Education Choices in Competing Neighborhoods: Rational vs Coarse Expectations*

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Abstract

We propose a model of education choice in which students differ in their ability and rejection cost. In less knowledgeable neighborhoods, students estimate their admission chance based on the aggregate acceptance rate among the applicants in the neighborhood. When seats at elite colleges are scarce, we show that a neighborhood relying on rational expectations that competes with a neighborhood relying on coarse expectations obtains almost all seats. We also characterize how seats are distributed when all neighborhoods use coarse expectations and neighborhoods differ in cost and ability distributions. Policy interventions such as quotas or the mixing of neighborhoods are discussed.

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1 INTRODUCTION

We develop a stylized model of education choice in which students from different neighborhoods compete for common seats at elite colleges. Students in each neighborhood differ in two characteristics, their ability (accessible through standardized test, say) and their investment cost in college-specific human capital, and they have to decide whether or not to apply to elite colleges. Whatever neighborhood applicants come from, those with highest ability get admitted up to the capacity constraint.

Depending on the neighborhood, students either rely on rational expectations to assess the prospects of being accepted when applying or else they rely on the (steady state) aggregate acceptance rate among the applicants in the neighborhood. In the latter case, they reason as if this local aggregate statistics applied to them. Students who rely on the local acceptance rate can be thought of as employing a heuristic based on word of mouth communication (about acceptance decisions) in which the neighborhood defines the natural boundary of the sampling considered in this heuristic. Students of such neighborhood are less well informed of the working of the education system than students of neighborhoods relying on rational expectations, and as such can be thought as being less well knowledgeable or integrated in the society.

Our main object of interest is in understanding how seats at elite college get allocated among neighborhoods as a function of how the characteristics of students are distributed and also as a function of whether the neighborhood relies on rational expectations or the local sampling heuristic. We are also interested in the welfare consequences of the induced education economy. We study this in the limiting case in which the number of seats at elite colleges is small (which fits in well with applications, see Blair and Smetters, 2021), and to fix ideas we consider the case of two neighborhoods assuming the two characteristics of students are independently distributed in each neighborhood.

Our main results are as follows. When neighborhood A relies on rational expectations and neighborhood B relies on the sampling heuristic, almost all seats end up in neighborhood A, irrespective of the distribution of student characteristics in the two neighborhoods. We believe this insight provides a simple formalization of how less knowledgeable neighborhoods end up getting very few seats at elite colleges irrespective of the ability merits of its students (as documented in Hoxby and Avery (2012); Hoxby and Turner (2015)).

When both neighborhoods rely on the local sampling heuristic, we provide a closed form characterization of the share of seats that go to either neighborhood as a function of the distribution of ability and costs in the two neighborhoods. We observe that the neighborhood with larger cost (i.e., the neighborhood with the cost distribution having lower density around 0) gets fewer seats per head than the neighborhood with smaller costs. We also observe that everything else equal (including the same distribution of costs and the same density of ability at the top), the neighborhood with larger mean ability gets fewer seats per head, which follows because in the neighborhood with larger mean ability more students apply thereby leading to a lower acceptance rate statistic. We also study the effect on redistribution, welfare as well as the average quality of admitted students of standard policy instruments such as quotas or the mixing of neighborhoods when all neighborhoods rely on the local sampling heuristic. We establish that when neighborhoods differ only in the distribution of costs, these policies have no effect on total welfare, despite allowing for redistribution between neighborhoods. We observe that reserving a number of seats in proportion to the size of the neighborhood is welfare-enhancing if the neighborhood with larger cost is the one with smaller ability variance — assuming an equal mean ability across neighborhoods. We note that quotas have a negative impact on the average ability of admitted students. Finally, we characterize the welfare impact of mixing neighborhoods, noting that in some cases mixing may be welfare-enhancing.

Related Literature At some broad level, our insights allow to shed light on empirical observations such as Hastings and Weinstein (2008) Hoxby and Turner (2015) or Kapor et al. (2020). On the theory side, our paper relates to several approaches in behavioral game theory (see in particular Eyster and Rabin (2005), (Jehiel, 2005)) to the extent that sampling students can be interpreted as missing the correlation between acceptance rate and ability. The approach closest to ours in the context of education is Manski (1993) who postulates an additive log-income equation, and assumes that students infer the returns to schooling by taking the conditional expectation of log-income. When students omit to condition on ability—e.g., because they do not observe the ability of their peers—he shows that more low-ability and less high-ability students enroll in college. Our model is different in that in our environment there is a strategic interaction across students due to the scarcity of seats in elite colleges, and students' expectations concern their admission chances which are affected by the application strategy of other students, but our analysis of the one-neighborhood case shares some similarity with Manski (1993) to the extent some high ability students refrain from applying and some low ability students apply and get rejected. However, our main contribution lies in the multi-neighborhood version of our model and the interaction of neighborhoods either using rational expectations or the sampling heuristic, which has no counterpart in Manski (1993).[1]

2 SETUP

We introduce a stylized model of career choice with strategic students and rationing at elite colleges. In this Section, we consider a single neighborhood with a mass of students normalized to 1 and a mass $q < 1$ of seats at elite colleges. We will focus on the case in which q is small, which fits in well with the consideration of elite colleges (Blair and Smetters, 2021). Students are parameterized by their ability $\theta \in [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}$, and by their cost $c \in [0, \overline{c}] \subseteq \mathbb{R}_+$, which is an investment cost in college-specific human capital incurred only if the student applies to the elite college and gets rejected. We assume that $\overline{\theta} > \underline{\theta} \geq 0$, $\overline{c} > 0$ and that students' types are distributed according to the cumulative distribution F on $N \equiv [\underline{\theta}, \overline{\theta}] \times [0, \overline{c}]$ with continuous density f that has full support on N.

