The careful reader will see from the proof that weaker forms of dependency also lead to inefficiencies. In Section 4 we generalize this proposition to any finite number of valuations (see Lemma 6).

[15] As suggested by Bob Wilson, inefficiencies require the presence of both rational and misspecified types due to our assumed symmetry on the distribution of types. In the absence of symmetry, one would expect inefficiencies to arise in SPA, even if there are no rational types, as long as signals are correlated.

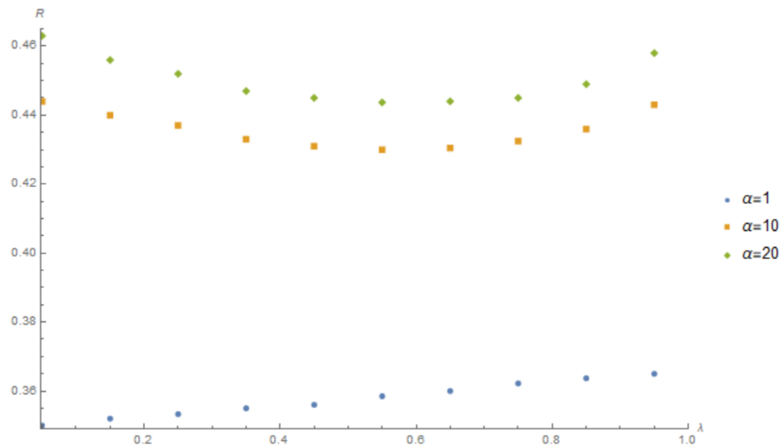

Figure 2: Revenue from the SPA as a function of λ: for $\alpha \in \{1, 10, 20\}$

Figure 2 plots the revenue as a function of λ for different values of α. Note that the comparison between different values of α with λ held fixed is not very informative since the joint distribution changes in a complicated way as α changes.

We see that for the case of weak correlation ($\alpha = 1$), revenue is increasing in the share of rational bidders. This suggests that the distortions in the misspecified type's bidding function adversely affect revenue. For highly correlated interim types, the pattern changes and revenue is U-shaped in the share of rational types. The initial decline is intuitive since the distortions in the m-types bid become larger if the share of rational types increases. Profits rise again if the share of rational types becomes so large that the presence of m-types becomes unlikely.

Figure 3 shows how efficiency changes depending on λ and α in Example 2. To make this comparable across different parameter sets, we normalize efficiency by the expected ex-post value achieved if the object is always allocated to the bidder with the highest interim type. Clearly when $\lambda = 0$ or 1, there is no inefficiency given that bidders of the same sophistication bid in the same way. Moreover, both when $\alpha = 0$ (the independent case) or $\alpha = \infty$ (perfect correlation) there is no inefficiency either. In the parametric example, we observe that the relative efficiency is U-shaped as a function of λ and α, as shown in Figure 4.

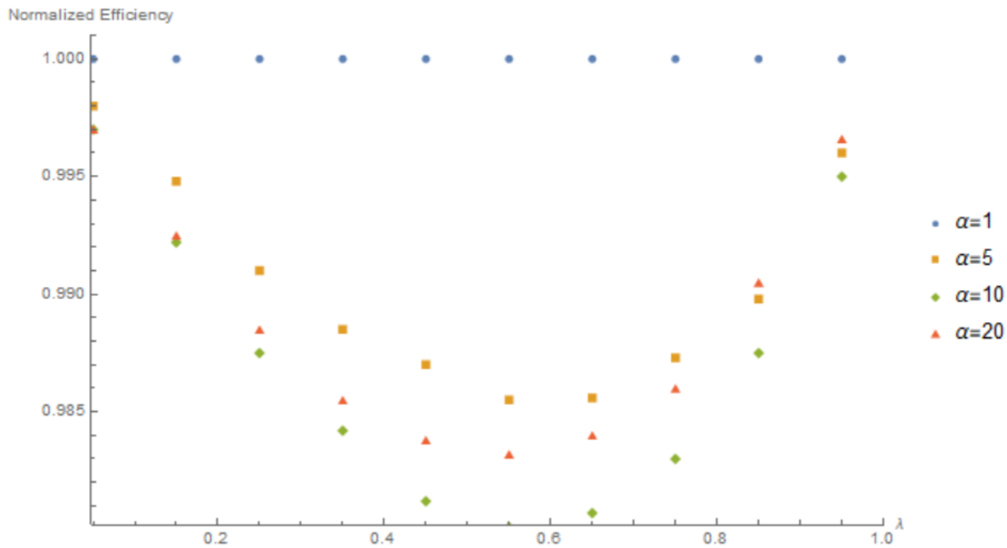

Figure 3: Efficiency of SPA as a function of λ: for $\alpha \in \{1, 5, 10, 20\}$

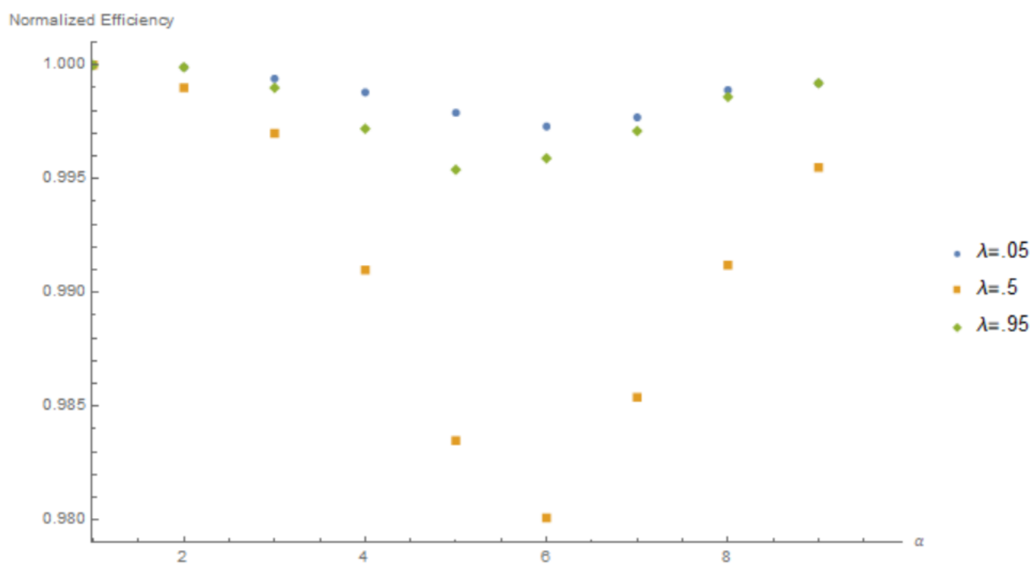

Figure 4: Efficiency of SPA as a function of $\alpha = 1/5 + 5^k$ where $k = 1, \ldots, 9$ is on the horizontal axis. $\lambda \in \{.05, .5, .95\}$.

3.2 First-price auction

In a first-price auction, we obtain the misspecified type's belief in a similar way as for the second-price auction:

$$H^{\text{FPA}}(b \mid v_i = 1) = \int_0^1 \left[\lambda F(\theta^r(b) \mid \tilde{\theta}_i) + (1-\lambda) F(\theta^m(b) \mid \tilde{\theta}_i)\right] \tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i,$$
$$H^{\text{FPA}}(b \mid v_i = 0) = \int_0^1 \left[\lambda F(\theta^r(b) \mid \tilde{\theta}_i) + (1-\lambda) F(\theta^m(b) \mid \tilde{\theta}_i)\right] (1-\tilde{\theta}_i) \frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i.$$

$b^r(\cdot)$ and $b^m(\cdot)$ now denote the bidding strategies of the rational and misspecified types in the symmetric equilibrium of the FPA, and the respective inverse functions are denoted by $\theta^r(\cdot)$ and $\theta^m(\cdot)$. The misspecified bidder's bid for type θ_i maximizes

$$\max_b \, (1-b)\, \theta_i H^{\text{FPA}}(b \mid v_i = 1) - b(1-\theta_i) H^{\text{FPA}}(b | v_i = 0). \tag{3}$$

Again we obtain a differential equation for $b^m(\theta_i)$. In contrast to the second price auction, however, we cannot assume that rational bidders bid their expected valuations. Instead they maximize

$$\max_b \, (\theta_i - b)\left(\lambda F(\theta^r(b)|\theta_i) + (1-\lambda) F(\theta^m(b)|\theta_i)\right)$$

In this optimization, rational bidders behave as if using the correct distribution f, the correct share of rational types in the population, and the equilibrium bidding strategies of both the rational and the misspecified types when determining their optimal bids. The first-order condition for the rational type's problem yields a second differential equation. To compute an equilibrium, we need to solve the system of two ODEs with the boundary condition $(b^m(0), b^r(0)) = (0, 0)$. This proves challenging even for the distributions in our parametric example, since the system has a singular point at the boundary condition. However, we obtain a similar inefficiency result as we had for the SPA.

Proposition 2. *Consider any joint distribution $f(\theta_1, \theta_2)$ such that $Corr[\theta_1, \theta_2] \neq 0$. If $\lambda \in (0, 1)$ then the symmetric equilibrium of the first-price auction is inefficient.*

3.3 Comparison

We can compute the bidding equilibrium for both auction formats for the case of only rational bidders ($\lambda = 1$) and only misspecified bidders $\lambda = 0$. Figure 5 shows in Example

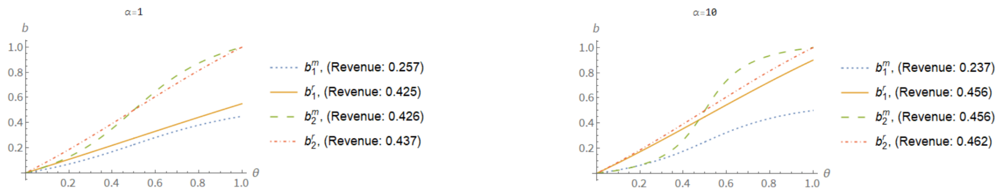

Figure 5: SPA bid-function $b^m(\theta_i)$, $\alpha \in \{1, 10\}$ (left to right)

2 the bid functions b_k^s where $k = 1, 2$ denotes first- or second-price auctions and $s = m, r$ denotes the misspecified or rational type.