[1] Another reference that studies biased belief formation in education choices is Streufert (2000). While the mechanism he studies and the induced bias are different in the two papers (in Streufert, it is related to the change of composition of the neighborhood aftre the education choice), another key difference is the equilibrium aspect of our approach, which makes our approach closer to Manski than Streufert.

Students choose among two occupations: going directly on the labor market (or a non-selective vocational training) L, or applying to selective colleges H. That is, their action set is $A = \{L, H\}$. For a student of ability θ, the gross utility of attending an elite college is $U^H(\theta) = \theta$, whereas we assume that the utility of going directly on the labor market is $U^L(\theta) = 0$. The net utility of a student with type (θ, c) is thus given by:

- If student (θ, c) goes on the labor market L her utility is 0.
- If student (θ, c) applies to H and obtains a seat, her utility is θ.
- If student (θ, c) applies to H but does not get a seat, she goes on the labor market and her utility is $-c$.

Even though we have formulated the model in terms of investment cost in college-specific human capital, the same model can easily accommodate other forms of costs, i.e. application costs and/or tuition fees in case of admission by appropriately redefining the variables.[2] In our results which are focused on the case in which q is small, only the distribution of (θ, c) for c close to 0 will matter. Thus, our analysis should be viewed as meaningful, even if one has in mind that c may not be substantial in practice.[3]

The model is completed as follows. First, students decide whether or not to apply to H as a function of their type. Then elite colleges fill in their q seats by accepting the applicants with highest θ. Our model implicitly assumes that elite colleges perfectly observe the ability θ of applicants, which makes the difference between the rational expectations equilibrium and our notion of equilibrium –referred to as local sampling equilibrium– strongest. We leave the extension to the case in which colleges screen applicants based on imperfect signals about applicants' ability for future research.

[2]Specifically, refer to c_a as the application cost, c_f as a tuition fee to be paid in case of acceptance and c_r as an investment cost in college-specific human capital to be paid only in case of rejection as formulated above. By identifying θ in the main model with $\theta - c_a - c_f$ and c with $c_a + c_r$, it readily verified that the more general model can be reduced to the simplified formulation adopted above.

[3]Moving away from the case in which q is small, one may mention that c may be significant in some cases, if applying to elite colleges require some preparation that cannot be re-used for other purposes.

We start presenting our description of the rational expectations equilibrium and the local sampling equilibrium for general cumulative distributions $F(\theta, c)$. Later on, we will assume that θ and c are independently distributed (even if, for our main results, the independence is only needed locally around $c = 0$).

A strategy profile $\sigma : N \longrightarrow \Delta A$ is a (measurable) function mapping the set of types into application probabilities. We let $\sigma(\theta, c) \in [0, 1]$ denote the probability that student (θ, c) applies to H. A key object that drives the choice of student (θ, c) is the subjective probability this student assigns to obtaining a seat at an elite college conditional on applying to H. In the rational case, this probability depends on θ. In the sampling equilibrium, this probability is constant (at a value that is endogenously determined, see below). To cover both cases, denote by $p(\theta)$ the subjective probability assigned by a type (θ, c)- student to being accepted when applying to H. Based on $p(\theta)$, student (θ, c) applies to H whenever:[4]

$$p(\theta)\theta - (1 - p(\theta))c \geq 0$$

This leads to the following definition.

DEFINITION 1. *σ is optimal given subjective beliefs $p(\cdot)$ if*

$$\sigma(\theta, c) = \begin{cases} 1 & \textit{when} \quad c \leq \frac{p(\theta)}{1-p(\theta)}\theta \\ 0 & \textit{when} \quad c > \frac{p(\theta)}{1-p(\theta)}\theta \end{cases}$$

For any strategy profile, let $\theta(\sigma)$ denote the cutoff at H such that any student with ability $\theta \geq \theta(\sigma)$ who applies to H is admitted. It is defined as follows: $\theta(\sigma) = \underline{\theta}$ when

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_0^{\overline{c}} \sigma(\theta, c) f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta < q$$

[4]For completeness, we assume that the student applies to H when indifferent, but how indifferences are resolved plays no role in the analysis.

Otherwise, $\theta(\sigma)$ is uniquely defined as the largest θ^* such that

$$\int_{\theta^*}^{\overline{\theta}} \int_0^{\overline{c}} \sigma(\theta, c) f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta = q$$

Rational Expectations Equilibrium

Admission beliefs $p^R(\cdot)$ are rational when they are consistent with the admission cutoff, given the strategy profile. That is,

$$p^R(\theta) = \begin{cases} 1 & \text{when} \quad \theta \geq \theta(\sigma) \\ 0 & \text{when} \quad \theta < \theta(\sigma) \end{cases}$$

Rational expectations equilibrium is thus a strategy σ^R that is optimal given beliefs p^R where the beliefs p^R are required to be consistent with σ^R.

It is readily verified that there exists a unique rational expectations equilibrium, which is characterized as follows. Let θ^* solves

$$\int_{\theta^*}^{\overline{\theta}} \int_0^{\overline{c}} f(c, \theta) \, \mathrm{d}c \, \mathrm{d}\theta = q \iff \theta^* = H^{-1}(1 - q)$$

where $H(\cdot)$ denotes the cdf of the marginal distribution of θ. In the unique Rational Expectations Equilibrium, a student with type (θ, c) applies to H whenever $\theta \geq \theta^*$ and does not apply whenever $\theta < \theta^*$. The rational expectations equilibrium induces perfect assortative matching as students sort across occupations based on their ability. Namely, high-ability students (those with ability θ above θ^*) go to elite colleges, and low-ability students (those with ability θ below θ^*) go on the labor market. No student applying to H gets rejected.