To illustrate the role of correlation, the functions are shown for $\alpha \in \{1, 10\}$. Comparing FPA and SPA in the rational case, we see the familiar revenue ranking that the SPA yields higher revenue than the FPA with correlated types. This revenue ranking is preserved in the case of misspecified bidders. Interestingly, with misspecified bidders, the gap between SPA and FPA becomes more pronounced if values are more correlated. This conforms well with the intuition for the distortions in the bid function: In the SPA low types underbid and high types overbid. In the FPA, the same forces lead the low types to underbid. But this allows the higher types to shade their bids more and the incentive to overbid does not compensate for this force. This leads to much lower bids for misspecified types compared to the rational equilibrium if the correlation is high.

Finally, we want to compare the efficiency of the SPA and FPA. This comparison is not interesting in the purely rational or purely misspecified cases since the symmetric equilibrium implies that both auction formats are fully efficient. A comparison in the mixed case is challenging because we are not able to compute the equilibrium in the FPA. To suggest some possible results, we can consider the best response of a misspecified type to the purely rational equilibrium. This allows us to show how efficiency changes if we inject a small share of misspecified types in a rational population. To illustrate such an approach, we have numerically computed how much efficiency is lost in expectation if bidder 1 uses the purely rational strategy and bidder 2 uses the misspecified response assuming $\alpha = 1.5$. This number gives the rate at which efficiency decreases if we decrease λ slightly from $\lambda = 1$. We find a marginal loss of .0035 for the FPA and .0088 for the SPA. This means that the SPA is less efficient than the FPA in this limiting case.

4 Auction-like Mechanisms

In this part, we consider a general class of auction-like mechanisms, in which bidders can place a one-dimensional bid $b \in B \subset \mathbb{R}$, and which allocates the object to the bidder with highest bid (possibly adjusted by a bonus or malus). We assume that bidders may choose not to participate in the auction in which case their utility is zero.[16]

Definition 2. An *auction-like mechanism* is given by $M = \left[B, (W_i)_{i=1,2}, (L_i)_{i=1,2}, \phi_1\right]$. $B = [\underline{b}, \bar{b}]$ is the set of *feasible bids*. The *allocation rule* $\phi_1 : B \to B$ is a strictly increasing function. The object is allocated to bidder 1 if $b_1 > \phi_1(b_2)$, to bidder two if $b_1 < \phi_1(b_2)$, and with probability 1/2 if $b_1 = \phi_1(b_2)$. We denote the inverse by $\phi_2 = \phi_1^{-1}$. The *payment rules* are $W_i : B \times B \to \mathbb{R}_0^+$, and $L_i : B \times B \to \mathbb{R}_0^+$, which specify the payment bidder i has to make as a function of the bids, if she wins or loses, respectively. We assume that for each $i \in \{1, 2\}$, both functions W_i, L_i are weakly increasing in bidder i's own bid.
An auction-like mechanism is *smooth* if for $i \in \{1, 2\}$, ϕ_i, W_i, and L_i, are continuously differentiable with derivatives that can be continuously extended to the boundary of B.

The smoothness assumption is made for tractability. Almost all common auction formats are smooth auction-like mechanisms. Our main result is that if there are at least three possible ex-post valuations, then for generic type distributions, no smooth auction-like mechanism exists that has an efficient equilibrium.

To make this precise, we reformulate the types of agents. We denote the *interim valuation* of bidder i with type θ_i by

$$w_i(\theta_i) := \mathbb{E}\left[v_i|\theta_i\right].$$

Given the normalization $0 = v^1 < \ldots < v^K = 1$, we have $w_i(\theta_i) \in [0, 1]$. For each $w_i \in [0, 1]$, we denote the set of types θ_i that have interim valuation w_i by

$$\Theta_i(w_i) := \left\{\theta_i \in \Theta_i | \mathbb{E}\left[v_i|\theta_i\right] = w_i\right\}.$$

For $w_i \in \{0, 1\}$ this set is a singleton; and for all $w_i \in (0, 1)$, there exists a diffeomorphism $x_i(\cdot; w_i) : \Theta_i(w_i) \to [0, 1]^{K-2}$, where $K = |V|$ is the number of ex-post valuations. We can

[16] Our definition of auction-like mechanisms is similar to that in Deb and Pai (2017) who restrict attention to symmetric auctions (in which only the winner makes a payment) to analyze the extent to which such anonymous auctions can allow for discrimination in asymmetric settings.

therefore write the type of bidder i as $(w_i, x_i) \in [0,1]^{K-1}$. While w_i is the payoff-relevant part of the type, for $w_i \in (0,1)$, x_i can be used to recover the belief $f(\theta_j | x_i^{-1}(x_i; w_i))$ about bidder j's type. Abusing notation we use $f(w_1, x_1, w_2, x_2)$ to denote the joint density of the buyers' types and assume that this density is smooth and strictly positive.[17]

Our main result is that for generic distributions, smooth auction-like mechanisms do not have efficient equilibria. To state this formally, we let $\mathcal{M}_+^d([0,1]^{2K-2})$ be the set of probability measures on $[0,1]^{2K-2}$ that admit continuous and strictly positive densities $f(w_1, x_1, w_2, x_2)$. We endow $\mathcal{M}_+^d([0,1]^{2K-2})$ with the uniform topology for densities. For given V and λ, let $\mathcal{I}(V, \lambda) \subset \mathcal{M}_+^d([0,1]^{2K-2})$ be the set of prior distributions for which all equilibria of any smooth auction-like mechanism are inefficient.

Theorem 1. *Suppose $K = |V| \geq 3$ and $\lambda \in (0,1)$. Then for generic type distributions, there exists no smooth auction-like mechanism with an efficient equilibrium. Formally, $\mathcal{I}(V, \lambda)$ is a residual subset of $\mathcal{M}_+^d([0,1]^{2K-2})$, that is, it contains a countable intersection of open and dense subsets of $\mathcal{M}_+^d([0,1]^{2K-2})$.*

The notion of genericity used here is the same as in Gizatulina and Hellwig (2017), who show the genericity of full surplus extraction. The key step in the proof is to show that in the presence of rational bidders, efficiency requires that the mechanism is a second-price auction. The reason is that to achieve efficiency, the bid in an auction-like mechanism must be a function of w_i only. If there are more than two ex-post valuations, for each $w_i \in (0,1)$, the set $\Theta(w_i)$ is a manifold of dimension $K - 2 \geq 1$, and all types in $\Theta(w_i)$ have identical interim expected valuations but different beliefs. We show that for generic distributions, the requirement that the bid is independent of the rational type's belief, implies that the mechanism must be a second-price auction.[18] We then complete the proof by extending the result of Proposition 1 to more than two ex-post valuations (see Lemma 6 below), showing that in a second-price auction the misspecified type does not bid truthfully, which rules

[17] Such a change of variable can be done while preserving the smoothness of $f(w_1, x_1, w_2, x_2)$ because $\theta_i \to \sum_k \theta_i^k v^k$ is a smooth function of θ_i (for example, one can think of x_i as consisting of $(\theta_i^k, k \geq 2)$).

[18] The intuition for this is that in any auction in which the payment of the winner would depend non-trivially on the winner's own bid, the optimal equilibrium bid would require some shading that depends non-trivially on the belief, as in the first-price auction. To ensure that the shading is the same for all beliefs as generated by variations of x_i, a second-price auction must be used.

out an efficient equilibrium.[19,20]

Remark 1 (Precise signal). The inefficiencies identified in Theorem 1 would vanish in second-price auctions if for every bidder i the signal θ_i was always very informative of the ex post value v_i, as in such a case bidders (whether rational or novice) would approximately bid their ex post value. Thus, the noisy character of θ_i is essential for the derivation of inefficiencies (as is the correlation between θ_i and θ_j).

Remark 2 (Two ex-post valuations). With only two ex-post valuations ($K = 2$), our proof does not apply. While the analysis of standard auctions in Section 3 suggests that bid functions of rational and misspecified types in auction-like mechanisms differ, it is an open question whether auction-like mechanism offer enough flexibility in choosing the payment rules so that types of both sophistication can be incentivized to use an identical bid function when $K = 2$.

Remark 3 (More than two bidders). The restriction to two bidders has been made for simplicity. With more than two bidders, we can consider misspecified types who have access to data from past auctions with observations of the form $(b_1, v_1, \ldots, b_N, v_N)$, where N is the number of bidders. Such bidders will now rely on $h(b_{-i}|v_i)$, the density of $b_{-i} = (b_j)_{j \neq i}$ conditional on v_i, to form their beliefs about how variables of interest are distributed. We can define auction-like mechanisms that award the object to the highest bidder and specify payments as a function of all bids. We conjecture that the key argument in our proof—namely that efficiency requires the use of a second-price auction also works with more than two bidders, as long as there are at least three ex-post valuations. Moreover, an analogous result to Proposition 1 and Lemma 6 implies that misspecified types do not use the rational bid function in any equilibrium of the second-price auction.

Remark 4 (Heterogeneous Signal Precision). So far we have assumed that whether the bidder is novice or rational, he receives a signal θ about his ex post value of equal precision. In some applications, it may be argued that a novice bidder may receive less precise signal

[19] It is readily verified that one cannot be arbitrarily close to an efficient allocation as this would require that the payment rule is close to that of the second-price auction (to ensure nearby efficiency for the allocation among rational bidders) and such an auction rule would not lead novice bidders to bid close to their expected valuation (thereby leading to welfare significant losses in the allocation between rational and novice bidders).

[20] In a second-price auction, inefficiencies would typically arise even without rational bidders when there are three or more ex post values (since a novice bidder i would not in general bid in the same way for different signals θ_i corresponding to the same interim expected value w_i). But, our argument for using a second-price auction makes use of the presence of rational bidders.

about his ex post value. It should be clear that the same inefficiency result as in Theorem 1 would arise in this more general scenario, as the novice bidder would still not bid his expected value in a SPA, and a SPA would be required to ensure efficiency among the rational bidders.

4.1 Proof of Theorem 1

Regular Equilibria of Simple Mechanisms. First, we show that it suffices to consider *regular equilibria* of *simple mechanisms*. We call a smooth auction-like mechanism *simple* if it is of the form $M = [[0,1], (W_i), (L_i), Id]$, where $\phi = Id$ denotes the identity so that the allocation rule is symmetric. We call an equilibrium *regular* if it is symmetric and the bid of each generalized type (w_i, x_i, s_i) is given by a continuous and strictly increasing function $b(w_i)$ with range $b([0,1]) = [0,1]$. In other words, the bid only depends on the interim valuations, but not on the identity, sophistication, or belief x_i, of the bidder. Note that a regular equilibrium of a simple mechanism is efficient. We denote the strictly increasing and continuous inverse of $b(\cdot)$ by $\psi : [0,1] \to [0,1]$.