Local Sampling Equilibrium

In the local sampling equilibrium, students form their expectation based the aggregate admission rate of applicants in their neighborhood. The idea is that they sample from their neighborhood those who applied in the previous generation and derive the acceptance rate among those. They then use this estimate to decide whether or not to apply to elite colleges as a function of their type. A sampling equilibrium is a steady state of such an environment.

When there is a single neighborhood as considered here, the steady state takes the following simple form. Call p^S the aggregate acceptance rate in the neighborhood. In a sampling equilibrium, the strategy σ^S has to be optimal given the admission belief $p(\cdot) = p^S$, and p^S must be consistent with σ^S in the sense that

$$p^S = q \Big/ \int_{\underline{\theta}}^{\overline{\theta}} \int_0^{\overline{c}} \sigma^S(\theta, c) f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta.$$

It is readily verified that a local sampling equilibrium always exists. If the acceptance rate were believed to be 1, everyone would apply but this would lead to rationing (because $q < 1$) and thus positive rejection rate. If the acceptance rate were very small, only those with very small c would apply, they would all be accepted, thereby leading to an effective acceptance rate equal to 1. The theorem of intermediate values guarantees the existence of at least one p^s such that the optimal strategy σ^S given p^S yields an effective acceptance rate of p^S.

Compared to the rational expectations equilibrium, we observe two types of inefficiencies in a local sampling equilibrium. Referring to $\theta^* = \theta(\sigma^S)$ as the admission threshold in a local sampling equilibrium, we have that (θ, c)-students with $\theta > \theta^*$and $c > \frac{p^S}{1-p^S}\theta$ do not apply to elite colleges when had they applied they would have been accepted and (θ, c)-students with $\theta < \theta^*$ and $c < \frac{p^S}{1-p^S}\theta$ apply to elite colleges and get rejected.

To quantify inefficiencies and measure the quality of admitted students, we introduce two aggregate measures for arbitrary admission beliefs $p(\cdot)$ (that can thus be applied to both the rational expectations and local sampling equilibrium).

First, we define the aggregate welfare as

$$W = \int_{\theta^*}^{\overline{\theta}} \int_0^{c^H(\theta, p(\theta))} \theta f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta - \int_{\underline{\theta}}^{\theta^*} \int_0^{c^H(\theta, p(\theta))} c f(\theta, c) \, \mathrm{d}c \, \mathrm{d}\theta$$

where $c^H(\theta, p(\theta)) = \frac{p(\theta)}{1-p(\theta)}\theta$ is the cost below which student (θ, c) applies to H conditional on admission chances $p(\theta)$. The first term describes the welfare of admitted students. The second term reflects the welfare of rejected applicants while the welfare of non-applicants is 0.

Second, we define the average quality of admitted students as

$$M = \frac{\int_{\theta^*}^{\overline{\theta}} \int_{\underline{0}}^{c^H(\theta,p(\theta))} \theta f(\theta, c) \,\mathrm{d}c \,\mathrm{d}\theta}{\int_{\theta^*}^{\overline{\theta}} \int_{0}^{c^H(\theta,p(\theta))} f(\theta, c) \,\mathrm{d}c \,\mathrm{d}\theta}.$$

2.1 Small Number of Seats

In the rest of the paper, we assume that the number q of seats is small and that θ and c are independently distributed with a smooth joint density $f(\theta, c) = h(\theta)g(c)$ with full support that we re-normalize to be $[0,1] \times [0,1]$.[5] In this case, we are able to provide closed form approximations to the sampling equilibrium which is shown to be unique.

Let $p(q)$ denote the aggregate acceptance rate when the mass of seats is q. Given that a (θ, c)-student applies if $c < \frac{p}{1-p}\theta$, this acceptance rate $p(q) = p$ is characterized by

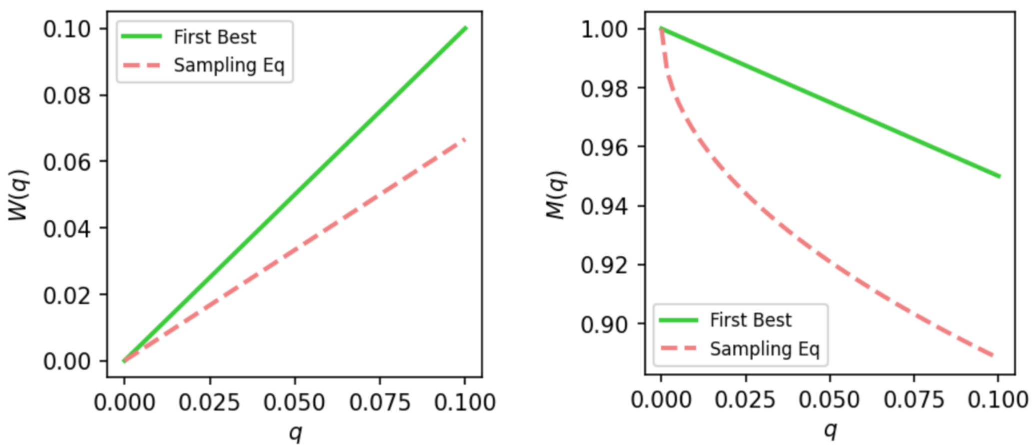
$$p = q \Big/ \int_0^1 h(\theta) G\left(\frac{p}{1-p}\theta\right) \,\mathrm{d}\theta.$$

In the next Proposition, we establish that the sampling equilibrium is unique when q is small enough and we provide approximations to the admission cutoff type $\theta^*(q)$, the admission rate $p(q)$ as well as the welfare $W(q)$ and the average ability of admitted students $M(q)$ in terms of the mass q of seats for arbitrary densities h and g. All proofs appear in Appendix.