Lemma 1. *Let $\tilde{M} = [\tilde{B}, (\tilde{W}_i), (\tilde{L}_i), \tilde{\phi}_1]$ be a smooth auction-like mechanism with an efficient equilibrium $(\tilde{b}_1(w_1, x_1, s_1), \tilde{b}_2(w_2, x_2, s_2))$. Then there exists a simple mechanism $M = [[0,1], (W_i), (L_i), Id]$, with a regular (and hence efficient) equilibrium.*

Proof. The proofs of all Lemmas can be found in the Appendix. □

In light of Lemma 1, it suffices to consider regular equilibria of simple mechanisms. The intuition behind this result is that in an efficient mechanism with a symmetric allocation rule,[21] all bidders must use the same bids as function of their interim valuation. The proof shows that mechanisms for which the bidding function has discontinuities, these jumps can be removed in a way that preserves the smoothness of the payment rules. Lemma 1 falls short of the revelation principle because the full revelation argument may not preserve the smoothness of the payment rules if the equilibrium of the original mechanism is non-smooth.

Second-Price Auctions. Next we derive a condition on the payment rules and equilibrium bid function that characterizes regular equilibria of the second-price auction. We

[21] Clearly, a mechanism with an asymmetric allocation rule can be made symmetric by a simple monotonic transformation.

denote the equilibrium difference in utility between winning and losing of a bidder with bid $b = b(w_i)$, whose bid is tied with the opponent by

$$\delta_i(b) = \psi(b) - (W_i(b,b) - L_i(b,b)) \,.$$

In a regular equilibrium of the SPA, the rational type bids truthfully ($b(w) = w$), and the payment rules satisfy $W_i(b,b) = b$ and $L_i \equiv 0$, so that $\delta_i(b) = 0$ for all $b \in [0,1]$. The following Lemma shows the converse result.

Lemma 2. *Consider a simple mechanism $M = [[0,1], (W_i), (L_i), Id]$ with a regular equilibrium. If $\delta_i(b) = 0$, for $i \in \{1,2\}$ and all $b \in [0,1]$, then M is a second-price auction—that is, for all $i \in \{1,2\}$, $L_i(b_i, b_j) = 0$ for all $b_i \le b_j$ and $W_i(b_i, b_j(w_j)) = w_j$ whenever $b_i \ge b_j(w_j)$.*

Differentiability of the Bidding Strategy. To show that $\delta_i(b) = 0$ for all bids we derive an implication of $\delta_i(b) > 0$ and show that it is violated generically. In the derivations we will use first-order conditions. The following Lemma shows that the inverse of the bid function, $\psi(b)$ is differentiable if $\delta_i(b) > 0$. The Lemma is based on the proof of Lemma 7 in Persico and Lizzeri (2000).

Lemma 3. *If $\delta_i(b_0) > 0$ for some $b_0 \in [0,1]$, then there exists a non-empty interval $(\alpha, \beta) \subset [0,1]$, with $b_0 \in (\alpha, \beta)$, such that ψ is continuously differentiable on (α, β), and $\psi'(b) > 0$ and $\delta_i(b) > 0$ for all $b \in (\alpha, \beta)$.*

For generic distributions, efficiency requires $M = SPA$. Next, we show that $\delta_i(b) > 0$ implies that a condition similar to the full-surplus extraction condition (McAfee and Reny, 1992) must be violated, and prove results analogous to Gizatulina and Hellwig (2017), to show that for generic prior densities $f(w_1, x_1, w_2, x_2)$, we must have $\delta_i(b) = 0$ for all $b \in [0,1]$, $i \in \{1,2\}$, and any regular equilibrium of a simple mechanism.

We begin by deriving an implication of $\delta_i(b) > 0$. Fix $b \in (0,1)$ such that $\delta_i(b) > 0$ and consider a rational bidder i with type (w_i, x_i), where $w_i = \psi(b)$ and $x_i \in X$ is arbitrary. In a regular equilibrium, this type maximizes (where we use $j \neq i$ to denote the opponent):

$$\max_{b' \in [0,1]} \int_0^{\psi(b')} \left(\psi(b) - W_i(b', b(w_j))\right) f(w_j|\psi(b), x_i) dw_j - \int_{\psi(b')}^1 L_i(b', b(w_j)) f(w_j|\psi(b), x_i) dw_j$$

Given Lemma 3, we can differentiate the objective function with respect to b', and obtain the first-order condition, which must hold for $b' = b$:

$$f(\tilde{w}_j = \psi(b)|\tilde{w}_i = \psi(b), x_i) = \int_0^1 \frac{\partial P_i(b, b(w_j))/\partial b_i}{\delta_i(b)\psi'(b)} f(w_j|\tilde{w}_i = \psi(b), x_i) dw_j, \tag{4}$$

where we simplify notation by denoting the payment of bidder i as follows

$$P_i(b_i, b_j) := \mathbf{1}_{\{b_i > b_j\}} W_i(b_i, b_j) + \mathbf{1}_{\{b_i < b_j\}} L_i(b_i, b_j).$$

Multiplying (4) by $f(\tilde{w}_i = \psi(b), x_i)/f(\tilde{w}_i = \tilde{w}_j = \psi(b))$, and using

$$f(x_i|\tilde{w}_i = \tilde{w}_j = \psi(b)) f(\tilde{w}_i = \tilde{w}_j = \psi(b)) = f(x_i, \tilde{w}_i = \tilde{w}_j = \psi(b)),$$

we obtain for all $x_i \in X_i$:

$$f(x_i|\tilde{w}_i = \tilde{w}_j = \psi(b)) = \int_0^1 m(b, \psi(b), w_j) f(x_i|\tilde{w}_i = \psi(b), w_j) dw_j, \tag{5}$$

where

$$m(b, \psi(b), w_j) = \frac{\partial P_i(b, b(w_j))/\partial b_i f(\tilde{w}_i = \psi(b), w_j)}{\delta_i(b)\psi'(b) f(\tilde{w}_i = \tilde{w}_j = \psi(b))}.$$

Since we consider a simple mechanism and prior densities $f \in \mathcal{M}_+^d([0,1]^{2K-2})$, and $\psi'(b) > 0$, the term $m(b, \psi(b), w_j)$ is finite and non-negative. For fixed b, $m(b, \psi(b), \cdot)$ is in fact a probability density on $[0, 1]$.[22]

Condition (5) states that the density $f(\cdot|\tilde{w}_i = \tilde{w}_j = \psi(b))$ can be expressed as a positive linear combination of the densities $f(\cdot|\tilde{w}_i = \psi(b), w_j)$ for $w_j \in [0, 1]$, with positive weights on $w_j \neq \psi(b)$. By virtually the same proof as for Theorem 2.4 in GH17, we can show that for generic distributions (5) is violated.

To state the result we need several definitions that mimic GH17. Let $\mathcal{M}_+^d(X)$ be the set of absolutely continuous probabilities measures on X with strictly positive and continuous densities, endowed with the topology induced by the sup-norm for density functions on X; let $\mathcal{C}([0,1], \mathcal{M}_+^d(X))$ be the set of continuous mappings from $[0, 1]$ to $\mathcal{M}_+^d(X)$, endowed with the topology of uniform convergence; and let $\mathcal{M}([0, 1])$ be the set of probability measures on $[0, 1]$, endowed with a topology that is metrizable by a metric that is a convex function

[22] Integrating both sides of (5) over X we see that $\int_0^1 m(b, \psi(b), w_j) dw_j = 1$.

on $\mathcal{M}([0,1]) \times \mathcal{M}([0,1])$. Finally let $\mathcal{E}(w_i) \subset \mathcal{C}([0,1], \mathcal{M}_+^d(X))$ be the set of continuous mappings that map $w \in [0,1]$ to densities $g(\cdot|w) \in \mathcal{M}_+^d(X)$ that satisfy the following condition: For all $\mu \in \mathcal{M}([0,1])$:

$$g(x_i|w_i) = \int_0^1 g(x_i|w')\mu(dw'),\ \forall x_i \in X \qquad \Longrightarrow \qquad \mu = \delta_{w_i} \tag{6}$$

where $\delta_{w_i} \in \mathcal{M}([0,1])$ is the Dirac measure with a mass-point on w_i.

Lemma 4 (see Theorem 2.4 in Gizatulina and Hellwig, 2017)**.** *For any $w_i \in (0,1)$, the set $\mathcal{E}(w_i)$ is a residual subset of $\mathcal{C}([0,1], \mathcal{M}_+^d(X))$, that is, it is a countable intersection of open and dense subsets of $\mathcal{C}([0,1], \mathcal{M}_+^d(X))$.*

The implication of this Lemma is that for fixed $w_i \in (0,1)$, and generic functions $w_j \mapsto f(\cdot|w_i, w_j)$ that map w_j to conditional densities $f(\cdot|w_i, w_j)$, any simple mechanism with a regular equilibrium must satisfy $\delta_i(b(w_i)) = 0$.

This Lemma is insufficient for our purposes for two reasons. First, we need to show that *for generic priors*, the function that maps w_j to the conditional density $f(x_i|w_i, w_j)$ is an element of $\mathcal{E}(w_i)$, and second we need to show this *for all* w_i. To this end, for $i \in \{1,2\}$ let $\mathcal{W}_i$ be a countable and dense subset of $(0,1)$. We show that for generic prior densities $f(w_1, x_1, w_2, x_2)$, the mapping that maps $w_j \in [0,1]$ to the conditional density $f_i(\cdot|w_i, w_j)$ is an element of $\mathcal{E}_i(w_i)$ for all $w_i \in \mathcal{W}_i$ and all $i \in \{1,2\}$. For the following Lemma, recall that $\mathcal{M}_+^d([0,1]^{2K-2})$ denotes the set of priors with strictly positive and continuous densities.

Lemma 5 (see Theorem 2.7 in Gizatulina and Hellwig, 2017)**.** *For $i \in \{1,2\}$, let $\mathcal{W}_i$ be a countable and dense subset of $(0,1)$. Let $\mathcal{F}$ be the set of prior densities in $\mathcal{M}_+^d([0,1]^{2K-2})$ such that for all $i \in \{1,2\}$ and $w_i \in \mathcal{W}_i$, the mapping $w_j \mapsto f(\cdot|w_i, w_j)$ is an element of $\mathcal{E}(w_i)$. Then $\mathcal{F}$ is a residual subset of $\mathcal{M}_+^d([0,1]^{2K})$, that is it contains a countable intersection of open and dense subsets of $\mathcal{M}_+^d([0,1]^{2K})$.*

This Lemma implies that for generic prior densities $f(w_1, x_1, w_2, x_2)$, any regular equilibrium of a simple mechanism must satisfy $\delta_i(b(w_i)) = 0$ for all $w_i \in \mathcal{W}_i$. Since the functions $b(\cdot)$ and $\delta_i(\cdot)$ are continuous and $\mathcal{W}_i$ is dense, this implies $\delta_i(b) = 0$ for all $b \in [0,1]$. By Lemma 2, this implies that for generic distributions, if a simple mechanism has a regular equilibrium, then it must be the second-price auction.