PROPOSITION 1. *When q is small, there is a unique sampling equilibrium. Moreover*

$$\begin{aligned}
p(q) &= \left(\frac{q}{g(0)E(\theta)}\right)^{1/2} + o(q^{1/2}) \\
\theta^*(q) &= 1 - \frac{1}{h(1)}\left(\frac{E(\theta)q}{g(0)}\right)^{1/2} + o(q^{1/2}) \\
W(q) &= \left(1 - \frac{E(\theta^2)}{2E(\theta)}\right) q + o(q) \\
M(q) &= 1 - \frac{1}{2h(1)}\left(\frac{E(\theta)q}{g(0)}\right)^{1/2} + o(q^{1/2})
\end{aligned}$$

[5]For our formal results to hold, we only need the independence to hold for c close to 0.



Note that in the first-best (or rational expectations) case, we have

$$\begin{aligned}
p^{FB} &= 1 \\
\theta^{FB}(q) &= 1 - \frac{q}{h(1)} + o(q) \\
W^{FB}(q) &= q + o(q) \\
M^{FB}(q) &= 1 - \frac{q}{2h(1)} + o(q)
\end{aligned}$$

Compared to the first best, we note that in the sampling equilibrium the admission cutoff, the welfare and average quality are smaller. The welfare loss is proportional to $\frac{E(\theta^2)}{2E(\theta)}$, meaning that it is larger when the distribution of ability is skewed towards high ability students. This is due to the fact that equilibrium mismatch is more costly (from a welfare perspective) when there are more high ability students. We observe that the welfare is always increasing in the number of seats. By contrast, the average ability is always decreasing in the number of seats, as expected. In a sampling equilibrium, the average quality decreases faster than in a rational expectation equilibrium, since we deviate from 1 by $\sqrt{q}$ which is greater than q for small values of q. The drop in average quality is also proportional to $\sqrt{E(\theta)}$ in a sampling equilibrium, whereas it is independent from the type distribution in a rational expectation equilibrium. For comparison, we plot the welfare and the average quality as a function of seats q in a sampling equilibrium and a rational expectation equilibrium for a uniform distribution.

3 Competing Neighborhoods

Our main object of interest is the case of multiple neighborhoods competing for the same positions. A given neighborhood either relies on rational expectations or else forms expectations about the admission probability using the aggregate probability within the neighborhood. In the latter case, the neighborhood plays a role only in shaping the admission rate that students use to make their application choice. The fact that students from the various neighborhoods compete for the same seats creates a linkage between the various neighborhoods as the threshold ability θ^* above which students get admitted has to be the same across neighborhoods. This linkage in turn induces externalities across neighborhoods the effects of which are the main subject of interest of this Section.

To formalize the questions of interest, consider a two-neighborhood setup. Neighborhood $i = 1, 2$ consists of a mass m_i of students with (θ_i, c_i). We let $f_i(\theta_i, c_i)$ denote the mass-normalized distribution of (θ_i, c_i). That is, (θ_i, c_i) is distributed according to $m_i f_i(\theta_i, c_i)$. In neighborhood i, we assume that either all students rely on rational expectations or that they all rely on the (local) aggregate admission rate among applicants in that neighborhood. We let $\tau_i = R$ when neighborhood i relies on rational expectations and $\tau_i = S$ when the neighborhood relies on the aggregate acceptance rate. Rational expectations arise in a neighborhood if students there are exposed to precise statistics (or if the social network of individual students is big enough to allow them to form such precise estimates). Otherwise, the neighborhood relies on the coarse statistic about the aggregate admission rate in the neighborhood, which is viewed as being accessible through word of mouth communication in the neighborhood.

Consider first neighborhood i in isolation, assume there is a mass q_i of seats available for students in this neighborhood and that students follow strategy σ_i (q_i will be endogenized in equilibrium). We let $\theta^{\tau_i}(\sigma_i, q_i)$ be the corresponding threshold admission ability in this neighborhood. It is computed as shown in Section 3 using there the mass-normalized mass of seats

q_i/m_i and using the rational expectations expression when $\tau_i = R$ and the sampling equilibrium expressions when $\tau_i = S$. An equilibrium is formally defined as follows.

DEFINITION 2. *A local sampling equilibrium with competing neighborhoods $i = 1, 2$ (with characteristics f_i and τ_i) and total mass q of seats is a strategy profile (σ_1, σ_2) such that there exist q_1, q_2 satisfying*

1. *σ_i is a local sampling equilibrium in the neighborhood i with a mass q_i of seats;*
2. *$q_1 + q_2 = q$ and,*
3. *$\theta^{\tau_1}(\sigma_1, q_1) = \theta^{\tau_2}(\sigma_2, q_2)$.*

We consider the case in which θ and c are independently distributed in each neighborhood, i.e. $f_i(\theta, c) = h_i(\theta)g_i(c)$ for $i = 1, 2$, and we focus on the case in which the mass q of seats is small.

In our first main result, we assume that one neighborhood relies on rational expectations while the other relies on the local aggregate admission rate. We show that the former neighborhood receives almost all seats.