Bidding Strategy of the Misspecified Type in the Second-Price Auction. So far we have made use of the rational type's first-order condition to show that efficiency cannot be achieved with an auction-like mechanism other than the SPA. To conclude the proof of Theorem 1 we show that for generic distributions, misspecified types do not use $b(w) = w$ in a SPA.

Lemma 6. *Let $\lambda \in (0, 1)$ and suppose that $\mathbb{E}_f \left[\theta_i^K | w_j \leq b\right] \neq \frac{\mathbb{E}_f\left[\theta_i^K\right]}{\mathbb{E}_f\left[\theta_i^1\right]} \mathbb{E}_f \left[\theta_i^1 | w_j \leq b\right]$ for some $i \in \{1, 2\}$ and $b \in [0, 1]$. In any equilibrium of the second price auction where the rational types bid truthfully, some types (θ_i, m_i) place a bid that is different from their interim valuation.*

It is easy to see that the subset of prior densities for which there exists $i \in \{1, 2\}$ and $b \in [0, 1]$ such that $\mathbb{E}_f \left[\theta_i^K | w_j \leq b\right] \neq \frac{\mathbb{E}_f\left[\theta_i^K\right]}{\mathbb{E}_f\left[\theta_i^1\right]} \mathbb{E}_f \left[\theta_i^1 | w_j \leq b\right]$ is open and dense $\mathcal{M}_+^d([0, 1]^{2K})$ so that its intersection with $\mathcal{F}$ is residual by Lemma 5. This concludes the proof of Theorem 1.

5 Discussion

In this Section we discuss how our main result is affected when losers' ex post valuations are not precisely accessible in the data or when more general mechanisms can be used.

5.1 Non-observability of losers' valuations

While we believe it is natural to assume that a relatively precise assessment of winners' ex post valuations is accessible after the auction, it may be argued that in some applications, forming estimates about losers' valuations is harder to the extent that it would require engaging in counterfactual exercises trying to assess what would have been the net benefit of losers had they won the auction. We now suggest that the same modeling of the decision rule of novice bidders would apply under plausible specifications of such a more general scenario.

Specifically, instead of assuming that (b_i, b_j, v_i, v_j) is accessible after the auction, we now assume that when bidder i is the winner only (b_i, b_j, v_i, ϕ_j) is accessible where $\phi_j = (\phi_j^1, ..., \phi_j^K) \in \Delta V$ is a noisy signal held (after the auction) about the ex post valuation bidder j would have obtained had he won the auction and ϕ_j^k represents the probability assigned to $v_j = v^k$ according to ϕ_j. In the face of such data, it seems then natural to complete the missing value of v_j with the distribution over V induced by ϕ_j. That is, substitute the observed data (b_i, v_i, b_j, ϕ_j) with each of $(b_i, v_i, b_j, v_j = v^k)$ with probability ϕ_j^k. From the obtained dataset, one can construct the empirical cumulative distributions $H_i(b \mid v^k)$ and $H_j(b \mid v^k)$, and proceed as in Section 2 for the derivation of a steady-state.

It should be mentioned that as long as the noisy signal ϕ_j about v_j in the above construction is unbiased in the sense that conditional on (b_i, b_j, v_i) its distribution generates the same distribution over the ex post valuation v_j of the loser as the correct one,[23] then the procedure just mentioned would lead to exactly the same decision rule for novice bidders as the one considered in the main model (this is so because, under the assumption that signals ϕ_j are unbiased, it is readily verified it would give rise to the same cumulative distributions $H_i(b \mid v^k)$ and $H_j(b \mid v^k)$).

Thus, we conclude that our modeling of novice bidders does not hinge on the assumption that losers' valuations are precisely observed after the auction but rather on the assumption that ex post, outside observers get unbiased estimates of those.

We might in some applications be willing to go further and consider the case in which outside observers would get biased estimates of losers' ex post valuations. The heuristic just proposed would allow us to consider such an extension and now the decision rule of novice bidders would also depend on the exact distributional shape of the bias of the estimate. We note that the same conclusion as in Theorem 1 would still hold in such a case, since efficiency among rational bidders would require using a Second-Price Auction, and novice bidders under this alternative scenario would not bid their expected valuation in this auction (except possibly for highly non-generic specifications of the bias in the estimate).

5.2 More general mechanisms

We have focused on a class of auction-like mechanisms in which bids are one-dimensional and a higher bid increases the chance of winning the auction. This is a natural class that covers virtually all practically relevant auction formats. In the working paper version, we explore whether more elaborate mechanisms could help improve efficiency. We note in our basic setup that since no two different types would have the same belief about the distribution of the opponent's interim type (for generic distributions), scoring rule mechanisms of the type considered in Johnson, Pratt, and Zeckhauser (1990) would allow the designer to

[23]This would arise, for example, if the estimate took the form of observing $v_j + \varepsilon_j$ where ε_j would be drawn from some distribution, say normal centered around 0, and the distribution of ε_j would be independent from the distribution of any other variable as introduced in the model.

elicit the interim type and approximate any allocation goal of her choice such as efficiency. However, we note that such a conclusion would not be robust to the inclusion of richer specifications of cognitive limitations which would, under plausible formulations, lead different interim types to have the same beliefs about the distribution of their opponent's interim type. Independently of this, such mechanisms are fragile, as stressed in the robust mechanism design literature.

6 Conclusion

This paper has revisited the possibility of efficient auctions when some bidders form their beliefs about others' bidding strategies based on accessible data from similar auctions which consist only of ex post values and bids. Our main impossibility result obtained in a private value setting demonstrates a novel source of potential inefficiency related to the cognitive limitation that is induced by missing data on the signals observed at the time of the auction.

The insight is obtained after noting that data-driven bidders reason as if they were in an interdependent value environment. But, note that in the proposed approach, the resulting misspecification disappears when the signals are independently distributed across bidders or when they are very precise, giving indication as to when we should expect to see inefficiencies in data-driven equilibria of second-price auctions with private values.

Beyond the general impossibility result, our proposed model could be used to analyze the degree of inefficiency induced by novice bidders as a function of the underlying correlation between valuations and the precision of the signals at the time of the auction. It could also be used to revisit empirical approaches to auctions with the premise that bidding behaviors are governed by data-driven equilibria instead of the Bayes Nash equilibrium (as assumed in most of the empirical literature, see Perrigne and Vuong, 2022).

A Omitted Proofs

A.1 Proof of Proposition 1

Proof of Proposition 1. If $\lambda \in (0,1)$, efficiency would require that $b^m(\theta_i) = \theta_i$ which implies

$$H^{\text{SPA}}(b \mid v_i = 1) = \int_0^1 F(b \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{\mathbb{E}[\tilde{\theta}_i]} d\tilde{\theta}_i,$$
$$H^{\text{SPA}}(b \mid v_i = 0) = \int_0^1 F(b \mid \tilde{\theta}_i)(1-\tilde{\theta}_i) \frac{f(\tilde{\theta}_i)}{\mathbb{E}[1-\tilde{\theta}_i]} d\tilde{\theta}_i.$$

Moreover, we must have

$$\theta_i \in \arg\max_b \left\{ \theta_i H^{\text{SPA}}(b \mid v_i = 1) - \theta_i \int_0^b x dH^{\text{SPA}}(x|v_i = 1) - (1-\theta_i) \int_0^b x dH^{\text{SPA}}(x|v_i = 0) \right\}$$

Differentiating the objective function and setting $b = \theta_i$ yields

$$(1-\theta_i)\theta_i \left[H^{\text{SPA}\prime}(\theta_i \mid v_i = 1) - H^{\text{SPA}\prime}(\theta_i \mid v_i = 0) \right]$$

We have

$$\begin{aligned}
&H^{\text{SPA}\prime}(\theta_i \mid v_i = 1) - H^{\text{SPA}\prime}(\theta_i \mid v_i = 0) \\
=& \int_0^1 f(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{\mathbb{E}[\tilde{\theta}_i]} d\tilde{\theta}_i - \int_0^1 f(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i) \frac{f(\tilde{\theta}_i)}{\mathbb{E}[1-\tilde{\theta}_i]} d\tilde{\theta}_i \\
=& \int_0^1 \left[\frac{\tilde{\theta}_i}{\mathbb{E}[\tilde{\theta}_i]} - \frac{1-\tilde{\theta}_i}{1-\mathbb{E}[\tilde{\theta}_i]} \right] f(\theta_i \mid \tilde{\theta}_i) f(\tilde{\theta}_i) d\tilde{\theta}_i \\
=& f(\theta_i) \int_0^1 \left[\frac{\tilde{\theta}_i}{\mathbb{E}[\tilde{\theta}_i]} - \frac{1-\tilde{\theta}_i}{1-\mathbb{E}[\tilde{\theta}_i]} \right] f(\tilde{\theta}_i|\theta_i) d\tilde{\theta}_i \\
=& f(\theta_i) \left[\frac{\mathbb{E}[\tilde{\theta}_i|\theta_j = \theta_i]}{\mathbb{E}[\tilde{\theta}_i]} - \frac{1-\mathbb{E}[\tilde{\theta}_i|\theta_j = \theta_i]}{1-\mathbb{E}[\tilde{\theta}_i]} \right]
\end{aligned}$$

Hence, for bidding θ_i to be optimal for the misspecified type we must have for all θ_i:

$$\frac{\mathbb{E}[\tilde{\theta}_i|\theta_j = \theta_i]}{\mathbb{E}[\tilde{\theta}_i]} - \frac{1 - \mathbb{E}[\tilde{\theta}_i|\theta_j = \theta_i]}{1 - \mathbb{E}[\tilde{\theta}_i]} = 0$$
$$\iff \mathbb{E}[\tilde{\theta}_i|\theta_j = \theta_i] = \mathbb{E}[\tilde{\theta}_i]$$

If the last line holds for all θ_i we must have

$$\begin{aligned}
& \int_0^1 \tilde{\theta}_i f(\tilde{\theta}_i|\theta_j) d\tilde{\theta}_i = \mathbb{E}[\tilde{\theta}_i], \qquad \forall \theta_j, \\
\iff \qquad & \int_0^1 \tilde{\theta}_i \theta_j f(\tilde{\theta}_i, \theta_j) d\tilde{\theta}_i = \mathbb{E}[\tilde{\theta}_1] \theta_j f(\theta_j), \qquad \forall \theta_j, \\
\implies \qquad & E\left[\tilde{\theta}_i \theta_j\right] = \left(\mathbb{E}[\tilde{\theta}_i]\right)^2.
\end{aligned}$$

The last line implies that we must have $\text{Corr}[\theta_1, \theta_2] = 0$ if the misspecified types first-order condition is satisfied for $b = \theta_i$ for all θ_i. Therefore, if $\text{Corr}[\theta_1, \theta_2] \neq 0$, there are types for which a misspecified bidder will not bid θ_i and since $b^r(\theta_j) = \theta_j$ for all types and $\lambda \in (0, 1)$, the allocation will be inefficiency for some type profiles. □

A.2 Proof of Proposition 2

Proof of Proposition 2. An efficient allocation requires that $b^r(\theta_i) = b^m(\theta_i) = b(\theta_i)$ for all $\theta_i \in [0, 1]$. We denote the inverse of $b(\cdot)$ by θ.