PROPOSITION 2. *Assume that $\tau_1 = R$ and $\tau_2 = S$. In the limit as q goes to 0, neighborhood 1 gets almost all seats as compared with neighborhood 2. That is, $\lim\limits_{q\to 0}\frac{q_2}{q_1} = 0$. Moreover since welfare is increasing in q (locally around 0) as established in the one neighborhood case, neighborhood 1 is favored in terms of welfare per capita.*

This result follows from the characterization of the admission threshold obtained in the previous Section. In the case of coarse expectations, the admission threshold deviates from 1 in a term proportional to the square root of the mass of seats obtained in the neighborhood. In the case of rational expectations, the admission threshold deviates from 1 in a term proportional to the mass of seats obtained in the neighborhood. To equate the two thresholds it should be that almost all seats are allocated to neighborhood 1. Viewing neighborhood 1 as the more established one so as to explain why this neighborhood relies on correct estimates about admission chances, our result provides an explanation as to why less established neighborhoods may be hurt in terms of effective access to elite colleges in the absence of any intervention.

In our next result, we assume that both neighborhoods rely on the local aggregate statistics about admission rates, that is, $\tau_i = S$ for $i = 1, 2$. The aggregate admission rate in neighborhood i is given by $p_i = \frac{q_i}{\mu_i}$ where μ_i is the mass of applicants in neighborhood i and q_i is the mass of seats obtained in neighborhood i. As in Section 2, a student in neighborhood i with characteristics (θ_i, c_i) applies to elite colleges whenever $c_i < \frac{p_i}{1-p_i}\theta_i$.

PROPOSITION 3. *As q gets small, the sampling equilibrium with competing neighborhoods and $\tau_i = S$ for $i = 1, 2$ is unique and characterized by*

$$\begin{aligned} q_i &= \frac{m_i h_i(1)^2 g_i(0)/E(\theta_i)}{m_1 h_1(1)^2 g_1(0)/E(\theta_1) + m_2 h_2(1)^2 g_2(0)/E(\theta_2)} q + o(q) \\ \frac{p_1}{p_2} &= \frac{E(\theta_2)}{E(\theta_1)} \frac{h_1(1)}{h_2(1)} + o(1) \end{aligned}$$

Proposition 3 allows us to see how the seats are distributed across neighborhoods as a function of the primitives m_i, g_i and h_i. We can establish the following comparative statics:

- When $h_1 = h_2$ (same distribution of θ_i), then $p_1 = p_2$ and $\frac{q_1}{m_1 g_1(0)} = \frac{q_2}{m_2 g_2(0)}$. Thus, if $g_1(0) > g_2(0)$, neighborhood 1 gets in relative share more seats than neighborhood 2. The neighborhood with a smaller opportunity cost (defined here as $\arg\max g_i(0) = 1$) applies more to elite colleges and obtains relatively more seats.

- When $h_1(1) = h_2(1)$ and $g_1(0) = g_2(0)$, if $E(\theta_1) > E(\theta_2)$, then $p_1 < p_2$ and $q_1/m_1 < q_2/m_2$. Namely, the neighborhood with a higher average quality is more pessimistic about admission chances and obtains fewer seats at elite colleges. As it turns out, when q is small, only students with θ_i around 1 get accepted. The mean ability $E(\theta_i)$ plays a role to the extent that all students apply to elite college when c is close to 0 irrespective of θ_i. This, in turn, explains the effect of $E(\theta_1) > E(\theta_2)$.

- When $E(\theta_1) = E(\theta_2)$ and $g_1(0) = g_2(0)$, if $h_1(1) < h_2(1)$, then $p_1 < p_2$ and $q_1/m_1 < q_2/m_2$. The interpretation is similar to the previous case.

3.1 Policy Instruments

In the rest of the paper, we assume that students in both neighborhoods rely on the aggregate admission rate in their respective neighborhood, i.e. $\tau_i = S$ for $i = 1, 2$, and we discuss the effect of two possible policy interventions. The first one consists in imposing quotas, pre-defining the number of seats each neighborhood. The second one consists in changing the compositions of the two neighborhoods by imposing some degree of mixing while leaving the equilibrium force determines the number of seats assigned to each neighborhood. When considering these interventions, we will discuss the effect in terms of welfare, in terms of expected quality of admitted students as well as a comparison of how the two neighborhoods benefit from the intervention.

Quotas. We investigate the effect on welfare of two types of quotas (assuming the mass q of seats is small) First, we consider allocating to each neighborhood a number of seats that is proportional to its size, i.e. $\frac{q_1}{m_1} = \frac{q_2}{m_2}$. Second, we consider reallocating seats to the neighborhood with the highest opportunity costs (defined here as the neighborhood with smaller cost density around 0).

PROPOSITION 4. *We assume that q is small enough and we consider the following policies:*

1. *Giving a number of seats to each neighborhood in proportion to its size is welfare improving compared to laissez faire if*

$$\arg\min_i \frac{h_i(1)^2 g_i(0)}{E(\theta_i)} = \arg\min_i \frac{E(\theta_i^2)}{E(\theta_i)}.$$

2. *Giving more seats to the neighborhood with higher opportunity cost (interpreted as neighborhood $\arg\min_i g_i(0)$) is welfare improving compared to laissez faire if*

$$\arg\min_i g_i(0) = \arg\min_i \frac{E(\theta_i^2)}{E(\theta_i)}.$$

Whether quotas are welfare improving or not depends only on the distribution of ability, not the distribution of costs. Moreover, if two neighborhoods have the same mean ability but one neighborhood has lower variance, reserving seats for this neighborhood is welfare improving.