The rational type's bid solves

$$\max_b \, (\theta_i - b) \, F(\theta(b)|\theta_i)$$

The FOC yields

$$\begin{aligned}
& -F(\theta_i|\theta_i) + (\theta_i - b(\theta_i)) \, f(\theta_i|\theta_i) \theta_i'(b(\theta_i)) = 0 \\
\iff \qquad & b'(\theta_i) = (\theta_i - b(\theta_i)) \frac{f(\theta_i|\theta_i)}{F(\theta_i|\theta_i)}.
\end{aligned} \tag{7}$$

The solution with boundary condition $b(0) = 0$ is

$$b(\theta_i) = \int_0^{\theta_i} x e^{-\int_x^{\theta_i} \frac{f(y|y)}{F(y|y)} dy} \frac{f(x|x)}{F(x|x)} dx.$$

The misspecified type maximizes (3)

$$\max_b (1-b)\,\theta_i H^{\text{FPA}}(b \mid v_i=1) - b(1-\theta_i)H^{\text{FPA}}(b|v_i=0).$$

with

$$\begin{aligned}
H^{\text{FPA}}(b \mid v_i=1) &= \int_0^1 F(\theta(b) \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i, \\
H^{\text{FPA}}(b \mid v_i=0) &= \int_0^1 F(\theta(b) \mid \tilde{\theta}_i)(1-\tilde{\theta}_i) \frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i.
\end{aligned}$$

This yields

$$\begin{aligned}
&\theta_i H^{\text{FPA}}(b(\theta_i) \mid v_i=1) + (1-\theta_i)H^{\text{FPA}}(b(\theta_i)|v_i=0) \\
&= (1-b(\theta_i))\,\theta_i H^{\text{FPA}\prime}(b(\theta_i) \mid v_i=1) - b(\theta_i)(1-\theta_i)H^{\text{FPA}\prime}(b(\theta_i)|v_i=0)
\end{aligned}$$

Using

$$\begin{aligned}
H^{\text{FPA}\prime}(b \mid v_i=1) &= \theta'(b)\int_0^1 f(\theta(b) \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i = \theta'(b)\frac{E[\tilde{\theta}_i|\theta(b)]}{E[\tilde{\theta}_i]} f(\theta(b)) \\
H^{\text{FPA}\prime}(b \mid v_i=0) &= \theta'(b)\int_0^1 f(\theta(b) \mid \tilde{\theta}_i)(1-\tilde{\theta}_i) \frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i = \theta'(b)\frac{1-E[\tilde{\theta}_i|\theta(b)]}{1-E[\tilde{\theta}_i]} f(\theta(b))
\end{aligned}$$

we have

$$
\begin{aligned}
&\theta_i \int_0^1 F(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i + (1-\theta_i)\int_0^1 F(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i)\frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i \\
&= (1-b(\theta_i))\,\theta_i \theta'(b(\theta_i))\frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} f(\theta_i) - b(\theta_i)(1-\theta_i)\theta'(b(\theta_i))\frac{1-E[\tilde{\theta}_i|\theta_i]}{1-E[\tilde{\theta}_i]} f(\theta_i) \\
\iff \quad &\theta_i \int_0^1 F(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i + (1-\theta_i)\int_0^1 F(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i)\frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i \\
&= \frac{1-b(\theta_i)}{b'(\theta_i)}\theta_i \frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} f(\theta_i) - \frac{b(\theta_i)}{b'(\theta_i)}(1-\theta_i)\frac{1-E[\tilde{\theta}_i|\theta_i]}{1-E[\tilde{\theta}_i]} f(\theta_i) \\
\iff \quad b'(\theta_i) &= \frac{(1-b(\theta_i))\,\theta_i \frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} f(\theta_i) - b(\theta_i)(1-\theta_i)\frac{1-E[\tilde{\theta}_i|\theta_i]}{1-E[\tilde{\theta}_i]} f(\theta_i)}{\theta_i \int_0^1 F(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i + (1-\theta_i)\int_0^1 F(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i)\frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i} \\
&= \theta_i \frac{\frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} f(\theta_i)}{\theta_i \int_0^1 F(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i + (1-\theta_i)\int_0^1 F(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i)\frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i} \\
&\quad - b(\theta_i)\frac{\theta_i \frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} f(\theta_i) - (1-\theta_i)\frac{1-E[\tilde{\theta}_i|\theta_i]}{1-E[\tilde{\theta}_i]} f(\theta_i)}{\theta_i \int_0^1 F(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i + (1-\theta_i)\int_0^1 F(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i)\frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i} \\
&= (\theta_i - b(\theta_i))\frac{f(\theta_i|\theta_i)}{F(\theta_i|\theta_i)}
\end{aligned}
$$

Where the last line follows from (7). Matching coefficients, we get

$$
\frac{\frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} f(\theta_i)}{\theta_i \int_0^1 F(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i + (1-\theta_i)\int_0^1 F(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i)\frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i} = \frac{f(\theta_i|\theta_i)}{F(\theta_i|\theta_i)}
$$

and

$$
\frac{\theta_i \frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} f(\theta_i) - (1-\theta_i)\frac{1-E[\tilde{\theta}_i|\theta_i]}{1-E[\tilde{\theta}_i]} f(\theta_i)}{\theta_i \int_0^1 F(\theta_i \mid \tilde{\theta}_i)\tilde{\theta}_i \frac{f(\tilde{\theta}_i)}{E[\tilde{\theta}_i]} d\tilde{\theta}_i + (1-\theta_i)\int_0^1 F(\theta_i \mid \tilde{\theta}_i)(1-\tilde{\theta}_i)\frac{f(\tilde{\theta}_i)}{E[1-\tilde{\theta}_i]} d\tilde{\theta}_i} = \frac{f(\theta_i|\theta_i)}{F(\theta_i|\theta_i)}
$$

Combining these we have

$$\frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} = \theta_i \frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} - (1-\theta_i)\frac{1-E[\tilde{\theta}_i|\theta_i]}{1-E[\tilde{\theta}_i]}$$
$$\frac{E[\tilde{\theta}_i|\theta_i]}{E[\tilde{\theta}_i]} = \frac{1-E[\tilde{\theta}_i|\theta_i]}{1-E[\tilde{\theta}_i]}$$

This is the same condition as for the SPA which requires that $\mathrm{Corr}[\theta_1, \theta_2] = 0$. □

A.3 Proof of Lemma 1

Proof of Lemma 1. Consider the equilibrium of the original mechanism $\tilde{M}$. For each bidder i and each $s_i \in \{r, m\}$, we define a (non-empty) correspondence that contains all bids that types with expected valuation w_i use.

$$b_i^{s_i}(w_i) = \tilde{b}_i(w_i, X, s_i)$$

where $X = [0,1]^{K-2}$. We prove the lemma in three steps: (1) we obtain an efficient equilibrium of the original mechanism with single-valued correspondences (or functions) $\hat{b}_i^{s_i}$. (2) We show that these functions satisfy $\hat{b}_i^r(w) = \hat{b}_i^s(w) = \tilde{\phi}_i(\hat{b}_j^r(w)) = \tilde{\phi}_i(\hat{b}_j^r(w))$, and a change of variable allows us to construct a mechanism $\check{M} = \left(B, \left(\check{W}_i\right), \left(\check{L}_i\right), Id\right)$ that has an efficient equilibrium in which $\check{b}_i^r(w) = \check{b}_i(w) = \check{b}_j^r(w) = \check{b}_j^r(w) = \check{b}(w)$. (3) We remove jump continuities in $\check{b}(w)$ and normalize the range of $\check{b}(w)$ to obtain a mechanism $M = ([0,1], (W_i), (L_i), Id)$ so that the (normalized) continuous part of $\check{b}(w)$ is an efficient equilibrium. We show that removing the discontinuities does not destroy the smoothness of the simple mechanism M.

Step 1: First, note that efficiency requires that the correspondences $b_i^{s_i}$ for $i \in \{1,2\}$ must be strictly increasing, meaning any selection must be strictly increasing. We denote the point-wise infimum and supremum of the correspondence by $\underline{b}_i^{s_i}(w) = \inf b_i^{s_i}(w_i)$ and $\overline{b}_i^{s_i}(w) = \sup b_i^{s_i}(w_i)$. Note that the infimum $\underline{b}_i^{s_i}(w)$ is strictly increasing if any selection from $b_i^{s_i}(w)$ is strictly increasing.

Suppose for some w_i, $b_i^r(w_i)$ is not single-valued. Efficiency and the fact that for every $b_i \in [\underline{b}_i^{s_i}(w), \overline{b}_i^{s_i}(w)]$, $\left(b_j^{s_j}\right)^{-1}(\phi_j(b_i)) \subset \{w_i\}$, that is, any bid in the closed interval between the between the infimal and supremal bid that bidder i with interim value w_i places in equilibrium is either not placed by bidder j or it is placed by a bidder with the same interim value. We can include the infimum (and supremum) since $w_j \in \left(b_j^{s_j}\right)^{-1}(\phi_2(\underline{b}_i^{s_i}(w)))$ for some $w_j < w_i$ would imply that there exists $w_i' \in (w_j, w_i)$ such that $b_i' < \underline{b}_i^{s_i}(w)$ for some $b_i' \in b_i^{s_i}(w_i')$, which violates efficiency.