In the next Proposition, we assume that the two neighborhoods have the same size and we establish that when the mass q of seats is small enough, assigning to each neighborhood the same mass $q/2$ of seats always deteriorates the average quality of admitted students as compared with the laissez faire.[6]

PROPOSITION 5. *We assume that q is small enough and that the two neighborhoods are of equal size $m_1 = m_2 = 1$. Reserving the same mass of seats $q/2$ to each neighborhood reduces the average quality of admitted students.*

The conventional wisdom is that quotas may deteriorate the quality of admitted students because without quotas, seats are allocated efficiently. This intuition holds true in the rational expectations paradigm in which the first-best allocation obtains. Somehow unexpectedly, the same conclusion that quotas have always a negative impact on the average quality of admitted students holds true also in our sampling equilibrium environment, despite the fact that seats are not allocated efficiently.

Of course, another effect of quotas is that the neighborhood receiving fewer per capita seats in laissez faire benefits in terms of relative welfare from a policy that assigns seats in proportion to the size of the neighborhood. Altogether, this observation together with Propositions 4 and 5 can be used to assess the pros and cons of quotas in a sampling equilibrium environment.

Mixed Neighborhoods. We investigate whether moving students from the high cost neighborhood to the low cost neighborhood (and vice versa) increases welfare. Unlike quotas which do not change students' social network, this intervention exactly aims at reducing inequalities of social capital. We consider random reallocation, i.e. from two initial neighborhoods with distributions F_i and F_j we draw new neighborhoods from the following com-

[6] Formally, the overall average quality of admitted students is defined as $M = \frac{q_1 M_1 + q_2 M_2}{q_1 + q_2}$ where M_i is the average quality of admitted students in neighborhood i and as before q_i is the mass of seats in neighborhood i.

pound distributions:

$$\tilde{F}_i = \alpha F_i + (1-\alpha) F_j$$
$$\tilde{F}_j = \alpha F_j + (1-\alpha) F_i$$

The parameter α scales the equalization across neighborhoods: for $\alpha = 1$ there is no reallocation of students, and for $\alpha = \frac{1}{2}$ the new neighborhoods have equal cost distributions. In order to preserve the independence while mixing, it should be that the heterogeneity is either only on g_i or only on h_i.

PROPOSITION 6. *1. If $h_1 = h_2$, then mixing has no effect on aggregate welfare for small enough q.*

2. If $m_1 = m_2$, $g_1 = g_2$, a complete mixing (i.e., $\alpha = \frac{1}{2}$) is welfare enhancing for small enough q whenever $1 - \frac{E(\theta_1^2)+E(\theta_2^2)}{2(E(\theta_1)+E(\theta_2))}$ is no smaller than

$$\frac{h_1(1)^2/E(\theta_1)}{h_1(1)^2/E(\theta_1) + h_2(1)^2/E(\theta_2)} \left(1 - \frac{E(\theta_1^2)}{2E(\theta_1)}\right) + \frac{h_2(1)^2/E(\theta_2)}{h_1(1)^2/E(\theta_1) + h_2(1)^2/E(\theta_2)} \left(1 - \frac{E(\theta_2^2)}{2E(\theta_2)}\right)$$

3. If $m_1 = m_2$, $g_1 = g_2$, $E[\theta_1] = E[\theta_2]$, and $h_1(1) = h_2(1)$ then mixing has no effect on aggregate welfare.

4. If $m_1 = m_2$, $g_1 = g_2$, $E[\theta_1] \neq E[\theta_2]$, $E[\theta_1^2] = E[\theta_2^2]$, and $h_1(1) = h_2(1)$ then aggregate welfare is monotonically increasing in mixing α.

Note that we maintain fixed either h or g across neighborhoods to preserve the independence between ability and cost in the compound distribution. We observe that mixing may sometimes be good for total welfare when the two neighborhoods have different distributions of ability. The result that when neighbors differ in average ability, then mixing has a positive effect on welfare may be viewed in the perspective of the Moving to Opportunity experiment showing that moving disadvantaged students to more advantaged neighborhoods improves college attendance and efficiency (Chetty et al., 2016; Chetty and Hendren, 2018a,b).[7]

[7] The effect of mixing on the average quality of admitted students is somehow cumber-

4 Conclusion

Our study has revealed how neighborhoods relying on the local sampling heuristic may be hurt relative to neighborhoods relying on rational expectations. It has also initiated some formal analysis of the welfare effect of policy instruments such as quotas or affirmative actions when all students rely on the local sampling heuristic. We leave for future research a more general exploration of this including the possibility that the sampling window considered for estimation purposes may not be the same across students of the same neighborhood.

some and can go either way in general.

PROOFS

Existence of Local Sampling Equilibria. Consider the following scheme:

$$p \longmapsto \sigma^{BR}(p,\cdot) \longmapsto b(\sigma^{BR},\cdot) \longmapsto p(b)$$

By Tychonoff's theorem, the scheme is compact-valued $p(b) \in [0,1]^{\Theta}$. Hence to obtain a fixed point, we just need to prove that the scheme is continuous. Fix a subjective belief map $p : \Theta \longrightarrow [0,1]$. The action space is binary and the subjective admission chances p enter payoffs linearly, hence σ^{BR} is the following measurable threshold strategy:

$$\sigma^{BR}(p,\cdot) = \begin{cases} 1 & \text{if} \quad p(\cdot) \geq \gamma(\cdot) \\ 0 & \text{if} \quad p(\cdot) < \gamma(\cdot) \end{cases}$$

where $\gamma(\theta,c) = \frac{c}{\theta+c}$. Take any converging sequence $p_n \longrightarrow p$. We need to show that $p \longmapsto \sigma^{BR}(p,\cdot)$ is continuous in the L^1-weak topology, namely

$$\int \sigma^{BR}(p_n,(\theta,c))\,\mathrm{d}F \longrightarrow \int \sigma^{BR}(p,(\theta,c))\,\mathrm{d}F.$$

We have

$$\int \sigma^{BR}(p_n,(\theta,c))\,\mathrm{d}F = \int \mathbf{1}\left\{p_n(\theta) \geq \gamma(\theta,c)\right\}\,\mathrm{d}F.$$

Therefore, continuity follows from Lebesgue's dominated convergence theorem. We now show the continuity of $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$. By Berge's maximum theorem, $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$ is upper-hemicontinuous. The loss function $|\theta - \tilde{\theta}|$ is strictly quasi-convex, hence $\sigma^{BR} \longmapsto b(\sigma^{BR},\cdot)$ is continuous. Finally, the continuity of $b \longmapsto p(b)$ follows directly from the integrability of p together with the continuity of the functions $\max\{\cdot,\cdot\}$ and $\min\{\cdot,\cdot\}$. Therefore, by the Schauder fixed point theorem the set of local sampling equilibria is nonempty. □

Proof of Proposition 1

Step 1. ($p \to 0$ as $q \to 0$): By contradiction, suppose that there exists b

with $p > b > 0$ for all q, then all (θ, c) such that $c < \frac{b}{1-b}\theta$ apply to H. The mass of applicants $m(q)$ is no smaller than $m^* = \Pr(c < \frac{b}{1-b}\theta)$ (and $m^* > 0$ where use is made of the full support assumption). But then $p(q) = \frac{q}{m(q)} \to 0$ and we get a contradiction.

Step 2. (Approximation of p in terms of θ^*): We have

$$\begin{aligned} p &= \frac{\int_{\theta^*}^1 h(\theta) G(\frac{p}{1-p}\theta) d\theta}{\int_0^1 h(\theta) G(\frac{p}{1-p}\theta) d\theta} \\ &\approx \frac{\int_{\theta^*}^1 h(\theta) \frac{p}{1-p}\theta g(0) d\theta}{\int_0^1 h(\theta) \frac{p}{1-p}\theta g(0) d\theta} \\ &= \frac{\int_{\theta^*}^1 h(\theta)\theta d\theta}{\int_0^1 h(\theta)\theta d\theta} \\ &= \frac{\int_{\theta^*}^1 h(\theta)\theta d\theta}{E(\theta)} \end{aligned}$$

using a 1st order Taylor approximation of G around 0. This approximation also implies that $\theta^* \to 1$ as $q \to 0$ to ensure that $p \to 0$ (from Step 1). This in turn implies that (using $h(\theta) \approx h(1)$ when θ is in $(\theta^*, 1)$, with $h(1) > 0$ because of full support):

$$p \approx \frac{h(1)}{E(\theta)} \frac{1 - (\theta^*)^2}{2}.$$

Step 3. (Approximation of θ^* in terms of q):

$$\begin{aligned} q &= \int_{\theta^*}^1 h(\theta) G\left(\frac{p}{1-p}\theta\right) d\theta \\ &\approx \int_{\theta^*}^1 h(\theta) p g(0) \theta d\theta \\ &\approx p g(0) h(1) \frac{1 - (\theta^*)^2}{2} \end{aligned}$$

where the first approximation uses $\frac{p}{1-p} \approx p$ and $G(p\theta) \approx p\theta g(0)$, and the second approximation uses $h(\theta) \approx h(1)$ for $\theta \in (\theta^*, 1)$.

Writing θ^* in terms of q, we get

$$\theta^* = 1 - \frac{1}{h(1)} \left(\frac{E(\theta)}{g(0)} \right)^{1/2} q^{1/2} + o(q^{1/2}).$$

Step 4. (Approximation of p in q): From steps 3 and 4 we get

$$p = \left(\frac{E(\theta)}{g(0)} \right)^{1/2} q^{1/2} + o(q^{1/2}).$$

Step 5. (Approximation of $W(q)$): We have

$$\begin{aligned}
W(q) &= \int_{\theta^*}^{1} \theta h(\theta) G\left(\frac{p}{1-p}\theta \right) d\theta - \int_0^{\theta^*} h(\theta) \left[\int_0^{\frac{p\theta}{1-p}} cg(c)dc \right] d\theta \\
&\approx g(0)ph(1)(1-\theta^*) - \int_0^{\theta^*} g(0)\frac{p^2\theta^2}{2} h(\theta)d\theta \\
&\approx g(0)ph(1)(1-\theta^*) - \frac{g(0)E(\theta^2)}{2} p^2 \\
&\approx q - \frac{E(\theta^2)}{2E(\theta)} q \\
&= \left(1 - \frac{E(\theta^2)}{2E(\theta)} \right) q + o(q)
\end{aligned}$$

using the above approximations.

Step 6. (Approximation of $M(q)$): We have

$$\begin{aligned}
M(q) &= \frac{\int_{\theta^*}^1 \theta h(\theta) G(\frac{p}{1-p}\theta) d\theta}{\int_{\theta^*}^1 h(\theta) G(\frac{p}{1-p}\theta) d\theta} \\
&\approx \frac{\left(1-\theta^{*3}\right)/3}{\left(1-\theta^{*2}\right)/2} \\
&= 1 - \frac{1}{2h(1)} \left(\frac{E(\theta)}{g(0)} \right)^{1/2} q^{1/2} + o(q^{1/2})
\end{aligned}$$

using the above approximations. $\square$

Proof of Proposition 2.