Since the probability that $w_j = w_i$ conditional on (w_i, x_i) is zero for all $x_i \in X_i$, the rational type is indifferent between all bids in $[\underline{b}_i^{s_i}(w), \overline{b}_i^{s_i}(w)]$. We set $\hat{b}_i(w_i, x_i, r) := \hat{b}_i^r(w_i) := \underline{b}_i^r(w)$. Similar steps show that we can set $\hat{b}_i(w_i, x_i, m) := \hat{b}_i^m(w_i) := \underline{b}_i^m(w)$.

Since the probability that $E[v_i|\theta_i] = w_i$ is zero, and there are at most countably many discontinuities, this modification of $\tilde{b}_i$ to $\hat{b}_i$ does not change the incentives of bidder j so that we have constructed a new equilibrium in which the correspondences of bidder i are single valued. We can apply the same modification to the strategy of bidder j. Clearly these modification preserve efficiency since $b_j^{s_j}(w_j) < \phi_2(\inf \tilde{b}_i^r(w_i))$ whenever $w_j < w_i$.

Step 2: We have shown in Step 1 that there exists an efficient equilibrium of $\tilde{M}$ that is given by the function $\hat{b}_i^s(w)$, $i \in \{1, 2\}$, $s \in \{r, m\}$. Clearly, efficiency requires that $\hat{b}_i^r(w) = \hat{b}_i^m(w) = \phi_i(\hat{b}_j^r(w)) = \phi_i(\hat{b}_j^m(w)) =: \hat{b}_i(w)$ for almost every w. The only exceptions are a countable set of interim values where all functions have a jump-discontinuity. Here we can redefine $\hat{b}_i^r(w) = \hat{b}_i^m(w) = \hat{b}_i(w) := \lim_{w' \uparrow w} \min \left\{\hat{b}_i^r(w'), \hat{b}_i^m(w'), \phi_i(\hat{b}_j^r(w')), \phi_i(\hat{b}_j^m(w'))\right\}$ for $i \neq j$, so that $\hat{b}_i^r(w) = \hat{b}_i^m(w) = \phi_i(\hat{b}_j^r(w)) = \phi_i(\hat{b}_j^m(w)) = \hat{b}_i(w)$ for every w, and $\hat{b}_i(w)$ is left-continuous.

The bids of bidder i are contained in $\hat{R}_i = [\hat{b}_i(0), \hat{b}_i(1)]$. We now define a new mechanism with $\check{B} = [0, 1]$, $\check{\phi}(w) = w$ and $\check{W}_i, \check{L}_i : [0, 1]^2 \to \mathbb{R}$ given by:

$$\check{W}_i(\check{b}_i, \check{b}_j) = \tilde{W}_i\left(\hat{b}_i(0) + \check{b}_i|\hat{R}_i|,\, \tilde{\phi}_j\left(\hat{b}_i(0) + \check{b}_j|\hat{R}_i|\right)\right),$$
$$\check{L}_i(b_i, b_j) = \tilde{L}_i\left(\hat{b}_i(0) + \check{b}_i|\hat{R}_i|,\, \tilde{\phi}_j\left(\hat{b}_i(0) + \check{b}_j|\hat{R}_i|\right)\right).$$

The new mechanism has an equilibrium given by the functions $\check{b}_i^s(w) = (\hat{b}_i(w) - \hat{b}_i(0))/|\hat{R}_i|$ and $\check{b}_j^s(w) = (\hat{b}_i(w) - b_i(p))/|\hat{R}_i|$. This equilibrium allocates to the bidder with the highest valuation since $\check{b}_i^s(w) > \check{b}_j^s(w)$ if and only if $\hat{b}_i(w) > \phi_i(\hat{b}_i(w))$ and the original mechanism was efficient. This implies that all bidding functions are the same: $\check{b}_i^s(w) = \check{b}_j^s(w) =: \check{b}(w)$ for $s \in \{r, m\}$. Moreover $\check{W}_i$ and $\check{L}_i$ are $\mathcal{C}^1$ since $\tilde{\phi}_j$ is continuously differentiable.

Step 3: The bidding function $\check{b}(w)$ is strictly increasing and can therefore be decomposed as $\check{b}(w) = \check{b}^C(w) + \check{b}^J(w)$, where $\check{b}^C(w)$ is continuous and $\check{b}^J(w)$ is constant except for a countable number of jump-discontinuities. We can modify the definition of $\check{M}$ and obtain a new smooth auction-like mechanism M with a symmetric equilibrium in which

$b(w) = \check{b}^C(w)/\left(\check{b}^C(1) - \check{b}^C(0)\right)$.

The function $b(w_i)$ specifies an equilibrium in the mechanism given by:

$$\begin{aligned}W_i(b_1, b_2) &= \check{W}_i(\check{b}((\check{b}^C)^{-1}(b_1(b^C(1) - b^C(0)))), \check{b}((\check{b}^C)^{-1}(b_2(b^C(1) - b^C(0))))), \\ L_i(b_1, b_2) &= \check{L}_i(\check{b}((\check{b}^C)^{-1}(b_1(b^C(1) - b^C(0)))), \check{b}((\check{b}^C)^{-1}(b_2(b^C(1) - b^C(0))))).\end{aligned}$$

Next, we show that W and L are continuously differentiable. In the mechanism defined in step 2, a rational bidder chooses b_i to maximize

$$\int_0^{\check{b}^{-1}(b_i)} \left(w_i - \check{W}_i(b_i, \check{b}(w_j))\right) dF(w_j|w_i, x_i) - \int_{\check{b}^{-1}(b_i)}^1 \check{L}_i(b_i, \check{b}(w_j)) dF(w_j|w_i, x_i),$$

where $F(w_j'|w, x_i)$ is the probability that $w_j \leq w_j'$, conditional on bidder i's type (w_i, x_i).

Consider a rational bidder with type $w_i = \hat{w} + \varepsilon$, where $\hat{w}$ is a discontinuity in the equilibrium bidding function $\check{b}$ of original mechanism. Placing a bid $b' \in [\check{b}(\hat{w}), \check{b}(\hat{w}_+))$ instead of $\check{b}(w_i)$ must not be profitable:

$$\begin{aligned}&\int_0^{\check{b}^{-1}(\check{b}(w_i))} \left(w_i - \check{W}_i(\check{b}(w_i), \check{b}(w_j))\right) dF(w_j|w_i, x_i) - \int_{\check{b}^{-1}(\check{b}(w_i))}^1 \check{L}_i(\check{b}(w_i), \check{b}(w_j)) dF(w_j|w_i, x_i) \\ \geq &\int_0^{\check{b}^{-1}(b')} \left(w_i - \check{W}_i(b', \check{b}(w_j))\right) dF(w_j|w_i, x_i) - \int_{\check{b}^{-1}(b')}^1 \check{L}_i(b', \check{b}(w_j)) dF(w_j|w_i, x_i)\end{aligned}$$

This can be rewritten as

$$\begin{aligned}&\int_0^{\hat{w}} \left(w_i - \check{W}_i(\check{b}(w_i), \check{b}(w_j))\right) dF(w_j|w_i, x_i) - \int_{\hat{w}}^1 \check{L}_i(\check{b}(w_i), \check{b}(w_j)) dF(w_j|w_i, x_i) \\ &+ \int_{\hat{w}}^{\hat{w}+\varepsilon} \left(w_i - \check{W}_i(\check{b}(w_i), \check{b}(w_j))\right) dF(w_j|w_i, x_i) + \int_{\hat{w}}^{\hat{w}+\varepsilon} \check{L}_i(\check{b}(w_i), \check{b}(w_j)) dF(w_j|w_i, x_i) \\ \geq &\int_0^{\hat{w}} \left(w_i - \check{W}_i(b', \check{b}(w_j))\right) dF(w_j|w_i, x_i) - \int_{\hat{w}}^1 \check{L}_i(b', \check{b}(w_j)) dF(w_j|w_i, x_i)\end{aligned}$$

The second term in on the left-hand side vanishes as $\varepsilon \to 0$ since $\check{W}_i$ and $\check{L}_i$ are bounded.

Hence we must have

$$\int_0^{\hat{w}} \left(\check{W}_i(b', \check{b}(w_j)) - \check{W}_i(\check{b}(\hat{w}_+), \check{b}(w_j))\right) dF(w_j|\hat{w}, x_i) + \int_{\hat{w}}^1 \left(\check{L}_i(b', \check{b}(w_j)) - \check{L}_i(\check{b}(\hat{w}_+), \check{b}(w_j))\right) dF(w_j|\hat{w}, x_i) \geq 0$$

Since $b' < \check{b}(w_i)$, and $\check{W}_i$ and $\check{L}_i$ are non-decreasing in the first argument, this implies that $\check{W}_i(b', \check{b}(w_j)) = \check{W}_i(b_i, \check{b}(w_j))$ and $\check{L}_i(b', \check{b}(w_j)) = \check{L}_i(b_i, \check{b}(w_j))$ all $b' \in [\check{b}(\hat{w}), \check{b}(\hat{w}_+)]$ and almost every w_j. By continuity of $\check{W}_i$ and $\check{L}_i$ the equalities must hold for all w_j. Hence since $\check{W}_i$ and $\check{L}_i$ are continuously differentiable, $\partial\check{W}_i(b', \check{b}(w_j))/\partial b_i = 0$ and $\partial\check{L}_i(b', \check{b}(w_j))/\partial b_i = 0$ for all w_j and all $b' \in [\check{b}(\hat{w}), \check{b}(\hat{w}_+)]$ and also $\partial\check{W}_i(b', \check{b}(w_j))/\partial b_j = \partial\check{W}_i(\check{b}(\hat{w}), \check{b}(w_j))/\partial b_j = \partial\check{W}_i(\check{b}_+(\hat{w}), \check{b}(w_j))/\partial b_j$ and $\partial\check{L}_i(b', \check{b}(w_j))/\partial b_j = \partial\check{L}_i(\check{b}(\hat{w}), \check{b}(w_j))/\partial b_j = \partial\check{L}_i(\check{b}(\hat{w}_+), \check{b}(w_j))/\partial b_j$ for all $b' \in [\check{b}(\hat{w}), \check{b}(\hat{w}_+)]$ and all w_j. Hence continuous differentiability is preserved by the elimination of the gaps. □

A.4 Proof of Lemma 2

Proof of Lemma 2. We first show that for all i and $b_i, b_j \in [0, 1]$: $\partial W_i(b_i, b_j)/\partial b_i = 0$ if $b_j < b_i$, and $\partial L_i(b_i, b_j)/\partial b_i = 0$ if $b_j > b_i$.

Since $\delta_i(b) = 0$ for all $b \in [0, 1]$ we have that ψ

$$\psi'(b) = \frac{\partial W_i(b, b)}{\partial b_i} + \frac{\partial W_i(b, b)}{\partial b_j} - \frac{\partial L_i(b, b)}{\partial b_i} - \frac{\partial L_i(b, b)}{\partial b_j} < \infty$$

where finiteness follows from the assumption that W_i and L_i are continuously differentiable.

Now suppose that for some $w_i \in (0, 1)$, $\int_0^1 \frac{\partial P_i(b(w_i), b(w_j))}{\partial b_i} f(w_j|w_i, x_i) dw_j > 0$. The same derivation leading to (8) in the proof of Lemma 3, together with $\delta_i(b(w_i)) = 0$ implies that

$$\liminf_{b \nearrow b(w_i)} \frac{\psi(b(w_i)) - \psi(b)}{b(w_i) - b} = \infty.$$

This contradicts $\psi'(b(w_i)) < \infty$. Hence $\int_0^1 \frac{\partial P_i(b(w_i), b(w_j))}{\partial b_i} f(w_j|w_i, x_i) dw_j = 0$ for all $w_i \in [0, 1]$. Since $\partial P(b_i, b_j)/\partial b_i \geq 0$ by assumption, we therefore have $\partial P_i(b_0, b(w_j))/\partial b_i = 0$ for almost every w_j and by continuity of $\partial W_i/\partial b_i$, $\partial L_i/\partial b_i$ and b, this holds for all w_j. Therefore $\partial_{b_i} W_i(b_0, b) = 0$ if $b < b_0$, and $\partial_{b_i} L_i(b_0, b) = 0$ if $b > b_0$.

To conclude the proof, note that individual rationality together with $L_i(b_i, b_j) \geq 0$

requires that $L_i(0, b_j) = 0$ for all b_j.[24] Since $\partial L_i(b_i, b_j)/\partial b_i = 0$ if $b_j > b_i$, this implies that $L_i(b_i, b_j) = 0$ for all $b_i \leq b_j$. Next, $\delta_i(b(w_i)) = 0$ implies $W_i(b_i(w), b_i(w)) = w_i + L_i(b_i(w), b_i(w)) = w_i$, and since $\partial W_i(b_i, b_j)/\partial b_i = 0$, $W_i(b_i, b_j(w_j)) = w_j$ whenever $b_i \geq b_j(w_j)$. $\square$

A.5 Proof of Lemma 3

Proof of Lemma 3. Consider a rational bidder i with types $(w_0, x_i) \in [0,1]^{K-1}$ and any sequence of valuations $w_i^n \nearrow w_0$. w_i^n prefers to bid $b^n = b(w_i^n)$ over bidding $b_0 = b(w_0)$. Therefore

$$\begin{aligned}
&\int_0^{\psi(b^n)} \left(w_i^n - W_i(b^n, b(w_j))\right) f(w_j|w_i^n, x_i) dw_j - \int_{\psi(b^n)}^1 L_i(b^n, b(w_j)) f(w_j|w_i^n, x_i) dw_j \\
\geq &\int_0^{\psi(b_0)} \left(w_i^n - W_i(b_0, b(w_j))\right) f(w_j|w_i^n, x_i) dw_j - \int_{\psi(b_0)}^1 L_i(b_0, b(w_j)) f(w_j|w_i^n, x_i) dw_j
\end{aligned}$$

$$\begin{aligned}
\iff \; &\frac{1}{b - b^n} \int_0^{\psi(b^n)} \left(W_i(b_0, b(w_j)) - W_i(b^n, b(w_j))\right) f(w_j|w_i^n, x_i) dw_j \\
&+ \frac{1}{b - b^n} \int_{\psi(b^n)}^1 \left(L_i(b_0, b(w_j)) - L_i(b^n, b(w_j))\right) f(w_j|w_i^n, x_i) dw_j \\
\geq &\frac{1}{b - b^n} \int_{\psi(b^n)}^{\psi(b_0)} \left(w_i^n - W_i(b_0, b(w_j)) + L_i(b_0, b(w_j))\right) f(w_j|w_i^n, x_i) dw_j
\end{aligned}$$

Taking the lim sup on both sides we get

$$\int_0^1 \frac{\partial P_i(b_0, b(w_j))}{\partial b_i} f(w_j|w_0, x_i) dw_j \geq \delta_i(b_0) f(w_0|w_0, x_i) \limsup_{n \to \infty} \frac{\psi(b_0) - \psi(b^n)}{b_0 - b^n}$$

[24] Notice that this holds independent of our normalization that $v^1 = 0$, since the lowest type never wins the object in a regular equilibrium.

where $P_i(b_i, b_j) = W_i(b_i, b_j) + L_i(b_i, b_j)$. Similarly, w_0 prefers to bid b_0 over b^n for all $n \in \mathbb{N}$:

$$\begin{aligned}
&\int_0^{\psi(b_0)} (w_0 - W_i(b_0, b(w_j)))\, f(w_j|w_0, x_i)dw_j - \int_{\psi(b_0)}^1 L_i(b_0, b(w_j)) f(w_j|w_0, x_i)dw_j \\
\geq &\int_0^{\psi(b^n)} (w_0 - W_i(b^n, b(w_j)))\, f(w_j|w_0, x_i)dw_j - \int_1^{\psi(b^n)} (L_i(b^n, b(w_j)))\, f(w_j|w_0, x_i)dw_j
\end{aligned}$$

$$\begin{aligned}
\iff &\frac{1}{b_0 - b^n} \int_{\psi(b^n)}^{\psi(b_0)} (w_0 - W_i(b_0, b(w_j)) + L_i(b_0, b(w_j)))\, f(w_j|w_0, x_i)dw_j \\
&\geq \frac{1}{b_0 - b^n} \int_0^{\psi(b^n)} (W_i(b_0, b(w_j)) - W_i(b^n, b(w_j)))\, f(w_j|w_0, x_i)dw_j \\
&+ \frac{1}{b - b^n} \int_{\psi(b^n)}^1 (L_i(b_0, b(w_j)) - L_i(b^n, b(w_j)))\, f(w_j|w_i^n, x_i)dw_j
\end{aligned}$$

Taking the lim inf on both sides we get

$$\delta_i(b_0) f(w_0|w_0, x_i) \liminf_{n\to\infty} \frac{\psi(b_0) - \psi(b^n)}{b_0 - b^n} \geq \int_0^1 \frac{\partial P_i(b_0, b(w_j))}{\partial b_i} f(w_j|w_0, x_i)dw_j.$$

Hence, for $\delta_i(b_0) > 0$ we have

$$\liminf_{n\to\infty} \frac{\psi(b_0) - \psi(b^n)}{b_0 - b^n} \geq \frac{\int_0^1 \frac{\partial P_i(b_0, b(w_j))}{\partial b_i} f(w_j|w_0, x_i)dw_j}{\delta_i(b_0) f(w_0|w_0, x_i)} \geq \limsup_{n\to\infty} \frac{\psi(b_0) - \psi(b^n)}{b_0 - b^n} \tag{8}$$

Notice that so far we have considered the case that $w^n < w_0$. The same steps apply for the case that the sequence satisfies $w^n > w_0$. Hence condition (8) applies for both cases. We have

$$\psi'(b_0) = \psi'_-(b_0) = \psi'_+(b_0) = \frac{\int_0^1 \frac{\partial P_i(b_0, b(w_j))}{\partial b_i} f(w_j|\psi(b_0), x_i)dw_j}{\delta_i(b_0) f(\psi(b_0)|\psi(b_0), x_i)}. \tag{9}$$

Hence $\psi(b_0)$ is differentiable at b_0. Since $\delta_i(b)$ is continuous, there exists ε such that $\delta_i(b) > 0$ for all $b \in B_\varepsilon(b_0)$. Since the right-hand side of (9) is continuous in b_0, ψ is continuously differentiable on $B_\varepsilon(b_0)$. Since ψ is strictly increasing there must be $b' \in B_\varepsilon(b_0)$ such that $\psi'(b') > 0$ and since ψ' is continuous, there exist $\alpha < b' < \beta$ such that $(\alpha, \beta) \subset B_\varepsilon(b_0)$ and ψ is continuously differentiable with $\psi'(b) > 0$ for $b \in (\alpha, \beta)$. □

A.6 Proof of Lemma 4

The proof follows the same steps as the proof of Theorem 2.4 in GH17, except that instead of considering continuous mappings from T_i to the space of all measures on T_{-i}, $\mathcal{M}(T_{-i})$, we consider continuous mappings from $[0,1]$ to the space of all absolutely continuous measures on $X = [0,1]^{K-2}$ with strictly positive and continuous density, which we denoted by $\mathcal{M}^d_+(X)$.

Restricting attention to $\mathcal{M}^d_+(X)$ instead of the space of all measures $\mathcal{M}(X)$, requires a straightforward modification of the constructions of the functions $\boldsymbol{g}$ and the measures $\beta_1, \ldots, \beta_K$ in footnote 20 of GH17. First we take the functions g^k to be functions $g^k : X \to [0,2]$ with $g^k(x^k) = 2$ and $g^k(x) = 0$ for $x \notin B^k$. This allows us to construct perturbations of the measures β^0_k which need to be elements $\mathcal{M}^d_+(X)$ for our purposes, by setting $\beta_k = (1-\varepsilon)\beta^0_k + \varepsilon\tilde{\beta}_k$ where the measure $\tilde{\beta}_k$ has a density $\tilde{f}_k$ that satisfies $\tilde{f}_k(x)$ for $x \notin B^k$ and $\int_X g^k(x)\tilde{f}_k(x)dx = 1$. Then, with $\varepsilon \neq -z/(1-z)$ for all negative eigenvalues of the matrix $\left(\int_X g^k(x)\beta^0_\ell(dx)\right)_{k,\ell}$, the vectors $\int_X \boldsymbol{g}(x)\beta_k(dx)$ for $k = 1, \ldots, K$ are linearly independent. The remaining steps in the proof are virtually unchanged.

A.7 Proof of Lemma 5

The proof follows Theorem 2.7 in GH17 and uses results from Section 5.4 in Gizatulina and Hellwig (2014).

First note that for elements of $\mathcal{M}^d_+([0,1]^{2K})$, marginal and conditional densities are defined in the usual way. Moreover, for each w_i, the function that maps w_j to the conditional probability measure on X that is given by the density $f(x_i|w_i, w_j)$, is an element of $\mathcal{C}([0,1], \mathcal{M}^d_+(X))$ (see GH14).