From Proposition 1, we have that

$$\begin{aligned}
\theta^* &= 1 - \frac{1}{h_2(1)} \left(\frac{E[\theta_2] q_2}{m_2 g_2(0)} \right)^{1/2} + o(q_2^{1/2}) \\
&= 1 - \frac{q_1}{m_1 h_1(1)} + o(q_1)
\end{aligned}$$

Considering that $q_1 + q_2 = q$, this yields

$$\begin{aligned}
q_2 &= \frac{m_2 g_2(0) \left(h_2(1)\right)^2}{\left(m_1 h_1(1)\right)^2 E[\theta_2]} q^2 + o(q^2) \\
q_1 &= q + o(q)
\end{aligned}$$

implying that $\lim_{q\to 0} \frac{q_2}{q_1} = 0$, as required. □

Proof of Proposition 3. At equilibrium, the admission cutoff must be equal between neighborhood 1 and 2. Using the formulas derived in Proposition 1, we have

$$\begin{aligned}
1 - \frac{1}{h_1(1)} \left(\frac{E[\theta_1] q_1}{g_1(0) m_1} \right)^{\frac{1}{2}} &= 1 - \frac{1}{h_2(1)} \left(\frac{E[\theta_2] q_2}{g_2(0) m_2} \right)^{\frac{1}{2}} \\
\iff q_1 &= \frac{m_1}{m_2} \frac{g_1(0)}{g_2(0)} \left(\frac{h_1(1)}{h_2(1)} \right)^2 \frac{1/E[\theta_1]}{1/E[\theta_2]}
\end{aligned}$$

Using the fact that $q_2 = q - q_1$, and rearranging the above formula yields the desired result. We obtain p_1/p_2 using the closed form from Proposition 1. □

Proof of Proposition 4. From the one neighborhood case, we know that aggregate welfare is (at the first order):

$$W = \left(1 - \frac{E[\theta_1^2]}{2E[\theta_1]}\right) q_1 + \left(1 - \frac{E[\theta_2^2]}{2E[\theta_2]}\right) q_2$$

Therefore, aggregate welfare can be increased only by giving more seats to neighborhood $\arg\min_i \frac{E[\theta_1^i]}{E[\theta_i]}$. Now if we give seats proportionally to the size of the neighborhood $q_i = \frac{q_i}{m_i}$, this will benefit the neighborhood that currently has the smallest number of seats, i.e. $\arg\min_i \frac{h_i(1)^2 g_i(0)}{E[\theta_i]}$. Alternatively, giving more seats to the most disadvantaged neighborhood will benefit neighborhood $\arg\min_i g_i(0)$. $\square$

Proof of Proposition 5. Letting $A_i = h_i(0)^2 g_i(0)/E(\theta_i)$, the value of M after the intervention is

$$M^{AA} = 1 - \frac{1}{2}\left[\left(\frac{q}{2A_1}\right)^{1/2} + \left(\frac{q}{2A_2}\right)^{1/2}\right] + o(q^{1/2})$$

The laissez-faire value of M is

$$M^{LF} = 1 - \left(\frac{q}{A_1 + A_2}\right)^{1/2} + o(q^{1/2})$$

That $M^{LF} > M^{AA}$ follows from Jensen's inequality noting that $x \to x^{-1/2}$ is convex. $\square$

Proof of Proposition 6.

1. Aggregate welfare is the same as in the one neighborhood case and only depend on h_1, h_2.

2. Aggregate welfare when $\alpha = \frac{1}{2}$ writes

$$\frac{1}{2}\left(1 - \frac{\frac{1}{2}E[\theta_1^2] + \frac{1}{2}E[\theta_2^2]}{E[\theta_1] + E[\theta_2]}\right) q + \frac{1}{2}\left(1 - \frac{\frac{1}{2}E[\theta_1^2] + \frac{1}{2}E[\theta_2^2]}{E[\theta_1] + E[\theta_2]}\right) q$$
$$= 1 - \frac{E[\theta_1^2] + E[\theta_2^2]}{2(E[\theta_1] + E[\theta_2])}$$

   which should be no less than aggregate welfare when $\alpha = 0$.

3. In this case we have $q_1 = q_2 = q/2$. Hence aggregate welfare writes

$$\left(1 - \frac{\alpha E[\theta_1^2] + (1-\alpha)E[\theta_2^2]}{2E[\theta]}\right)\frac{q}{2} + \left(1 - \frac{\alpha E[\theta_2^2] + (1-\alpha)E[\theta_1^2]}{2E[\theta]}\right)\frac{q}{2}$$
$$= \left(2 - \frac{E[\theta_1^2] + E[\theta_2^2]}{2E[\theta]}\right)\frac{q}{2}$$

   which is independent of α.

4. In this case, aggregate welfare writes

$$q\Bigg[\left(1-\frac{E[\theta^2]}{2(\alpha E[\theta_1]+(1-\alpha)E[\theta_2])}\right)\frac{\frac{1}{\alpha E[\theta_1]+(1-\alpha)E[\theta_2]}}{\frac{1}{\alpha E[\theta_1]+(1-\alpha)E[\theta_2]}+\frac{1}{\alpha E[\theta_2]+(1-\alpha)E[\theta_1]}} + \left(1-\frac{E[\theta^2]}{2(\alpha E[\theta_2]+(1-\alpha)E[\theta_1])}\right)\frac{\frac{1}{\alpha E[\theta_2]+(1-\alpha)E[\theta_1]}}{\frac{1}{\alpha E[\theta_1]+(1-\alpha)E[\theta_2]}+\frac{1}{\alpha E[\theta_2]+(1-\alpha)E[\theta_1]}}\Bigg]$$

which can be verified is an increasing function of α.

□

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