Analog to the proof of Theorem 2.7 in GH17, we let $\mathcal{F}^i_{w_i} \subset \mathcal{M}^d_+([0,1]^{2K})$ be the set of priors such that the function $w_j \mapsto f(\cdot|w_i, w_j)$ is an element of $\mathcal{E}(w_i)$. The key step is to show that the residualness of $\mathcal{E}(w_i)$ in $\mathcal{C}([0,1], \mathcal{M}^d_+(X))$ implies the residualness of $\mathcal{F} = \bigcap_{i \in \{1,2\}, w_i \in \mathcal{W}_i} \mathcal{F}^i_{w_i}$ in $\mathcal{M}^d_+([0,1]^{2K})$. For each $i \in \{1,2\}$ and $w_i \in (0,1)$, let $\psi_{i,w_i} : \mathcal{M}^d_+([0,1]^{2K}) \to \mathcal{M}^d_+([0,1]) \times \mathcal{C}([0,1], \mathcal{M}^d_+(X))$ be the mapping that maps the prior to the conditional distribution $f(w_j|w_i)$ and the function $w_j \mapsto f(x_i|w_i, w_j)$. As shown in the proof of Lemma 5.9 in GH14, the maps ψ_{i,w_i} are continuous and open if $\mathcal{M}^d_+([0,1]^{2K})$ is endowed with the uniform topology for density functions. As in the proof of Theorem 2.7 in GH17, this implies that $\mathcal{F}^i_{w_i}$ is as residual subset of $\mathcal{M}^d_+([0,1]^{2K})$, that is it contains a countable intersection $\bigcap_{n \in \mathbb{N}} H_n(i, w_i)$ of open and dense sets $H_n(i, w_i) \subset \mathcal{M}^d_+([0,1]^{2K})$.

Clearly, $H = \bigcap_{i\in\{1,2\}} \bigcap_{w_i\in\mathcal{W}_i} \bigcap_{n(i,w_i)\in\mathbb{N}} H_{n(i,w_i)}(i,w_i)$ is a subset of $\mathcal{F}$. By a diagonal argument, H is a countable intersection of open and dense subsets of $\mathcal{M}_+^d([0,1]^{2K})$ and hence $\mathcal{F}$ is residual.

A.8 Proof of Lemma 6

Proof of Lemma 6. We have shown this for $|V| = 2$ in Proposition 1. For $|V| \geq 3$, we need to modify the proof. If m-types bid $b(w_i) = w_i$, we must have for all θ_i that

$$w_i = \mathbb{E}[v_i|\theta_i] \in \arg\max_b \left\{ \sum_{k=1}^{K} \theta_i^k \left(v_i^k H^{\mathrm{SPA}}(b \mid v_i^k) - \int_0^b z dH^{\mathrm{SPA}}(z|v_i^k) \right) \right\}.$$

The first-order condition is

$$\sum_{k=1}^{|V|} \theta_i^k \left(v_i^k - w_i \right) H^{\mathrm{SPA}\prime}(w_i \mid v_i^k) = 0$$

Considering the type $\theta_i = (1-b, 0, \dots, 0, b)$ for any $b \in (0,1)$, we have $w_i = b$, and the first-order condition simplifies to

$$H^{\mathrm{SPA}\prime}(b \mid v_i = 1) - H^{\mathrm{SPA}\prime}(b \mid v_i = 0) = 0$$

We have

$$\begin{aligned} H^{\mathrm{SPA}}(b \mid v_i^k) &= \frac{\mathbb{P}_f\left[b_j \leq b, v_i = v_i^k\right]}{\mathbb{P}_f\left[v_i = v_i^k\right]} = \frac{\int_{\Theta_i} \mathbb{P}_f\left[w_j \leq b \middle| \tilde{\theta}_i\right] \mathbb{P}_f\left[v_i = v_i^k \middle| \tilde{\theta}_i\right] f(\tilde{\theta}_i) d\tilde{\theta}_i}{\mathbb{E}_f\left[\theta_i^k\right]} \\ &= \frac{\int_{\Theta_i} F_{w_j}(b|\tilde{\theta}_i) \tilde{\theta}_i^k f(\tilde{\theta}_i) d\tilde{\theta}_i}{\mathbb{E}_f\left[\theta_i^k\right]} \\ H^{\mathrm{SPA}\prime}(b \mid v_i^k) &= \frac{\int_{\Theta_i} f_{w_j}(b|\tilde{\theta}_i) \tilde{\theta}_i^k f(\tilde{\theta}_i) d\tilde{\theta}_i}{\mathbb{E}_f\left[\theta_i^k\right]} \end{aligned}$$

Substituting this in the first-order condition, we get for all $b \in B$:

$$\begin{aligned}
&\frac{\int_{\Theta_i} f_{w_j}(b|\tilde{\theta}_i)\tilde{\theta}_i^K f(\tilde{\theta}_i)d\tilde{\theta}_i}{\mathbb{E}_f\left[\theta_i^K\right]} - \frac{\int_{\Theta_i} f_{w_j}(b|\tilde{\theta}_i)\tilde{\theta}_i^1 f(\tilde{\theta}_i)d\tilde{\theta}_i}{\mathbb{E}_f\left[\theta_i^1\right]} = 0 \\
\iff \qquad &\int_{\Theta_i}\left[\frac{\tilde{\theta}_i^K}{\mathbb{E}_f\left[\theta_i^K\right]} - \frac{\tilde{\theta}_i^1}{\mathbb{E}_f\left[\theta_i^1\right]}\right] f_{w_i}(\tilde{\theta}_i|w_j = b) f_{w_j}(b) d\tilde{\theta}_i = 0 \\
&\qquad \iff \qquad \frac{\mathbb{E}_f\left[\theta_i^K|w_j = b\right]}{\mathbb{E}_f\left[\theta_i^K\right]} = \frac{\mathbb{E}_f\left[\theta_i^1|w_j = b\right]}{\mathbb{E}_f\left[\theta_i^1\right]} \\
&\qquad \iff \qquad \mathbb{E}_f\left[\theta_i^K|w_j \leq b\right] = \frac{\mathbb{E}_f\left[\theta_i^K\right]}{\mathbb{E}_f\left[\theta_i^1\right]}\mathbb{E}_f\left[\theta_i^1|w_j \leq b\right]
\end{aligned}$$

For generic distributions, the last line is violated. □

References

[1] Bergemann, Dirk, and Steven Morris (2005): “Robust Mechansim Design,” Econometrica, 73, 1771–1813.

[2] Crémer, Jacques, and Richard McLean (1988): “Full Extraction of the Surplus in Baysian and Dominant Strategy Auctions,” Econometrica, 56(6), 1247–1257.

[3] Dasgupta, Partha, and Eric Maskin (2000): ”Efficient Auctions,” Quarterly Journal of Economics 115, 341-388.

[4] Deb, Rahul and Mallesh Pai. (2017) ”Discrimination via Symmetric Auctions,” American Economic Journal: Microeconomics, 9 (1): 275-314.

[5] Esponda, Ignacio, and Demian Pouzo (2016): “Berk Nash Equilibrium: A Framework for Modeling Agents with Misspecified Models,” Econometrica, 84, 1093–1130.

[6] Eyster, Erik, and Mattew Rabin (2005): “Cursed Equilibrium,” Econometrica, 73(5), 1623–72.

[7] Gizatulina, Alia, and Martin Hellwig (2014): “Beliefs, Payoff, Information: On the Robustness of the BDP Property in Models with Endogenous Beliefs,” Journal of Mathematical Economics, 51(3), 136–153.

[8] Gizatulina, Alia, and Martin Hellwig (2017): “The generic possibility of full surplus extraction in models with large type spaces,” Journal of Economic Theory, 170, 385–416

[9] Hendricks, Kenneth, and Robert Porter. (1988): “An empirical study of an auction with asymmetric information,” American Economic Review 78, 865–883.

[10] Hendricks, Kenneth, Pinkse, Joris, and Robert Porter (2003): “Empirical implications of equilibrium bidding in first-price, symmetric, common value auctions,” Review of Economic Studies 70, 115–145.

[11] Jaynes, Edwin Thompson (1957): ”Information Theory and Statistical Mechanics” Physical Review. Series II. 106 (4): 620-630.

[12] Jehiel, Philippe (2005): “Analogy-Based Expectation Equilibrium,” Journal of Economic Theory, 123(2), 81–104.

[13] Jehiel, Philippe (2022): “Analogy-based expectation equilibrium and related concepts: Theory, applications and beyond,” mimeo.

[14] Jehiel, Philippe, and Frédéric Koessler (2008): “Revisiting Games of Incomplete Information with Analogy-Based Expectations,” Games and Economic Behavior, 62, 533–557.

[15] Jehiel, Philippe, and Benny Moldovanu (2001): ”Efficient Design with Interdependent Valuations,” Econometrica 69, 1237-1259.

[16] Johnson, Scott., John. W. Pratt, and Richard J. Zeckhauser (1990): “Efficiency Despite Mutually Payoff-Relevant Private Information: The Finite Case,” Econometrica, 58(4), 873–900.

[17] Li, Shengwu. (2017): ”Obviously Strategy-Proof Mechanisms,” American Economic Review, 107 (11): 3257-87.

[18] Lizzeri, Alessandro., and Nicola Persico (2000): “Uniqueness and Existence of Equilibrium in Auctions with a Reserve Price,” Games and Economic Behavior, 30, 83–114.

[19] Maskin Eric (1992): ”Auctions and Privatization” In: Siebert H Privatization. J.C.B. Mohr Publisher ; pp. 115-136.

[20] McAfee, Preston R.., and Philip J. Reny (1992): “Correlated Information and Mechanism Design,” Econometrica, 60(2), 395–421.

[21] Milgrom, Paul and Robert Weber (1982): ”A Theory of Auctions and Competitive Bidding,” Econometrica 50, 1089-1122.

[22] Perrigne Isabelle, and Quang Vuong (2022): ”Econometric Methods for Auctions,” forthcoming Handbook of Econometrics.

[23] Spiegler, Ran (2016): “Bayesian Networks and Boundedly Rational Expectations,” Quarterly Journal of Economics, 131, 1243–1290.

[24] Wilson, Robert (1987): “Game-Theoretic Approaches to Trading Processes” in Advances in Economic Theory: Fifth World Congress of the Econometric Society, T.Bewley (ed.), Cambridge: Cambridge University Press.